Computer Algebra Independent Integration Tests

Summer 2023 edition

7-Inverse-hyperbolic-functions/7.5-Inverse-hyperbolic-secant/200-7.5.1-u-a+b-arcsech-c-x-^n

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [190]. This is test number [200].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (190)	0.00 (0)
Mathematica	100.00 (190)	0.00 (0)
Maple	81.05 (154)	18.95 (36)
Fricas	68.42 (130)	31.58 (60)
Maxima	33.16 (63)	66.84 (127)
Mupad	27.37 (52)	72.63 (138)
Sympy	25.79 (49)	74.21 (141)
Giac	23.68 (45)	76.32 (145)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

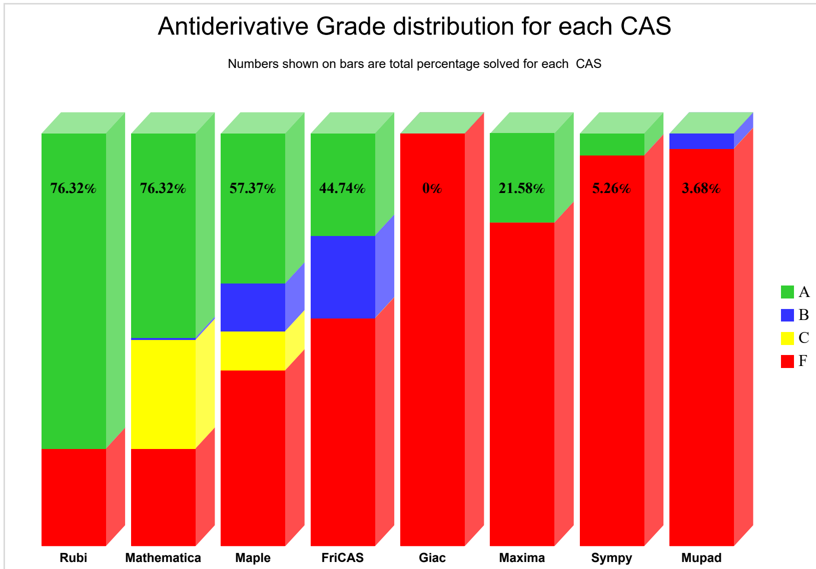
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

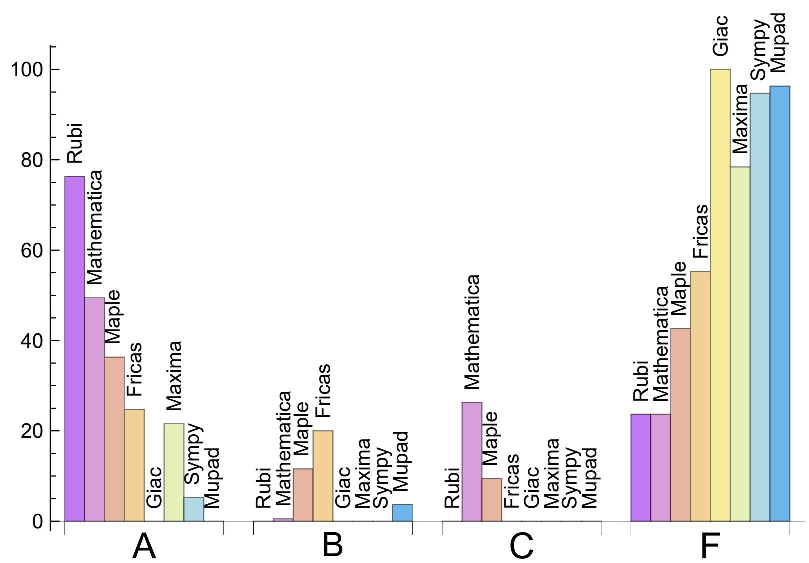
System	% A grade	% B grade	% C grade	% F grade
Rubi	76.316	0.000	0.000	23.684
Mathematica	49.474	0.526	26.316	23.684
Maple	36.316	11.579	9.474	42.632
Fricas	24.737	20.000	0.000	55.263
Maxima	21.579	0.000	0.000	78.421
Sympy	5.263	0.000	0.000	94.737
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	3.684	0.000	96.316

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	36	100.00	0.00	0.00
Fricas	60	95.00	5.00	0.00
Maxima	127	53.54	0.79	45.67
Sympy	141	85.82	14.18	0.00
Mupad	138	0.00	100.00	0.00
Giac	145	98.62	0.00	1.38

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.27
Giac	0.28
Fricas	0.30
Maxima	0.63
Mupad	4.39
Maple	7.10
Mathematica	7.21
Sympy	9.38

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	20.84	1.05	23.00	1.00
Mupad	30.00	1.23	27.00	1.17
Sympy	45.08	0.98	22.00	0.96
Rubi	200.77	1.00	138.50	1.00
Maxima	291.67	17.15	107.00	1.00
Maple	300.62	1.45	143.00	1.04
Fricas	307.53	2.09	135.50	1.50
Mathematica	382.47	1.59	139.50	1.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

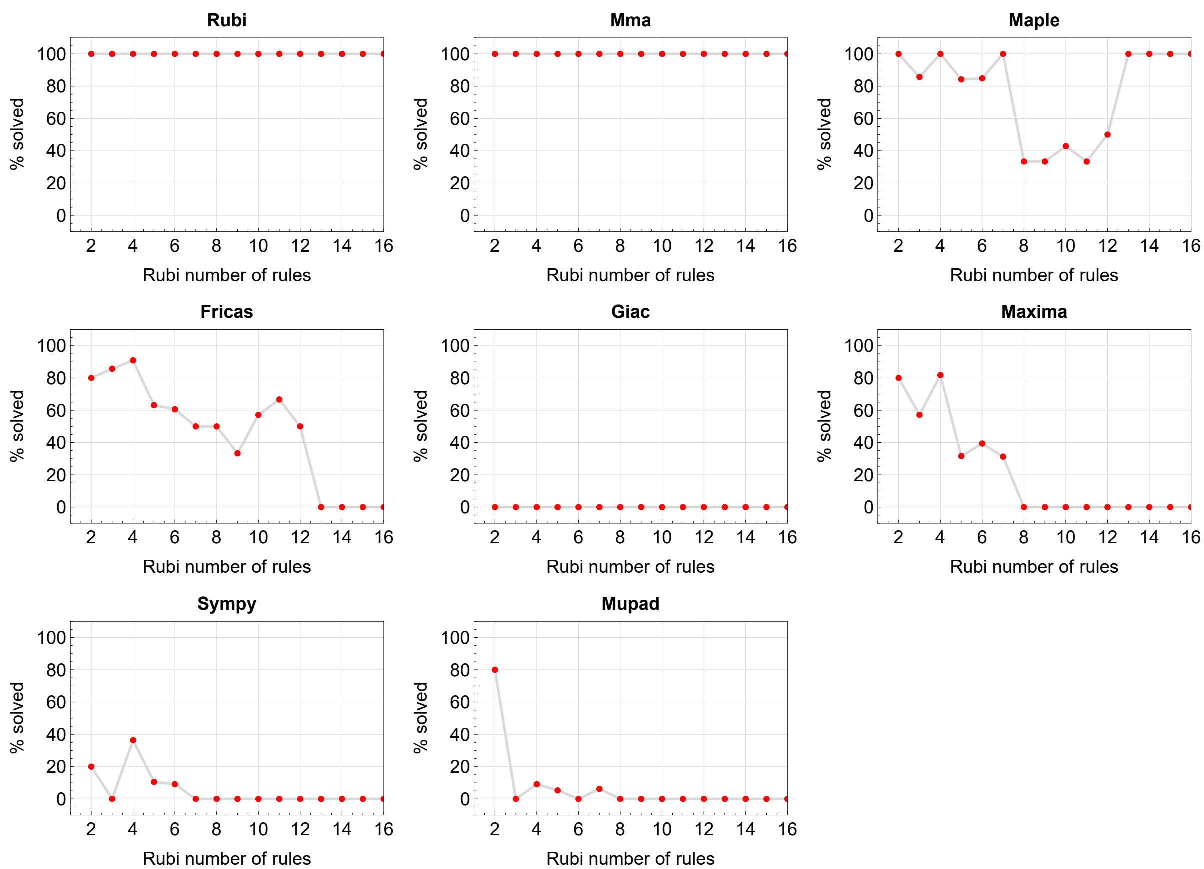


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

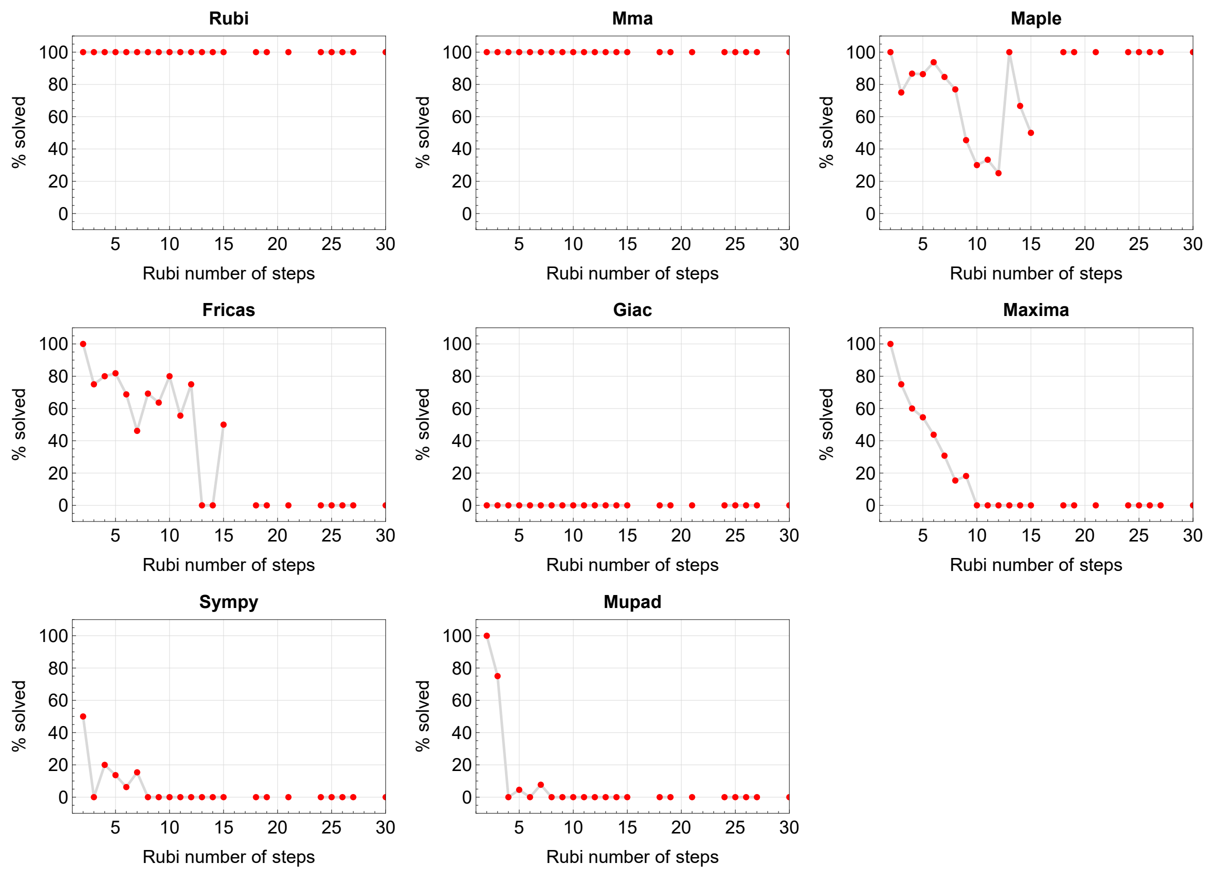


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

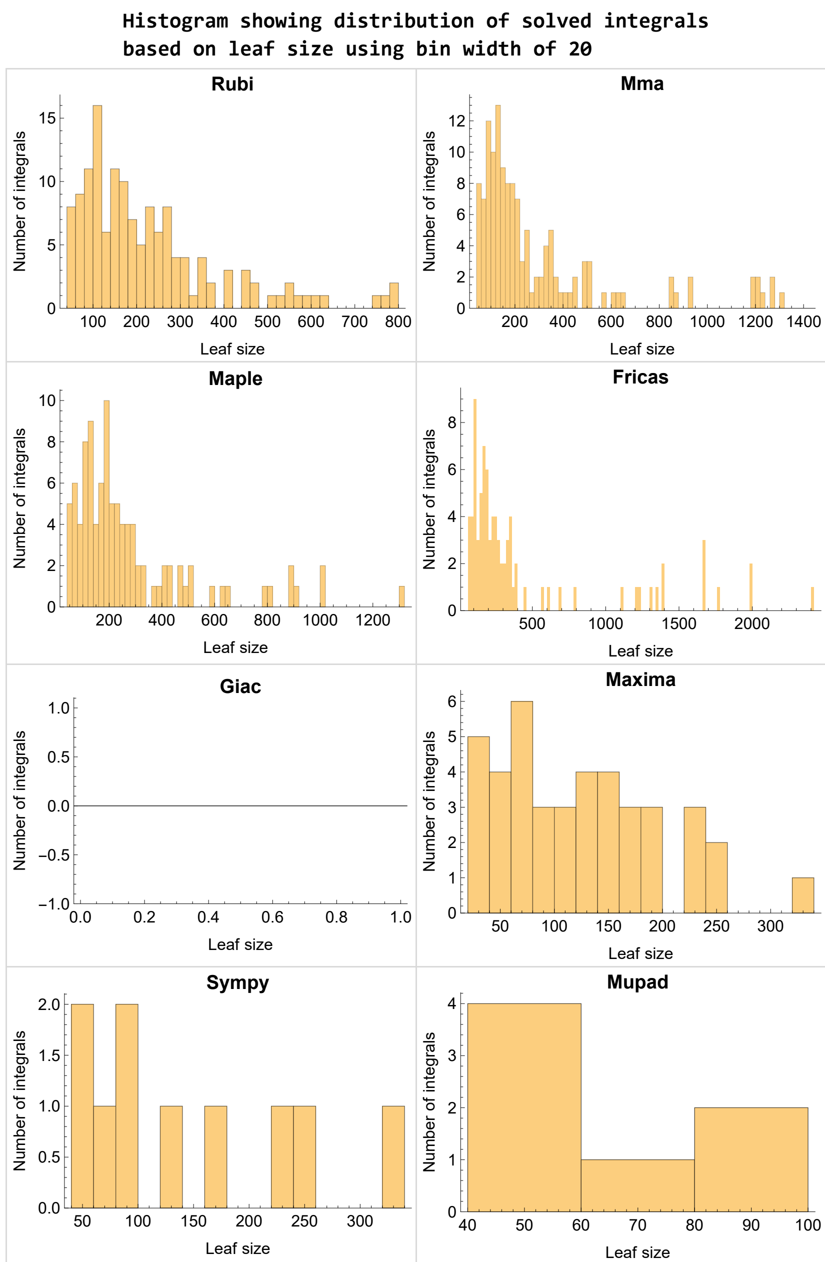


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

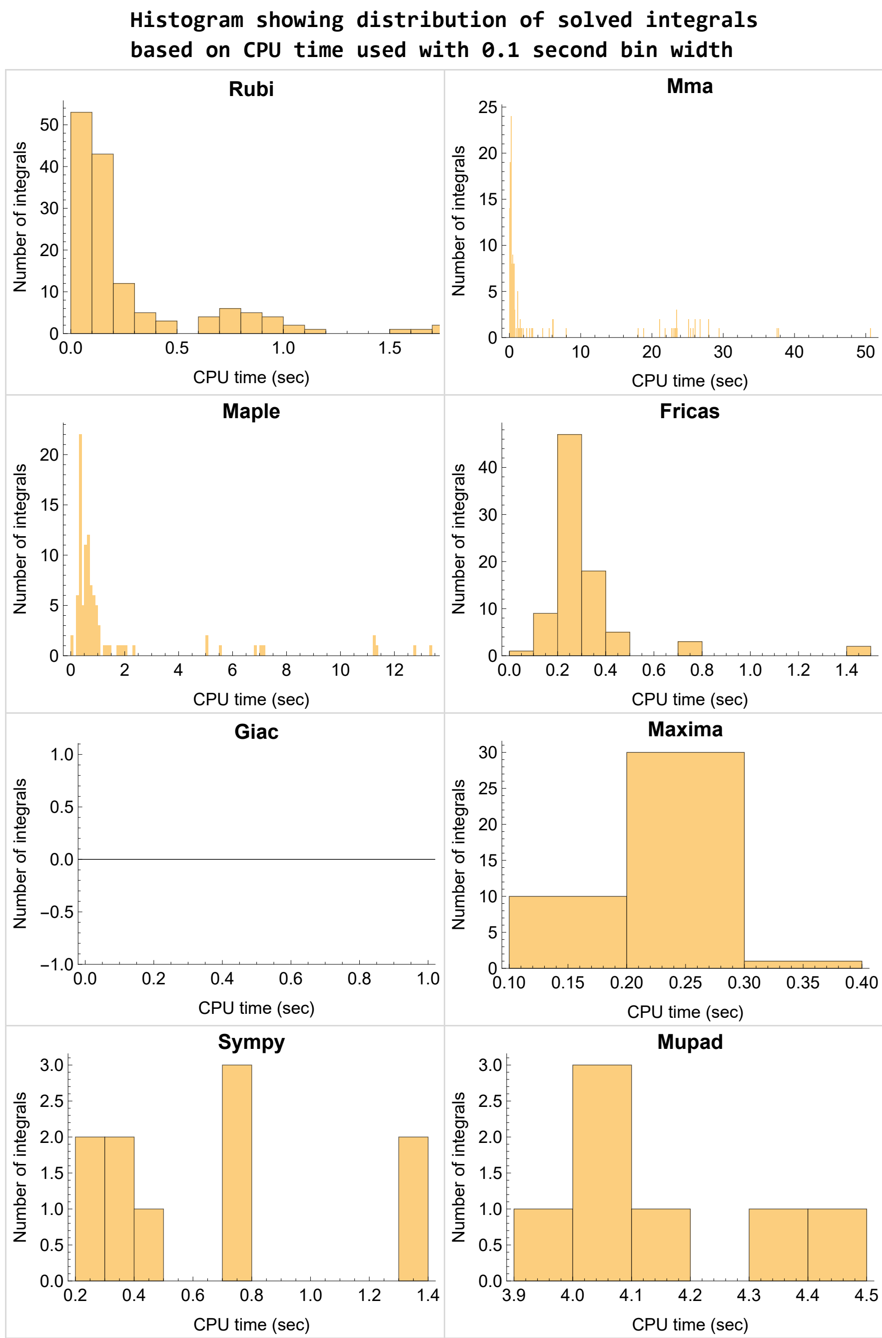


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

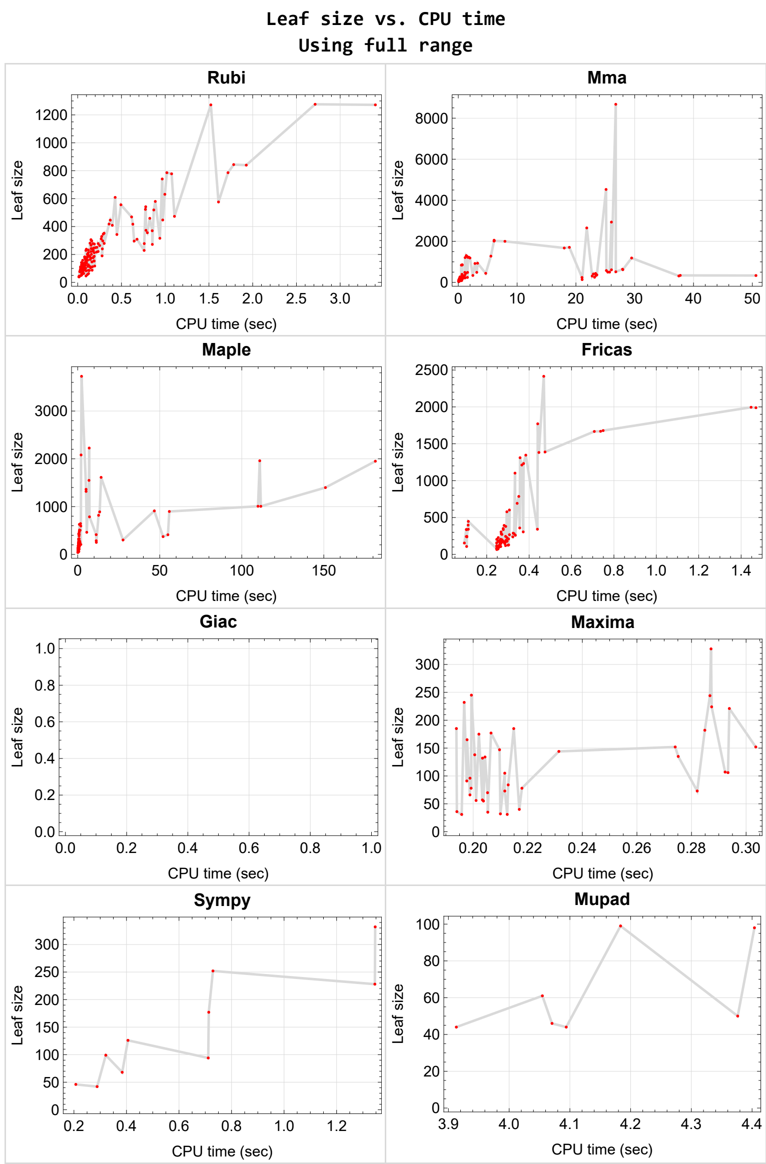


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 189, 190}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {81, 82, 83, 84, 85, 86, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129}

Maple {110, 111, 113, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

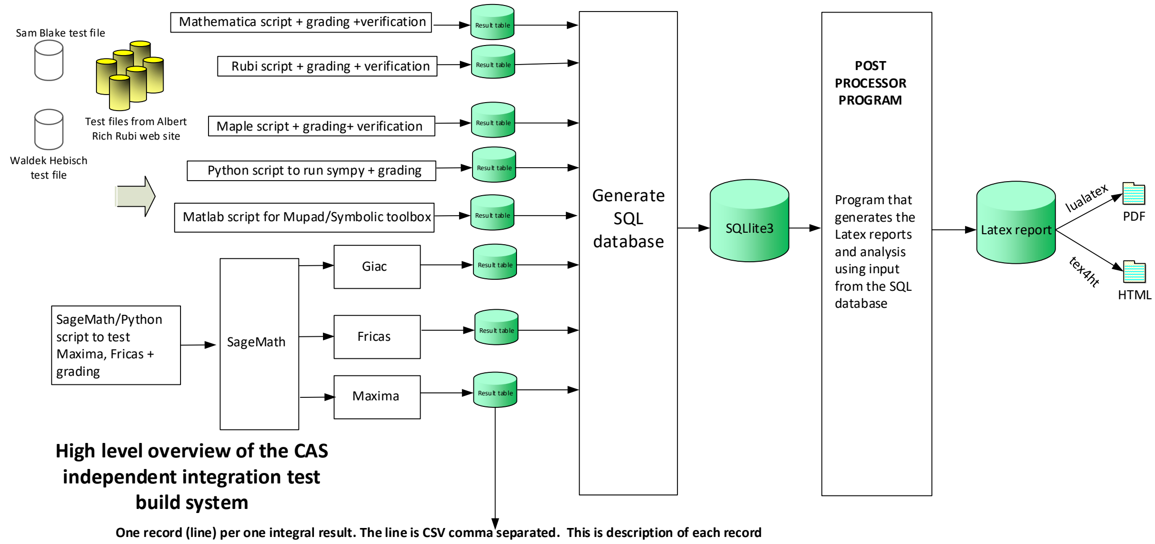
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 76, 77, 79, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 108, 109, 130, 131, 132, 140, 141, 150, 151, 152, 159, 160, 161, 168, 169, 170, 177, 178, 179, 186, 187, 188 }

B grade { 45 }

C grade { 19, 21, 23, 74, 75, 78, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 100, 101, 102, 103, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 138, 139, 148, 149, 157, 158, 166, 167, 175, 176 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36, 37, 39, 40, 41, 42, 44, 54, 55, 56, 60, 74, 75, 76, 77, 79, 82, 83, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109 }

B grade { 4, 33, 35, 38, 46, 47, 48, 49, 50, 61, 62, 66, 67, 68, 80, 81, 84, 85, 86, 117, 124, 125 }

C grade { 78, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129 }

F normal fail { 10, 12, 14, 43, 45, 71, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 8, 9, 16, 17, 18, 19, 20, 22, 27, 28, 29, 30, 31, 32, 39, 40, 41, 48, 49, 50, 88, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 130, 131, 138, 139, 140, 148, 149, 150, 157, 158, 166, 167, 186, 187 }

B grade { 4, 7, 21, 23, 24, 25, 33, 35, 38, 47, 74, 75, 76, 77, 79, 80, 89, 90, 91, 101, 102, 103, 117, 124, 125, 132, 141, 151, 152, 159, 160, 161, 168, 169, 170, 175, 176, 188 }

C grade { }

F normal fail { 1, 3, 5, 6, 10, 11, 12, 13, 14, 15, 26, 34, 36, 37, 42, 43, 44, 45, 46, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 78, 84, 85, 86, 98, 99, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 177, 178, 179 }

F(-1) timedout fail { 81, 82, 83 }

F(-2) exception fail { }

Maxima

A grade { 4, 7, 16, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 38, 47, 74, 75, 76, 77, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107 }

B grade { }

C grade { }

F normal fail { 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 26, 33, 34, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 78, 79, 98, 99, 108, 109, 111, 113, 115, 116, 117, 123, 126, 132, 141, 152, 161, 166, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

F(-1) timedout fail { 1 }

F(-2) exception fail { 80, 81, 82, 83, 84, 85, 86, 110, 112, 114, 118, 119, 120, 121, 122, 124, 125, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 167, 168, 171, 172, 173, 174 }

Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 188 }

F(-1) timedout fail { }

F(-2) exception fail { 186, 187 }

Mupad

A grade { }

B grade { 24, 25, 27, 28, 76, 77, 91 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

F(-2) exception fail { }

Sympy

A grade { 4, 20, 22, 24, 35, 95, 96, 97, 106, 107 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 131, 132, 138, 139, 140, 141, 148, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 177, 178, 179, 187, 188 }

F(-1) timedout fail { 86, 123, 124, 125, 126, 127, 128, 149, 168, 169, 170, 171, 172, 173, 174, 175, 176, 181, 182, 186 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	182	280	0	0	0	0	0
N.S.	1	1.00	1.11	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	0.424	0.617	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	77	150	0	125	0	0	0
N.S.	1	1.00	0.74	1.44	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.068	0.117	0.391	0.000	0.263	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	138	230	0	0	0	0	0
N.S.	1	1.00	1.18	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.269	0.545	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	100	40	106	42	0	0
N.S.	1	1.00	1.00	1.89	0.75	2.00	0.79	0.00	0.00
time (sec)	N/A	0.046	0.073	0.359	0.217	0.252	0.288	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	90	183	0	0	0	0	0
N.S.	1	1.00	1.43	2.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.216	0.398	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	136	0	0	0	0	0
N.S.	1	1.00	0.98	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.042	0.260	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	61	35	97	0	0	0
N.S.	1	1.00	0.86	1.24	0.71	1.98	0.00	0.00	0.00
time (sec)	N/A	0.038	0.099	0.278	0.205	0.258	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	54	77	0	106	0	0	0
N.S.	1	1.00	0.60	0.86	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.050	0.055	0.270	0.000	0.248	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	112	0	116	0	0	0
N.S.	1	1.00	0.72	1.10	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.061	0.115	0.831	0.000	0.247	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	281	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.654	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	188	236	0	0	0	0	0
N.S.	1	1.00	1.02	1.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	0.763	0.416	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	199	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	0.556	0.000	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	101	149	0	0	0	0	0
N.S.	1	1.00	0.99	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.477	0.360	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	128	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	84	181	0	0	0	0	0
N.S.	1	1.00	0.95	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.066	0.283	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	75	98	55	155	0	0	0
N.S.	1	1.00	0.90	1.18	0.66	1.87	0.00	0.00	0.00
time (sec)	N/A	0.055	0.093	0.293	0.204	0.247	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	147	126	0	174	0	0	0
N.S.	1	1.00	1.07	0.92	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.063	0.161	0.253	0.000	0.253	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	120	192	0	186	0	0	0
N.S.	1	1.00	0.67	1.07	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.092	0.133	0.829	0.000	0.257	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	143	134	135	183	0	0	0
N.S.	1	1.00	1.01	0.94	0.95	1.29	0.00	0.00	0.00
time (sec)	N/A	0.043	0.235	0.362	0.275	0.276	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	97	77	78	100	94	0	0
N.S.	1	1.00	0.89	0.71	0.72	0.92	0.86	0.00	0.00
time (sec)	N/A	0.035	0.110	0.312	0.199	0.255	0.711	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	123	114	106	174	0	0	0
N.S.	1	1.00	1.12	1.04	0.96	1.58	0.00	0.00	0.00
time (sec)	N/A	0.032	0.159	0.308	0.293	0.266	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	57	90	68	0	0
N.S.	1	1.00	1.00	0.88	0.74	1.17	0.88	0.00	0.00
time (sec)	N/A	0.022	0.100	0.315	0.203	0.253	0.384	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	103	92	73	162	0	0	0
N.S.	1	1.00	1.32	1.18	0.94	2.08	0.00	0.00	0.00
time (sec)	N/A	0.028	0.111	0.307	0.282	0.266	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	59	36	73	46	0	50
N.S.	1	1.00	1.27	1.31	0.80	1.62	1.02	0.00	1.11
time (sec)	N/A	0.017	0.075	0.305	0.194	0.253	0.207	0.000	4.376

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	80	42	31	119	0	0	44
N.S.	1	1.00	2.00	1.05	0.78	2.98	0.00	0.00	1.10
time (sec)	N/A	0.015	0.152	0.089	0.196	0.257	0.000	0.000	3.913

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	47	98	0	0	0	0	0
N.S.	1	1.00	0.84	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.070	0.408	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	42	54	32	66	0	0	46
N.S.	1	1.00	1.05	1.35	0.80	1.65	0.00	0.00	1.15
time (sec)	N/A	0.017	0.084	0.308	0.210	0.247	0.000	0.000	4.071

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	117	108	105	77	0	0	61
N.S.	1	1.00	1.24	1.15	1.12	0.82	0.00	0.00	0.65
time (sec)	N/A	0.030	0.091	0.316	0.212	0.249	0.000	0.000	4.055

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	74	73	56	79	0	0	0
N.S.	1	1.00	0.96	0.95	0.73	1.03	0.00	0.00	0.00
time (sec)	N/A	0.025	0.086	0.312	0.201	0.246	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	137	131	147	90	0	0	0
N.S.	1	1.00	1.09	1.04	1.17	0.71	0.00	0.00	0.00
time (sec)	N/A	0.043	0.120	0.305	0.210	0.255	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	81	73	89	0	0	0
N.S.	1	1.00	0.86	0.74	0.67	0.82	0.00	0.00	0.00
time (sec)	N/A	0.037	0.111	0.304	0.212	0.253	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	157	151	185	100	0	0	0
N.S.	1	1.00	0.99	0.96	1.17	0.63	0.00	0.00	0.00
time (sec)	N/A	0.056	0.181	0.314	0.215	0.249	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	212	224	0	244	0	0	0
N.S.	1	1.00	1.71	1.81	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.090	0.357	0.724	0.000	0.289	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	241	328	0	0	0	0	0
N.S.	1	1.00	1.72	2.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	1.554	0.863	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	112	165	84	205	99	0	0
N.S.	1	1.00	1.72	2.54	1.29	3.15	1.52	0.00	0.00
time (sec)	N/A	0.063	0.434	0.645	0.213	0.266	0.321	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	126	232	0	0	0	0	0
N.S.	1	1.00	1.62	2.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.334	0.540	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	116	241	0	0	0	0	0
N.S.	1	1.00	1.40	2.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.189	0.454	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	87	121	78	143	0	0	0
N.S.	1	1.00	1.43	1.98	1.28	2.34	0.00	0.00	0.00
time (sec)	N/A	0.055	0.251	0.449	0.218	0.263	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	183	191	0	165	0	0	0
N.S.	1	1.00	1.55	1.62	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.068	0.177	0.424	0.000	0.263	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	134	191	0	181	0	0	0
N.S.	1	1.00	1.10	1.57	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.081	0.271	0.809	0.000	0.263	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	268	263	0	204	0	0	0
N.S.	1	1.00	1.77	1.74	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.092	0.287	0.937	0.000	0.247	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	337	464	0	0	0	0	0
N.S.	1	1.00	1.51	2.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.174	2.476	0.924	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	440	0	0	0	0	0	0
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	1.268	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	219	318	0	0	0	0	0
N.S.	1	1.00	1.74	2.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.122	1.157	0.785	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	282	0	0	0	0	0	0
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.568	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	182	429	0	0	0	0	0
N.S.	1	1.00	1.60	3.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.242	0.578	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	165	225	144	228	0	0	0
N.S.	1	1.00	1.62	2.21	1.41	2.24	0.00	0.00	0.00
time (sec)	N/A	0.081	0.354	0.536	0.231	0.254	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	245	321	0	271	0	0	0
N.S.	1	1.00	1.50	1.97	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.089	0.529	0.502	0.000	0.272	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	256	387	0	305	0	0	0
N.S.	1	1.00	1.20	1.82	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.189	0.409	0.827	0.000	0.268	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	332	485	0	351	0	0	0
N.S.	1	1.00	1.37	2.00	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.155	0.742	0.986	0.000	0.278	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	18
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.50
time (sec)	N/A	0.014	2.986	0.342	0.292	0.259	0.261	0.257	4.184

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	16
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.60
time (sec)	N/A	0.006	2.555	0.237	0.261	0.257	0.266	0.256	3.776

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	16	20
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	1.14	1.43
time (sec)	N/A	0.022	0.350	0.214	0.321	0.251	0.671	0.262	3.868

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	54	0	0	0	0	0
N.S.	1	1.00	0.93	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.098	0.505	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	60	0	0	0	0	0
N.S.	1	1.00	0.89	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.089	0.684	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	110	0	0	0	0	0
N.S.	1	1.00	0.78	0.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.187	0.856	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	546	28	12	14	18
N.S.	1	1.00	1.17	1.00	45.50	2.33	1.00	1.17	1.50
time (sec)	N/A	0.015	19.987	0.366	0.607	0.263	0.569	0.260	4.319

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	535	26	12	12	16
N.S.	1	1.00	1.20	1.00	53.50	2.60	1.20	1.20	1.60
time (sec)	N/A	0.007	76.817	0.235	0.614	0.256	0.597	0.255	3.956

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	544	30	14	16	20
N.S.	1	1.00	1.14	1.00	38.86	2.14	1.00	1.14	1.43
time (sec)	N/A	0.021	7.066	0.225	0.512	0.277	1.181	0.266	3.969

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	164	0	0	0	0	0
N.S.	1	1.00	0.95	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	0.462	0.553	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	92	186	0	0	0	0	0
N.S.	1	1.00	1.08	2.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	0.590	0.760	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	250	420	0	0	0	0	0
N.S.	1	1.00	1.32	2.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	0.949	0.941	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	2818	42	12	14	18
N.S.	1	1.00	1.17	1.00	234.83	3.50	1.00	1.17	1.50
time (sec)	N/A	0.012	5.995	0.349	2.353	0.265	0.990	0.262	4.459

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	2771	40	12	12	16
N.S.	1	1.00	1.20	1.00	277.10	4.00	1.20	1.20	1.60
time (sec)	N/A	0.006	93.529	0.243	2.376	0.270	1.024	0.272	4.069

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	2638	45	14	16	20
N.S.	1	1.00	1.14	1.00	188.43	3.21	1.00	1.14	1.43
time (sec)	N/A	0.019	2.552	0.220	1.895	0.278	1.960	0.259	4.087

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	103	244	0	0	0	0	0
N.S.	1	1.00	0.90	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.288	0.612	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	122	277	0	0	0	0	0
N.S.	1	1.00	1.09	2.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.168	0.467	0.786	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	204	628	0	0	0	0	0
N.S.	1	1.00	0.85	2.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	0.575	1.005	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1450	44	15	18	22
N.S.	1	1.00	1.12	1.00	90.62	2.75	0.94	1.12	1.38
time (sec)	N/A	0.020	5.025	0.652	10.450	0.276	5.199	0.276	4.586

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	704	30	15	18	22
N.S.	1	1.00	1.12	1.00	44.00	1.88	0.94	1.12	1.38
time (sec)	N/A	0.019	2.741	0.498	4.826	0.272	2.119	0.274	4.252

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	97	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.38
time (sec)	N/A	0.021	0.430	0.813	0.306	0.266	0.421	0.256	3.732

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	616	32	15	18	22
N.S.	1	1.00	1.12	1.00	38.50	2.00	0.94	1.12	1.38
time (sec)	N/A	0.022	0.933	0.716	0.895	0.282	1.133	0.257	3.824

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	190	265	221	358	0	0	0
N.S.	1	1.00	0.72	1.00	0.84	1.36	0.00	0.00	0.00
time (sec)	N/A	0.251	0.384	0.581	0.294	0.356	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	147	199	152	280	0	0	0
N.S.	1	1.00	0.73	0.99	0.76	1.39	0.00	0.00	0.00
time (sec)	N/A	0.157	0.215	0.593	0.304	0.329	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	142	107	70	177	0	0	99
N.S.	1	1.00	1.00	0.75	0.49	1.25	0.00	0.00	0.70
time (sec)	N/A	0.086	0.341	0.307	0.205	0.296	0.000	0.000	4.183

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	80	42	31	119	0	0	44
N.S.	1	1.00	2.00	1.05	0.78	2.98	0.00	0.00	1.10
time (sec)	N/A	0.013	0.054	0.089	0.212	0.289	0.000	0.000	4.094

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	393	511	0	0	0	0	0
N.S.	1	1.00	1.72	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.759	0.630	1.097	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	222	207	0	578	0	0	0
N.S.	1	1.00	1.51	1.41	0.00	3.93	0.00	0.00	0.00
time (sec)	N/A	0.095	0.244	1.914	0.000	0.295	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	342	593	0	1212	0	0	0
N.S.	1	1.00	1.12	1.94	0.00	3.96	0.00	0.00	0.00
time (sec)	N/A	0.151	0.629	1.882	0.000	0.366	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	2653	818	0	0	0	0	0
N.S.	1	1.00	7.73	2.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	21.844	12.724	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	2938	413	0	0	0	0	0
N.S.	1	1.00	10.53	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	26.038	11.261	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	1707	286	0	0	0	0	0
N.S.	1	1.00	9.13	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.188	18.887	11.285	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	1675	251	0	0	0	0	0
N.S.	1	1.00	15.95	2.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	18.020	11.347	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	4527	890	0	0	0	0	0
N.S.	1	1.00	16.28	3.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	25.129	13.388	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	609	609	8675	1612	0	0	0	0	0
N.S.	1	1.00	14.24	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.428	26.779	14.253	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	222	18	15	18	22
N.S.	1	1.00	1.12	1.00	13.88	1.12	0.94	1.12	1.38
time (sec)	N/A	0.037	21.678	0.396	0.844	0.287	13.563	0.266	4.339

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	162	213	244	259	0	0	0
N.S.	1	1.00	0.71	0.93	1.07	1.13	0.00	0.00	0.00
time (sec)	N/A	0.094	0.399	0.608	0.287	0.335	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	144	171	182	238	0	0	0
N.S.	1	1.00	0.83	0.98	1.05	1.37	0.00	0.00	0.00
time (sec)	N/A	0.081	0.267	0.592	0.285	0.323	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	189	120	107	209	0	0	0
N.S.	1	1.00	1.69	1.07	0.96	1.87	0.00	0.00	0.00
time (sec)	N/A	0.045	0.383	0.342	0.292	0.303	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	127	112	66	182	0	0	98
N.S.	1	1.00	1.32	1.17	0.69	1.90	0.00	0.00	1.02
time (sec)	N/A	0.061	0.268	0.346	0.199	0.290	0.000	0.000	4.404

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	76	110	91	106	0	0	0
N.S.	1	1.00	0.60	0.87	0.72	0.84	0.00	0.00	0.00
time (sec)	N/A	0.062	0.142	0.346	0.198	0.266	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	101	129	132	128	0	0	0
N.S.	1	1.00	0.55	0.70	0.72	0.70	0.00	0.00	0.00
time (sec)	N/A	0.078	0.202	0.346	0.203	0.303	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	117	147	165	149	0	0	0
N.S.	1	1.00	0.49	0.62	0.69	0.63	0.00	0.00	0.00
time (sec)	N/A	0.099	0.254	0.359	0.198	0.267	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	126	139	177	168	228	0	0
N.S.	1	1.00	0.54	0.60	0.76	0.72	0.98	0.00	0.00
time (sec)	N/A	0.122	0.254	0.579	0.207	0.288	1.346	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	106	121	138	147	177	0	0
N.S.	1	1.00	0.59	0.67	0.77	0.82	0.98	0.00	0.00
time (sec)	N/A	0.111	0.217	0.605	0.201	0.285	0.713	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	85	194	96	125	126	0	0
N.S.	1	1.00	0.52	1.18	0.59	0.76	0.77	0.00	0.00
time (sec)	N/A	0.142	0.169	0.621	0.199	0.301	0.406	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	108	161	0	0	0	0	0
N.S.	1	1.00	0.36	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.645	0.221	0.974	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	170	168	0	0	0	0	0
N.S.	1	1.00	0.55	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	0.246	1.049	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	207	286	328	341	0	0	0
N.S.	1	1.00	0.75	1.04	1.19	1.24	0.00	0.00	0.00
time (sec)	N/A	0.190	0.464	0.738	0.287	0.439	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	174	208	224	305	0	0	0
N.S.	1	1.00	0.85	1.02	1.10	1.50	0.00	0.00	0.00
time (sec)	N/A	0.112	0.281	0.620	0.287	0.372	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	158	184	152	287	0	0	0
N.S.	1	1.00	0.89	1.04	0.86	1.62	0.00	0.00	0.00
time (sec)	N/A	0.102	0.239	0.617	0.274	0.325	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	149	201	134	267	0	0	0
N.S.	1	1.00	0.85	1.14	0.76	1.52	0.00	0.00	0.00
time (sec)	N/A	0.095	0.271	0.620	0.204	0.307	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	134	177	175	167	0	0	0
N.S.	1	1.00	0.63	0.83	0.82	0.78	0.00	0.00	0.00
time (sec)	N/A	0.122	0.277	0.612	0.202	0.278	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	160	209	232	199	0	0	0
N.S.	1	1.00	0.57	0.74	0.83	0.71	0.00	0.00	0.00
time (sec)	N/A	0.141	0.347	0.626	0.197	0.272	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	168	198	245	227	332	0	0
N.S.	1	1.00	0.60	0.71	0.88	0.82	1.19	0.00	0.00
time (sec)	N/A	0.175	0.275	0.735	0.199	0.296	1.347	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	139	283	185	192	252	0	0
N.S.	1	1.00	0.60	1.23	0.80	0.83	1.10	0.00	0.00
time (sec)	N/A	0.178	0.255	0.737	0.194	0.291	0.729	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	176	276	0	0	0	0	0
N.S.	1	1.00	0.48	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.852	0.417	1.229	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	223	250	0	0	0	0	0
N.S.	1	1.00	0.60	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.780	1.110	1.388	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	519	519	921	411	0	0	0	0	0
N.S.	1	1.00	1.77	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.869	2.836	54.985	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	459	459	860	506	0	0	0	0	0
N.S.	1	1.00	1.87	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.824	0.667	1.454	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	849	302	0	0	0	0	0
N.S.	1	1.00	1.81	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.616	0.671	27.625	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	417	417	841	2081	0	0	0	0	0
N.S.	1	1.00	2.02	4.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.630	0.520	2.056	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	523	933	372	0	0	0	0	0
N.S.	1	1.00	1.78	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.774	3.283	52.023	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	631	631	1278	786	0	0	0	0	0
N.S.	1	1.00	2.03	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.996	5.543	7.178	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	580	580	1208	644	0	0	0	0	0
N.S.	1	1.00	2.08	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.887	1.157	1.710	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	345	462	0	602	0	0	0
N.S.	1	1.00	2.35	3.14	0.00	4.10	0.00	0.00	0.00
time (sec)	N/A	0.163	1.102	5.515	0.000	0.307	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	542	542	1189	2226	0	0	0	0	0
N.S.	1	1.00	2.19	4.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.777	1.985	7.004	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	840	840	1270	1006	0	0	0	0	0
N.S.	1	1.00	1.51	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.927	1.465	111.627	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	786	786	1226	910	0	0	0	0	0
N.S.	1	1.00	1.56	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.019	1.790	46.703	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	786	786	1216	898	0	0	0	0	0
N.S.	1	1.00	1.55	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.718	1.641	55.842	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	844	844	1305	1007	0	0	0	0	0
N.S.	1	1.00	1.55	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.783	1.354	109.899	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	778	778	2000	1549	0	0	0	0	0
N.S.	1	1.00	2.57	1.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.073	7.932	6.870	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	486	1362	0	1346	0	0	0
N.S.	1	1.00	2.81	7.87	0.00	7.78	0.00	0.00	0.00
time (sec)	N/A	0.152	1.581	5.088	0.000	0.384	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	486	1318	0	1232	0	0	0
N.S.	1	1.00	2.24	6.07	0.00	5.68	0.00	0.00	0.00
time (sec)	N/A	0.216	1.114	5.096	0.000	0.374	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	741	741	2054	3727	0	0	0	0	0
N.S.	1	1.00	2.77	5.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.966	6.088	2.332	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1272	1272	2022	1960	0	0	0	0	0
N.S.	1	1.00	1.59	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.522	6.110	110.957	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1276	1276	2030	1398	0	0	0	0	0
N.S.	1	1.00	1.59	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.714	6.105	150.993	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1272	1272	2015	1950	0	0	0	0	0
N.S.	1	1.00	1.58	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.402	6.075	181.431	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	447	447	340	0	0	1995	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	4.46	0.00	0.00	0.00
time (sec)	N/A	0.972	37.746	0.000	0.000	1.447	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	329	329	365	0	0	1669	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	5.07	0.00	0.00	0.00
time (sec)	N/A	0.274	23.449	0.000	0.000	0.737	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	307	0	0	1382	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	6.25	0.00	0.00	0.00
time (sec)	N/A	0.234	22.759	0.000	0.000	0.446	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	1.00	1.17
time (sec)	N/A	0.072	8.491	0.250	0.000	0.263	4.151	0.297	4.789

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.075	9.094	0.242	0.000	0.266	4.037	0.292	4.544

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.068	19.221	0.214	0.000	0.257	7.943	0.281	4.563

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.026	6.935	0.210	0.000	0.250	2.041	0.290	4.304

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.061	2.615	0.235	0.000	0.253	2.602	0.279	4.589

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	312	312	576	0	0	242	0	0	0
N.S.	1	1.00	1.85	0.00	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.268	25.184	0.000	0.000	0.104	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	446	446	641	0	0	340	0	0	0
N.S.	1	1.00	1.44	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.372	27.925	0.000	0.000	0.113	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	418	418	313	0	0	1989	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	4.76	0.00	0.00	0.00
time (sec)	N/A	0.358	37.502	0.000	0.000	1.470	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	342	0	0	1667	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	5.61	0.00	0.00	0.00
time (sec)	N/A	0.286	23.438	0.000	0.000	0.708	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	1.00	1.17
time (sec)	N/A	0.083	10.530	0.250	0.000	0.259	23.893	0.306	4.458

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.083	8.691	0.720	0.000	0.261	19.948	0.268	4.627

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.96	1.00	1.17
time (sec)	N/A	0.091	19.894	0.830	0.000	0.265	69.279	0.292	4.963

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.95	1.00	1.20
time (sec)	N/A	0.033	8.307	0.807	0.000	0.261	20.249	0.291	4.390

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.068	14.907	0.257	0.000	0.252	18.369	0.288	4.800

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.070	18.912	0.313	0.000	0.268	20.322	0.302	4.607

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	409	409	620	0	0	338	0	0	0
N.S.	1	1.00	1.52	0.00	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.395	27.983	0.000	0.000	0.104	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	556	556	1187	0	0	447	0	0	0
N.S.	1	1.00	2.13	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.494	29.473	0.000	0.000	0.113	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	356	366	0	0	1679	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	0.796	23.560	0.000	0.000	0.749	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	251	406	0	0	1389	0	0	0
N.S.	1	1.00	1.62	0.00	0.00	5.53	0.00	0.00	0.00
time (sec)	N/A	0.218	22.963	0.000	0.000	0.475	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	239	0	0	1102	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	7.20	0.00	0.00	0.00
time (sec)	N/A	0.187	21.037	0.000	0.000	0.333	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	1.00	1.17
time (sec)	N/A	0.066	3.539	0.292	0.000	0.260	2.478	0.275	4.373

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	1.00	1.17
time (sec)	N/A	0.075	8.116	0.241	0.000	0.252	7.323	0.284	4.482

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.065	13.722	0.204	0.000	0.253	3.462	0.292	4.561

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.024	1.115	0.214	0.000	0.261	1.163	0.280	4.731

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	501	0	0	154	0	0	0
N.S.	1	1.00	2.27	0.00	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.150	25.414	0.000	0.000	0.095	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	346	346	612	0	0	244	0	0	0
N.S.	1	1.00	1.77	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.292	26.043	0.000	0.000	0.106	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	278	436	0	0	1771	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	6.37	0.00	0.00	0.00
time (sec)	N/A	0.761	23.343	0.000	0.000	0.441	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	249	0	0	1311	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	7.41	0.00	0.00	0.00
time (sec)	N/A	0.175	23.174	0.000	0.000	0.357	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	135	0	0	379	0	0	0
N.S.	1	1.00	1.55	0.00	0.00	4.36	0.00	0.00	0.00
time (sec)	N/A	0.165	21.060	0.000	0.000	0.290	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	1.00	1.17
time (sec)	N/A	0.075	11.494	0.261	0.000	0.257	18.227	0.264	4.952

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.96	1.00	1.17
time (sec)	N/A	0.087	15.275	0.725	0.000	0.251	77.562	0.272	4.750

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.076	19.490	0.811	0.000	0.248	40.456	0.295	4.661

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.072	7.967	0.873	0.000	0.252	9.141	0.283	4.514

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	334	0	0	107	0	0	0
N.S.	1	1.00	3.63	0.00	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.058	50.615	0.000	0.000	0.105	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	249	501	0	0	238	0	0	0
N.S.	1	1.00	2.01	0.00	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.201	25.727	0.000	0.000	0.107	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	272	272	348	0	0	2415	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	8.88	0.00	0.00	0.00
time (sec)	N/A	0.853	23.415	0.000	0.000	0.469	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	218	0	0	786	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	4.39	0.00	0.00	0.00
time (sec)	N/A	0.179	0.698	0.000	0.000	0.351	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	204	0	0	692	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	4.49	0.00	0.00	0.00
time (sec)	N/A	0.187	0.620	0.000	0.000	0.344	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.00	1.00	1.17
time (sec)	N/A	0.085	20.346	0.268	0.000	0.265	0.000	0.277	4.527

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	1.00	1.17
time (sec)	N/A	0.094	24.001	0.839	0.000	0.256	0.000	0.276	4.820

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.088	20.714	0.851	0.000	0.255	0.000	0.289	5.141

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.076	19.657	0.900	0.000	0.260	0.000	0.284	4.580

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	246	488	0	0	335	0	0	0
N.S.	1	1.00	1.98	0.00	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.181	3.139	0.000	0.000	0.105	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	266	517	0	0	397	0	0	0
N.S.	1	1.00	1.94	0.00	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.152	26.786	0.000	0.000	0.112	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	576	441	0	0	0	0	0	0
N.S.	1	0.97	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.609	4.667	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	352	322	0	0	0	0	0	0
N.S.	1	0.95	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.797	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	192	190	0	0	0	0	0	0
N.S.	1	0.93	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.129	0.533	0.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.26
time (sec)	N/A	0.055	3.766	1.323	0.321	0.264	8.615	0.258	3.915

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	1.26
time (sec)	N/A	0.053	7.643	1.289	0.350	0.266	0.000	0.271	4.034

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	42	0	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.68	0.00	1.00	1.16
time (sec)	N/A	0.078	1.478	1.315	0.320	0.272	0.000	0.324	4.118

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.070	0.223	0.497	0.310	0.272	8.730	0.299	4.037

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.071	1.537	0.456	0.326	0.274	3.271	0.285	4.102

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	45	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.80	0.96	1.00	1.16
time (sec)	N/A	0.078	1.933	1.340	0.325	0.261	22.929	0.283	4.664

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	473	473	213	0	0	393	0	0	0
N.S.	1	1.00	0.45	0.00	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	1.106	0.633	0.000	0.000	0.282	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	316	316	178	0	0	336	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.939	0.524	0.000	0.000	0.278	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	140	0	0	279	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.113	0.487	0.000	0.000	0.269	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	100	37	34	26	30
N.S.	1	1.00	1.08	0.92	3.85	1.42	1.31	1.00	1.15
time (sec)	N/A	0.061	0.854	0.258	0.789	0.252	2.718	0.268	5.322

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	124	39	36	26	30
N.S.	1	1.00	1.08	0.92	4.77	1.50	1.38	1.00	1.15
time (sec)	N/A	0.067	9.050	0.536	0.858	0.243	24.428	0.272	4.827

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [11] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	6	1.00	10	0.600
2	A	5	5	1.00	10	0.500
3	A	8	6	1.00	10	0.600
4	A	4	4	1.00	8	0.500
5	A	7	5	1.00	6	0.833
6	A	6	6	1.00	10	0.600
7	A	4	3	1.00	10	0.300
8	A	4	4	1.00	10	0.400
9	A	5	5	1.00	10	0.500
10	A	14	9	1.00	10	0.900
11	A	10	10	1.00	10	1.000
12	A	11	8	1.00	10	0.800
13	A	7	7	1.00	8	0.875
14	A	9	6	1.00	6	1.000
15	A	7	7	1.00	10	0.700
16	A	5	3	1.00	10	0.300
17	A	6	6	1.00	10	0.600
18	A	8	6	1.00	10	0.600
19	A	8	6	1.00	12	0.500
20	A	6	4	1.00	12	0.333
21	A	6	6	1.00	12	0.500
22	A	4	4	1.00	12	0.333
23	A	4	4	1.00	12	0.333
24	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	8	0.250
26	A	6	6	1.00	12	0.500
27	A	2	2	1.00	12	0.167
28	A	5	5	1.00	12	0.417
29	A	4	4	1.00	12	0.333
30	A	7	5	1.00	12	0.417
31	A	6	4	1.00	12	0.333
32	A	9	5	1.00	12	0.417
33	A	5	5	1.00	14	0.357
34	A	8	6	1.00	14	0.429
35	A	4	4	1.00	12	0.333
36	A	7	5	1.00	10	0.500
37	A	6	6	1.00	14	0.429
38	A	4	3	1.00	14	0.214
39	A	4	3	1.00	14	0.214
40	A	5	5	1.00	14	0.357
41	A	5	3	1.00	14	0.214
42	A	10	10	1.00	14	0.714
43	A	11	8	1.00	14	0.571
44	A	7	7	1.00	12	0.583
45	A	9	6	1.00	10	0.600
46	A	7	7	1.00	14	0.500
47	A	5	3	1.00	14	0.214
48	A	6	6	1.00	14	0.429
49	A	8	6	1.00	14	0.429
50	A	10	6	1.00	14	0.429
51	N/A	0	0	1.00	12	0.000
52	N/A	0	0	1.00	10	0.000
53	N/A	0	0	1.00	14	0.000
54	A	4	4	1.00	14	0.286
55	A	6	6	1.00	14	0.429
56	A	9	5	1.00	14	0.357
57	N/A	0	0	1.00	12	0.000
58	N/A	0	0	1.00	10	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	N/A	0	0	1.00	14	0.000
60	A	5	5	1.00	14	0.357
61	A	7	7	1.00	14	0.500
62	A	11	6	1.00	14	0.429
63	N/A	0	0	1.00	12	0.000
64	N/A	0	0	1.00	10	0.000
65	N/A	0	0	1.00	14	0.000
66	A	6	5	1.00	14	0.357
67	A	8	7	1.00	14	0.500
68	A	13	6	1.00	14	0.429
69	N/A	0	0	1.00	16	0.000
70	N/A	0	0	1.00	16	0.000
71	A	3	3	1.00	14	0.214
72	N/A	0	0	1.00	16	0.000
73	N/A	0	0	1.00	16	0.000
74	A	9	7	1.00	16	0.438
75	A	8	7	1.00	16	0.438
76	A	7	7	1.00	14	0.500
77	A	3	2	1.00	8	0.250
78	A	4	2	1.00	16	0.125
79	A	8	7	1.00	16	0.438
80	A	11	8	1.00	16	0.500
81	A	21	12	1.00	18	0.667
82	A	14	10	1.00	18	0.556
83	A	8	8	1.00	18	0.444
84	A	5	5	1.00	18	0.278
85	A	11	10	1.00	18	0.556
86	A	18	13	1.00	18	0.722
87	N/A	0	0	1.00	16	0.000
88	A	6	6	1.00	19	0.316
89	A	5	6	1.00	19	0.316
90	A	4	4	1.00	16	0.250
91	A	3	4	1.00	19	0.210
92	A	4	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	5	6	1.00	19	0.316
94	A	6	6	1.00	19	0.316
95	A	5	5	1.00	19	0.263
96	A	5	5	1.00	19	0.263
97	A	7	6	1.00	17	0.353
98	A	12	12	1.00	19	0.632
99	A	14	14	1.00	19	0.737
100	A	6	7	1.00	21	0.333
101	A	5	6	1.00	18	0.333
102	A	5	6	1.00	21	0.286
103	A	5	6	1.00	21	0.286
104	A	5	6	1.00	21	0.286
105	A	6	7	1.00	21	0.333
106	A	5	6	1.00	21	0.286
107	A	7	6	1.00	19	0.316
108	A	13	14	1.00	21	0.667
109	A	15	16	1.00	21	0.762
110	A	24	11	1.00	21	0.524
111	A	26	9	1.00	19	0.474
112	A	19	7	1.00	18	0.389
113	A	19	7	1.00	21	0.333
114	A	24	10	1.00	21	0.476
115	A	32	15	1.00	21	0.714
116	A	30	13	1.00	21	0.619
117	A	8	6	1.00	19	0.316
118	A	25	11	1.00	21	0.524
119	A	50	13	1.00	21	0.619
120	A	27	10	1.00	21	0.476
121	A	47	11	1.00	18	0.611
122	A	50	13	1.00	21	0.619
123	A	35	14	1.00	21	0.667
124	A	6	7	1.00	21	0.333
125	A	9	7	1.00	19	0.368
126	A	30	12	1.00	21	0.571
127	A	35	11	1.00	21	0.524

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	63	12	1.00	21	0.571
129	A	81	12	1.00	18	0.667
130	A	12	12	1.00	23	0.522
131	A	11	12	1.00	23	0.522
132	A	10	10	1.00	21	0.476
133	N/A	0	0	1.00	23	0.000
134	N/A	0	0	1.00	23	0.000
135	N/A	0	0	1.00	23	0.000
136	N/A	0	0	1.00	20	0.000
137	N/A	0	0	1.00	23	0.000
138	A	9	10	1.00	23	0.435
139	A	10	11	1.00	23	0.478
140	A	12	12	1.00	23	0.522
141	A	11	11	1.00	21	0.524
142	N/A	0	0	1.00	23	0.000
143	N/A	0	0	1.00	23	0.000
144	N/A	0	0	1.00	23	0.000
145	N/A	0	0	1.00	20	0.000
146	N/A	0	0	1.00	23	0.000
147	N/A	0	0	1.00	23	0.000
148	A	10	11	1.00	23	0.478
149	A	11	11	1.00	23	0.478
150	A	11	12	1.00	23	0.522
151	A	10	12	1.00	23	0.522
152	A	10	10	1.00	21	0.476
153	N/A	0	0	1.00	23	0.000
154	N/A	0	0	1.00	23	0.000
155	N/A	0	0	1.00	23	0.000
156	N/A	0	0	1.00	20	0.000
157	A	9	10	1.00	23	0.435
158	A	9	11	1.00	23	0.478
159	A	10	11	1.00	23	0.478
160	A	9	11	1.00	23	0.478

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
161	A	5	5	1.00	21	0.238
162	N/A	0	0	1.00	23	0.000
163	N/A	0	0	1.00	23	0.000
164	N/A	0	0	1.00	23	0.000
165	N/A	0	0	1.00	23	0.000
166	A	4	5	1.00	20	0.250
167	A	8	10	1.00	23	0.435
168	A	10	11	1.00	23	0.478
169	A	7	8	1.00	23	0.348
170	A	6	6	1.00	21	0.286
171	N/A	0	0	1.00	23	0.000
172	N/A	0	0	1.00	23	0.000
173	N/A	0	0	1.00	23	0.000
174	N/A	0	0	1.00	23	0.000
175	A	8	9	1.00	23	0.391
176	A	8	10	1.00	20	0.500
177	A	5	6	0.97	23	0.261
178	A	5	6	0.95	23	0.261
179	A	4	5	0.93	21	0.238
180	N/A	0	0	1.00	23	0.000
181	N/A	0	0	1.00	23	0.000
182	N/A	0	0	1.00	25	0.000
183	N/A	0	0	1.00	25	0.000
184	N/A	0	0	1.00	25	0.000
185	N/A	0	0	1.00	25	0.000
186	A	15	10	1.00	26	0.385
187	A	12	10	1.00	26	0.385
188	A	7	8	1.00	26	0.308
189	N/A	0	0	1.00	26	0.000
190	N/A	0	0	1.00	26	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^4 \text{sech}^{-1}(ax)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 164

$$\begin{aligned}\int x^4\text{sech}^{-1}(ax)^2\,dx = &-\frac{3x}{20a^4}-\frac{x^3}{30a^2}-\frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{20a^4}\\ &-\frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{10a^2}+\frac{1}{5}x^5\text{sech}^{-1}(ax)^2\\ &-\frac{3\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{10a^5}\\ &+\frac{3i\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}-\frac{3i\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}\end{aligned}$$

```
[Out] -3/20*x/a^4-1/30*x^3/a^2+1/5*x^5*arcsech(a*x)^2-3/10*arcsech(a*x)*arctan(1/
a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/a^5+3/20*I*polylog(2,-I*(1/a/x+(1/a/x-
1)^(1/2)*(1+1/a/x)^(1/2)))/a^5-3/20*I*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1
+1/a/x)^(1/2)))/a^5-3/20*x*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/a^
4-1/10*x^3*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used

= {6420, 5526, 4270, 4265, 2317, 2438}

$$\int x^4\operatorname{sech}^{-1}(ax)^2\,dx = -\frac{3\operatorname{sech}^{-1}(ax)\arctan\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{10a^5} + \frac{3i\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{3i\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{3x}{20a^4} - \frac{3x\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3}{30a^2} - \frac{x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\operatorname{sech}^{-1}(ax)^2$$

```
[In] Int[x^4*ArcSech[a*x]^2,x]
```

```
[Out] (-3*x)/(20*a^4) - x^3/(30*a^2) - (3*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*A
rcSech[a*x])/(20*a^4) - (x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*
x])/(10*a^2) + (x^5*ArcSech[a*x]^2)/5 - (3*ArcSech[a*x]*ArcTan[E^ArcSech[a*
x]])/(10*a^5) + (((3*I)/20)*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^5 - (((3*I)/
20)*PolyLog[2, I*E^ArcSech[a*x]])/a^5
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^2\text{sech}^5(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^5}\\
&= \frac{1}{5}x^5\text{sech}^{-1}(ax)^2 - \frac{2\text{Subst}\left(\int x\text{sech}^5(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{5a^5}\\
&= -\frac{x^3}{30a^2} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^2\\
&\quad - \frac{3\text{Subst}\left(\int x\text{sech}^3(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{10a^5}\\
&= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{10a^2}\\
&\quad + \frac{1}{5}x^5\text{sech}^{-1}(ax)^2 - \frac{3\text{Subst}\left(\int x\text{sech}(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{20a^5}\\
&= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{10a^2}\\
&\quad + \frac{1}{5}x^5\text{sech}^{-1}(ax)^2 - \frac{3\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{10a^5}\\
&\quad + \frac{(3i)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{20a^5}\\
&\quad - \frac{(3i)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{20a^5}
\end{aligned}$$

$$= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^2 - \frac{3\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{10a^5} + \frac{(3i)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{sech}^{-1}(ax)}\right)}{20a^5} - \frac{(3i)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{sech}^{-1}(ax)}\right)}{20a^5}$$

$$= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^2 - \frac{3\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{10a^5} + \frac{3i\,\text{PolyLog}\left(2, -ie^{\text{sech}^{-1}(ax)}\right)}{20a^5} - \frac{3i\,\text{PolyLog}\left(2, ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.11

$$\int x^4\text{sech}^{-1}(ax)^2\,dx$$

$$= \frac{-9ax - 2a^3x^3 - 9ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax) - 6a^3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax) + 12a^5x^5\text{sech}^{-1}(ax)^2 +}{}$$

```
[In] Integrate[x^4*ArcSech[a*x]^2,x]
```

```
[Out] (-9*a*x - 2*a^3*x^3 - 9*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x
] - 6*a^3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + 12*a^5*x^5
*ArcSech[a*x]^2 + (9*I)*ArcSech[a*x]*Log[1 - I/E^ArcSech[a*x]] - (9*I)*ArcS
ech[a*x]*Log[1 + I/E^ArcSech[a*x]] + (9*I)*PolyLog[2, (-I)/E^ArcSech[a*x]]
- (9*I)*PolyLog[2, I/E^ArcSech[a*x]])/(60*a^5)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{\left(-6\,\text{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,a^3x^3+12a^4x^4\,\text{arcsech}(ax)^2-9\,\text{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax-2a^2x^2-9\right)ax}{60}+\frac{3i\,\text{arcsech}(ax)}{}$
default	$\frac{\left(-6\,\text{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,a^3x^3+12a^4x^4\,\text{arcsech}(ax)^2-9\,\text{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax-2a^2x^2-9\right)ax}{60}+\frac{3i\,\text{arcsech}(ax)}{}$

```
[In] int(x^4*arcsech(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/60*(-6*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a^3*x
^3+12*a^4*x^4*arcsech(a*x)^2-9*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a
/x)^(1/2)*a*x-2*a^2*x^2-9)*a*x+3/20*I*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-1)^
(1/2)*(1+1/a/x)^(1/2)))-3/20*I*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(1/2)*(
1+1/a/x)^(1/2)))+3/20*I*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-
3/20*I*dilog(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))))
```

Fricas [F]

$$\int x^4 \text{sech}^{-1}(ax)^2\,dx = \int x^4\,\text{arsech}\,(ax)^2\,\,dx$$

```
[In] integrate(x^4*arcsech(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*arcsech(a*x)^2, x)
```

Sympy [F]

$$\int x^4 \text{sech}^{-1}(ax)^2\,dx = \int x^4\,\text{asech}^2\,(ax)\,dx$$

```
[In] integrate(x**4*asech(a*x)**2,x)
```

```
[Out] Integral(x**4*asech(a*x)**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int x^4 \text{sech}^{-1}(ax)^2\,dx = \text{Timed out}$$

```
[In] integrate(x^4*arcsech(a*x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int x^4 \text{sech}^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arsech}(ax)^2 \, dx$$

```
[In] integrate(x^4*arcsech(a*x)^2,x, algorithm="giac")
[Out] integrate(x^4*arcsech(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{sech}^{-1}(ax)^2 \, dx = \int x^4 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^2 \, dx$$

```
[In] int(x^4*acosh(1/(a*x))^2,x)
[Out] int(x^4*acosh(1/(a*x))^2, x)
```

3.2 $\int x^3 \text{sech}^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 104

$$\int x^3 \text{sech}^{-1}(ax)^2 \, dx = -\frac{x^2}{12a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{6a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 - \frac{\log(x)}{3a^4}$$

```
[Out] -1/12*x^2/a^2+1/4*x^4*arcsech(a*x)^2-1/3*ln(x)/a^4-1/3*(a*x+1)*arcsech(a*x)
*((-a*x+1)/(a*x+1))^(1/2)/a^4-1/6*x^2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1
))^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6420, 5526, 4270, 4269, 3556}

$$\int x^3 \text{sech}^{-1}(ax)^2 \, dx = -\frac{\log(x)}{3a^4} - \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{3a^4} - \frac{x^2}{12a^2} - \frac{x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{6a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^2$$

```
[In] Int[x^3*ArcSech[a*x]^2,x]
```

```
[Out] -1/12*x^2/a^2 - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(3*a^4)
- (x^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(6*a^2) + (x^4*Arc
Sech[a*x]^2)/4 - Log[x]/(3*a^4)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^2\text{sech}^4(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^4} \\
&= \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 - \frac{\text{Subst}\left(\int x\text{sech}^4(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{2a^4} \\
&= -\frac{x^2}{12a^2} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{6a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 \\
&\quad - \frac{\text{Subst}\left(\int x\text{sech}^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{3a^4}
\end{aligned}
$$

$$= -\frac{x^2}{12a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{6a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 + \frac{\text{Subst}\left(\int \tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{3a^4}$$

$$= -\frac{x^2}{12a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{6a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^2 - \frac{\log(x)}{3a^4}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

$$\int x^3\text{sech}^{-1}(ax)^2\,dx = -\frac{a^2x^2 + 2\sqrt{\frac{1-ax}{1+ax}}(2+2ax+a^2x^2+a^3x^3)\,\text{sech}^{-1}(ax) - 3a^4x^4\text{sech}^{-1}(ax)^2 + 4\log(x)}{12a^4}$$

```
[In] Integrate[x^3*ArcSech[a*x]^2,x]
```

```
[Out] -1/12*(a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(2 + 2*a*x + a^2*x^2 + a^3*x^3
)*ArcSech[a*x] - 3*a^4*x^4*ArcSech[a*x]^2 + 4*Log[x])/a^4
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{-\frac{\text{arcsech}(ax)}{3}+\frac{a^4x^4\,\text{arcsech}(ax)^2}{4}-\frac{\text{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,a^3x^3}{6}-\frac{\text{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax}{3}-\frac{a^2x^2}{12}+\frac{\ln\left(1+\left(\frac{1}{ax}+\sqrt{\right.\right.}{}}{a^4}$
default	$\frac{-\frac{\text{arcsech}(ax)}{3}+\frac{a^4x^4\,\text{arcsech}(ax)^2}{4}-\frac{\text{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,a^3x^3}{6}-\frac{\text{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax}{3}-\frac{a^2x^2}{12}+\frac{\ln\left(1+\left(\frac{1}{ax}+\sqrt{\right.\right.}{}}{a^4}$

```
[In] int(x^3*arcsech(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/3*arcsech(a*x)+1/4*a^4*x^4*arcsech(a*x)^2-1/6*arcsech(a*x)*(-(a*x
-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a^3*x^3-1/3*arcsech(a*x)*(-(a*x-1)/a/x)^
(1/2)*((a*x+1)/a/x)^(1/2)*a*x-1/12*a^2*x^2+1/3*ln(1+(1/a/x+(1/a/x-1)^(1/2)*
(1+1/a/x)^(1/2))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20

$$\int x^3 \mathrm{sech}^{-1}(ax)^2\,dx$$
$$= \frac{3\,a^4x^4\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - a^2x^2 - 2\,(a^3x^3+2\,ax)\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) - 4\,\log(x)}{12\,a^4}$$

```
[In] integrate(x^3*arcsech(a*x)^2,x, algorithm="fricas")
[Out] 1/12*(3*a^4*x^4*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - a^2
*x^2 - 2*(a^3*x^3 + 2*a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a
^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) - 4*log(x))/a^4
```

Sympy [F]

$$\int x^3 \mathrm{sech}^{-1}(ax)^2\,dx = \int x^3\,\mathrm{asech}^2\,(ax)\,dx$$

```
[In] integrate(x**3*asech(a*x)**2,x)
[Out] Integral(x**3*asech(a*x)**2, x)
```

Maxima [F]

$$\int x^3 \mathrm{sech}^{-1}(ax)^2\,dx = \int x^3\,\mathrm{arsech}\,(ax)^2\,dx$$

```
[In] integrate(x^3*arcsech(a*x)^2,x, algorithm="maxima")
[Out] integrate(x^3*arcsech(a*x)^2, x)
```

Giac [F]

$$\int x^3 \text{sech}^{-1}(ax)^2 \, dx = \int x^3 \operatorname{arsech}(ax)^2 \, dx$$

```
[In] integrate(x^3*arcsech(a*x)^2,x, algorithm="giac")
[Out] integrate(x^3*arcsech(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{sech}^{-1}(ax)^2 \, dx = \int x^3 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^2 \, dx$$

```
[In] int(x^3*acosh(1/(a*x))^2,x)
[Out] int(x^3*acosh(1/(a*x))^2, x)
```

3.3 $\int x^2 \text{sech}^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int x^2 \text{sech}^{-1}(ax)^2 \, dx = -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{2\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{3a^3} + \frac{i\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{3a^3} - \frac{i\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{3a^3}$$

```
[Out] -1/3*x/a^2+1/3*x^3*arcsech(a*x)^2-2/3*arcsech(a*x)*arctan(1/a/x+(1/a/x-1)^(
1/2)*(1+1/a/x)^(1/2))/a^3+1/3*I*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/
x)^(1/2)))/a^3-1/3*I*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a
^3-1/3*x*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6420, 5526, 4270, 4265, 2317, 2438}

$$\int x^2 \text{sech}^{-1}(ax)^2 \, dx = -\frac{2\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{3a^3} + \frac{i\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{3a^3} - \frac{i\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{3a^3} - \frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2$$

```
[In] Int[x^2*ArcSech[a*x]^2,x]
```

```
[Out] -1/3*x/a^2 - (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(3*a^2) +
 (x^3*ArcSech[a*x]^2)/3 - (2*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]])/(3*a^3) +
 ((I/3)*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^3 - ((I/3)*PolyLog[2, I*E^ArcSec
h[a*x]])/a^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
```

```
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^2 \text{sech}^3(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}$$
$$= \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{2\text{Subst}\left(\int x\text{sech}^3(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{3a^3}$$
$$= -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{\text{Subst}\left(\int x\text{sech}(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{3a^3}$$
$$= -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{2\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{3a^3} + \frac{i\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{3a^3} - \frac{i\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{3a^3}$$
$$= -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{2\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{3a^3} + \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{sech}^{-1}(ax)}\right)}{3a^3} - \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{sech}^{-1}(ax)}\right)}{3a^3}$$
$$= -\frac{x}{3a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{3a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^2 - \frac{2\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{3a^3} + \frac{i\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{3a^3} - \frac{i\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{3a^3}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int x^2\text{sech}^{-1}(ax)^2\,dx$$
$$= \frac{-ax - ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax) + a^3x^3\text{sech}^{-1}(ax)^2 + i\text{sech}^{-1}(ax)\log\left(1-ie^{-\text{sech}^{-1}(ax)}\right) - i\text{sech}^{-1}(a}{3a^3}$$

```
[In] Integrate[x^2*ArcSech[a*x]^2,x]
```

[Out] (-(a*x) - a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + a^3*x^3*ArcSech[a*x]^2 + I*ArcSech[a*x]*Log[1 - I/E^ArcSech[a*x]] - I*ArcSech[a*x]*Log[1 + I/E^ArcSech[a*x]] + I*PolyLog[2, (-I)/E^ArcSech[a*x]] - I*PolyLog[2, I/E^ArcSech[a*x]])/(3*a^3)

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.97

method	result
derivativedivides	$\frac{\frac{\left(a^2x^2\operatorname{arcsech}(ax)^2-\operatorname{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,ax-1\right)ax}{3}+\frac{i\operatorname{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\sqrt{1+\frac{1}{ax}}\right)\right)}{3}-\frac{i\operatorname{arcsech}(ax)\ln\left(1-i\right.}{}}{a^3}$
default	$\frac{\frac{\left(a^2x^2\operatorname{arcsech}(ax)^2-\operatorname{arcsech}(ax)\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,ax-1\right)ax}{3}+\frac{i\operatorname{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\sqrt{1+\frac{1}{ax}}\right)\right)}{3}-\frac{i\operatorname{arcsech}(ax)\ln\left(1-i\right.}{}}{a^3}$

[In] int(x^2*arcsech(a*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/a^3*(1/3*(a^2*x^2*arcsech(a*x)^2-arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a*x-1)*a*x+1/3*I*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-1/3*I*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))+1/3*I*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-1/3*I*dilog(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))))

Fricas [F]

$$\int x^2\text{sech}^{-1}(ax)^2\,dx=\int x^2\operatorname{arsech}\left(ax\right)^2\,dx$$

[In] integrate(x^2*arcsech(a*x)^2,x, algorithm="fricas")

[Out] integral(x^2*arcsech(a*x)^2, x)

Sympy [F]

$$\int x^2\text{sech}^{-1}(ax)^2\,dx=\int x^2\operatorname{asech}^2\left(ax\right)dx$$

[In] integrate(x**2*asech(a*x)**2,x)

[Out] Integral(x**2*asech(a*x)**2, x)

Maxima [F]

$$\int x^2 \mathrm{sech}^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arsech}(ax)^2 \, dx$$

```
[In] integrate(x^2*arcsech(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsech(a*x)^2, x)
```

Giac [F]

$$\int x^2 \mathrm{sech}^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arsech}(ax)^2 \, dx$$

```
[In] integrate(x^2*arcsech(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsech(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \mathrm{sech}^{-1}(ax)^2 \, dx = \int x^2 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^2 \, dx$$

```
[In] int(x^2*acosh(1/(a*x))^2,x)
```

```
[Out] int(x^2*acosh(1/(a*x))^2, x)
```

3.4 $\int x\text{sech}^{-1}(ax)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int x\text{sech}^{-1}(ax)^2\,dx = -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^2 - \frac{\log(x)}{a^2}$$

```
[Out] 1/2*x^2*arcsech(a*x)^2-ln(x)/a^2-(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1
/2)/a^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6420, 5526, 4269, 3556}

$$\int x\text{sech}^{-1}(ax)^2\,dx = -\frac{\log(x)}{a^2} - \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^2$$

```
[In] Int[x*ArcSech[a*x]^2,x]
```

```
[Out] -((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/a^2) + (x^2*ArcSech[a*
x]^2)/2 - Log[x]/a^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^2\text{sech}^2(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}\\
&= \frac{1}{2}x^2\text{sech}^{-1}(ax)^2 - \frac{\text{Subst}\left(\int x\text{sech}^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}\\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^2 + \frac{\text{Subst}\left(\int \tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}\\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^2 - \frac{\log(x)}{a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int x\text{sech}^{-1}(ax)^2\,dx = -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^2 - \frac{\log(x)}{a^2}$$

```
[In] Integrate[x*ArcSech[a*x]^2,x]
```

```
[Out] -((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/a^2) + (x^2*ArcSech[a*
x]^2)/2 - Log[x]/a^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.89

method	result	size
derivativedivides	$\frac{-2\,\mathrm{arcsech}(ax)+\frac{\mathrm{arcsech}(ax)\left(\mathrm{arcsech}(ax)a^2x^2-2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax+2\right)}{2}+\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^2\right)}{a^2}$	100
default	$\frac{-2\,\mathrm{arcsech}(ax)+\frac{\mathrm{arcsech}(ax)\left(\mathrm{arcsech}(ax)a^2x^2-2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax+2\right)}{2}+\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^2\right)}{a^2}$	100

```
[In] int(x*arcsech(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/a^2*(-2*arcsech(a*x)+1/2*arcsech(a*x)*(arcsech(a*x)*a^2*x^2-2*(-(a*x-1)/a
/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a*x+2)+ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^
(1/2))^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(49) = 98$.

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int x\mathrm{sech}^{-1}(ax)^2\,dx = \frac{a^2x^2\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2-2\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)-2\,\log(x)}{2\,a^2}$$

```
[In] integrate(x*arcsech(a*x)^2,x, algorithm="fricas")
[Out] 1/2*(a^2*x^2*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 2*a*x*
sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)
/(a*x)) - 2*log(x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x\mathrm{sech}^{-1}(ax)^2\,dx = \begin{cases} \frac{x^2\,\mathrm{asech}^2(ax)}{2}-\frac{\sqrt{-a^2x^2+1}\,\mathrm{asech}(ax)}{a^2}-\frac{\log(x)}{a^2} & \text{for } a\neq 0 \\ \infty x^2 & \text{otherwise} \end{cases}$$

```
[In] integrate(x*asech(a*x)**2,x)
[Out] Piecewise((x**2*asech(a*x)**2/2 - sqrt(-a**2*x**2 + 1)*asech(a*x)/a**2 - lo
g(x)/a**2, Ne(a, 0)), (oo*x**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int x\mathrm{sech}^{-1}(ax)^2\,dx = \frac{1}{2}\,x^2\,\mathrm{arsech}\left(ax\right)^2 - \frac{x\sqrt{\frac{1}{a^2x^2}-1}\,\mathrm{arsech}\left(ax\right)}{a} - \frac{\log\left(x\right)}{a^2}$$

```
[In] integrate(x*arcsech(a*x)^2,x, algorithm="maxima")
[Out] 1/2*x^2*arcsech(a*x)^2 - x*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x)/a - log(x)/a^
2
```

Giac [F]

$$\int x\mathrm{sech}^{-1}(ax)^2\,dx = \int x\,\mathrm{arsech}\left(ax\right)^2\,dx$$

```
[In] integrate(x*arcsech(a*x)^2,x, algorithm="giac")
[Out] integrate(x*arcsech(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{sech}^{-1}(ax)^2\,dx = \int x\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)^2\,dx$$

```
[In] int(x*acosh(1/(a*x))^2,x)
[Out] int(x*acosh(1/(a*x))^2, x)
```

3.5 $\int \operatorname{sech}^{-1}(ax)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 63

$$\int \operatorname{sech}^{-1}(ax)^2\,dx = x\operatorname{sech}^{-1}(ax)^2 - \frac{4\operatorname{sech}^{-1}(ax)\arctan\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{2i\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{2i\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

```
[Out] x*arcsech(a*x)^2-4*arcsech(a*x)*arctan(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2
))/a+2*I*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a-2*I*polylo
g(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6414, 5526, 4265, 2317, 2438}

$$\int \operatorname{sech}^{-1}(ax)^2\,dx = -\frac{4\operatorname{sech}^{-1}(ax)\arctan\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{2i\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{2i\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} + x\operatorname{sech}^{-1}(ax)^2$$

```
[In] Int[ArcSech[a*x]^2,x]
```

```
[Out] x*ArcSech[a*x]^2 - (4*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]])/a + ((2*I)*PolyL
og[2, (-I)*E^ArcSech[a*x]])/a - ((2*I)*PolyLog[2, I*E^ArcSech[a*x]])/a
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6414

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^2\text{sech}(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}\\
&= x\text{sech}^{-1}(ax)^2 - \frac{2\text{Subst}\left(\int x\text{sech}(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}\\
&= x\text{sech}^{-1}(ax)^2 - \frac{4\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a}\\
&\quad + \frac{(2i)\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}\\
&\quad - \frac{(2i)\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}
\end{aligned}
$$

$$= x\text{sech}^{-1}(ax)^2 - \frac{4\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a} + \frac{(2i)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\text{sech}^{-1}(ax)}\right)}{a} - \frac{(2i)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\text{sech}^{-1}(ax)}\right)}{a}$$

$$= x\text{sech}^{-1}(ax)^2 - \frac{4\text{sech}^{-1}(ax)\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a} + \frac{2i\,\text{PolyLog}\left(2, -ie^{\text{sech}^{-1}(ax)}\right)}{a} - \frac{2i\,\text{PolyLog}\left(2, ie^{\text{sech}^{-1}(ax)}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \text{sech}^{-1}(ax)^2\,dx = \frac{i\left(\text{sech}^{-1}(ax)\left(-iax\text{sech}^{-1}(ax) + 2\log\left(1 - ie^{-\text{sech}^{-1}(ax)}\right) - 2\log\left(1 + ie^{-\text{sech}^{-1}(ax)}\right)\right) + 2\,\text{PolyLog}\left(2, -ie\right.\right.}{a}$$

```
[In] Integrate[ArcSech[a*x]^2,x]
[Out] (I*(ArcSech[a*x]*((-I)*a*x*ArcSech[a*x] + 2*Log[1 - I/E^ArcSech[a*x]] - 2*L
og[1 + I/E^ArcSech[a*x]]) + 2*PolyLog[2, (-I)/E^ArcSech[a*x]] - 2*PolyLog[2
, I/E^ArcSech[a*x]]))/a
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.90

method	result
derivativedivides	$\frac{\text{arcsech}(ax)^2 ax+2i\,\text{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)-2i\,\text{arcsech}(ax)\ln\left(1-i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)+2i\,\text{di}}{a}$
default	$\frac{\text{arcsech}(ax)^2 ax+2i\,\text{arcsech}(ax)\ln\left(1+i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)-2i\,\text{arcsech}(ax)\ln\left(1-i\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)\right)+2i\,\text{di}}{a}$

```
[In] int(arcsech(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/a*(arcsech(a*x)^2*a*x+2*I*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1
/a/x)^(1/2)))-2*I*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2
)))+2*I*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-2*I*dilog(1-I*(1
/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))))
```

Fricas [F]

$$\int \operatorname{sech}^{-1}(ax)^2 \, dx = \int \operatorname{arsech}\left(ax\right)^2 \, dx$$

```
[In] integrate(arcsech(a*x)^2,x, algorithm="fricas")
[Out] integral(arcsech(a*x)^2, x)
```

Sympy [F]

$$\int \operatorname{sech}^{-1}(ax)^2 \, dx = \int \operatorname{asech}^2\left(ax\right) dx$$

```
[In] integrate(asech(a*x)**2,x)
[Out] Integral(asech(a*x)**2, x)
```

Maxima [F]

$$\int \operatorname{sech}^{-1}(ax)^2 \, dx = \int \operatorname{arsech}\left(ax\right)^2 \, dx$$

```
[In] integrate(arcsech(a*x)^2,x, algorithm="maxima")
[Out] x*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^2 - integrate(-(a^2*x^2*log(a)^2 +
(a^2*x^2 - 1)*log(x)^2 + (a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 - log(a
)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - 2*
(a^2*x^2*log(a) + (a^2*x^2*(log(a) + 1) + (a^2*x^2 - 1)*log(x) - log(a))*sq
rt(a*x + 1)*sqrt(-a*x + 1) + (a^2*x^2 - 1)*log(x) - log(a))*log(sqrt(a*x +
1)*sqrt(-a*x + 1) + 1) - log(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))/(a^
2*x^2 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 1), x)
```

Giac [F]

$$\int \operatorname{sech}^{-1}(ax)^2 \, dx = \int \operatorname{arsech}\left(ax\right)^2 \, dx$$

```
[In] integrate(arcsech(a*x)^2,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{-1}(ax)^2\,dx=\int \operatorname{acosh}\left(\frac{1}{a\,x}\right)^2\,dx$$

```
[In] int(acosh(1/(a*x))^2,x)
[Out] int(acosh(1/(a*x))^2, x)
```

3.6 $\int \frac{\text{sech}^{-1}(ax)^2}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 64

$$\int \frac{\text{sech}^{-1}(ax)^2}{x} dx = \frac{1}{3}\text{sech}^{-1}(ax)^3 - \text{sech}^{-1}(ax)^2 \log\left(1 + e^{2\text{sech}^{-1}(ax)}\right) - \text{sech}^{-1}(ax)\,\text{PolyLog}\left(2, -e^{2\text{sech}^{-1}(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3, -e^{2\text{sech}^{-1}(ax)}\right)$$

```
[Out] 1/3*arcsech(a*x)^3-arcsech(a*x)^2*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)-arcsech(a*x)*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)+1
/2*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.07 (sec), antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6420, 3799, 2221, 2611, 2320, 6724}

$$\int \frac{\text{sech}^{-1}(ax)^2}{x} dx = -\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2, -e^{2\text{sech}^{-1}(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3, -e^{2\text{sech}^{-1}(ax)}\right) + \frac{1}{3}\text{sech}^{-1}(ax)^3 - \text{sech}^{-1}(ax)^2 \log\left(e^{2\text{sech}^{-1}(ax)} + 1\right)$$

```
[In] Int[ArcSech[a*x]^2/x,x]
```

```
[Out] ArcSech[a*x]^3/3 - ArcSech[a*x]^2*Log[1 + E^(2*ArcSech[a*x])] - ArcSech[a*x
]*PolyLog[2, -E^(2*ArcSech[a*x])] + PolyLog[3, -E^(2*ArcSech[a*x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)$$

$$= \frac{1}{3}\text{sech}^{-1}(ax)^3 - 2\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(ax)\right)$$

$$
\begin{aligned}
&= \frac{1}{3}\text{sech}^{-1}(ax)^3 - \text{sech}^{-1}(ax)^2 \log\left(1 + e^{2\text{sech}^{-1}(ax)}\right) \\
&\quad + 2\text{Subst}\left(\int x \log\left(1 + e^{2x}\right)\,dx, x, \text{sech}^{-1}(ax)\right) \\
&= \frac{1}{3}\text{sech}^{-1}(ax)^3 - \text{sech}^{-1}(ax)^2 \log\left(1 + e^{2\text{sech}^{-1}(ax)}\right) \\
&\quad - \text{sech}^{-1}(ax)\,\text{PolyLog}\left(2, -e^{2\text{sech}^{-1}(ax)}\right) \\
&\quad + \text{Subst}\left(\int \text{PolyLog}\left(2, -e^{2x}\right)\,dx, x, \text{sech}^{-1}(ax)\right) \\
&= \frac{1}{3}\text{sech}^{-1}(ax)^3 - \text{sech}^{-1}(ax)^2 \log\left(1 + e^{2\text{sech}^{-1}(ax)}\right) \\
&\quad - \text{sech}^{-1}(ax)\,\text{PolyLog}\left(2, -e^{2\text{sech}^{-1}(ax)}\right) \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{\text{PolyLog}(2, -x)}{x}\,dx, x, e^{2\text{sech}^{-1}(ax)}\right) \\
&= \frac{1}{3}\text{sech}^{-1}(ax)^3 - \text{sech}^{-1}(ax)^2 \log\left(1 + e^{2\text{sech}^{-1}(ax)}\right) \\
&\quad - \text{sech}^{-1}(ax)\,\text{PolyLog}\left(2, -e^{2\text{sech}^{-1}(ax)}\right) + \frac{1}{2}\,\text{PolyLog}\left(3, -e^{2\text{sech}^{-1}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int \frac{\text{sech}^{-1}(ax)^2}{x}\,dx &= -\frac{1}{3}\text{sech}^{-1}(ax)^3 - \text{sech}^{-1}(ax)^2 \log\left(1 + e^{-2\text{sech}^{-1}(ax)}\right) \\
&\quad + \text{sech}^{-1}(ax)\,\text{PolyLog}\left(2, -e^{-2\text{sech}^{-1}(ax)}\right) \\
&\quad + \frac{1}{2}\,\text{PolyLog}\left(3, -e^{-2\text{sech}^{-1}(ax)}\right)
\end{aligned}
$$

```
[In] Integrate[ArcSech[a*x]^2/x,x]
```

```
[Out] -1/3*ArcSech[a*x]^3 - ArcSech[a*x]^2*Log[1 + E^(-2*ArcSech[a*x])] + ArcSech
[a*x]*PolyLog[2, -E^(-2*ArcSech[a*x])] + PolyLog[3, -E^(-2*ArcSech[a*x])]/2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.12

method	result
derivativedivides	$\frac{\operatorname{arcsech}(ax)^3}{3} - \operatorname{arcsech}(ax)^2 \ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^2\right) - \operatorname{arcsech}(ax)\operatorname{polylog}($
default	$\frac{\operatorname{arcsech}(ax)^3}{3} - \operatorname{arcsech}(ax)^2 \ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^2\right) - \operatorname{arcsech}(ax)\operatorname{polylog}($

```
[In] int(arcsech(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arcsech(a*x)^3-arcsech(a*x)^2*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)-arcsech(a*x)*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)+1
/2*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x}\,dx$$

```
[In] integrate(arcsech(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsech(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{asech}^2(ax)}{x}\,dx$$

```
[In] integrate(asech(a*x)**2/x,x)
```

```
[Out] Integral(asech(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x}\,dx$$

```
[In] integrate(arcsech(a*x)^2/x,x, algorithm="maxima")
[Out] integrate(arcsech(a*x)^2/x, x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x}\,dx$$

```
[In] integrate(arcsech(a*x)^2/x,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x}\,dx$$

```
[In] int(acosh(1/(a*x))^2/x,x)
[Out] int(acosh(1/(a*x))^2/x, x)
```

3.7 $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 49

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2} dx = -\frac{2}{x} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{x} - \frac{\operatorname{sech}^{-1}(ax)^2}{x}$$

```
[Out] -2/x-arcsech(a*x)^2/x+2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6420, 3377, 2718}

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2} dx = -\frac{\operatorname{sech}^{-1}(ax)^2}{x} + \frac{2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{x} - \frac{2}{x}$$

```
[In] Int[ArcSech[a*x]^2/x^2,x]
```

```
[Out] -2/x + (2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/x - ArcSech[a*x
]^2/x
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\left(a\text{Subst}\left(\int x^2\sinh(x)\,dx, x, \text{sech}^{-1}(ax)\right)\right)$$
$$= -\frac{\text{sech}^{-1}(ax)^2}{x} + (2a)\text{Subst}\left(\int x\cosh(x)\,dx, x, \text{sech}^{-1}(ax)\right)$$
$$= \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{x} - \frac{\text{sech}^{-1}(ax)^2}{x} - (2a)\text{Subst}\left(\int \sinh(x)\,dx, x, \text{sech}^{-1}(ax)\right)$$
$$= -\frac{2}{x} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{x} - \frac{\text{sech}^{-1}(ax)^2}{x}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^2}\,dx = -\frac{2-2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)+\text{sech}^{-1}(ax)^2}{x}$$

```
[In] Integrate[ArcSech[a*x]^2/x^2,x]
[Out] -((2 - 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + ArcSech[a*x]^2)
/x)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$a\left(-\frac{\operatorname{arcsech}(ax)^2}{ax}+2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)-\frac{2}{ax}\right)$	61
default	$a\left(-\frac{\operatorname{arcsech}(ax)^2}{ax}+2\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)-\frac{2}{ax}\right)$	61

```
[In] int(arcsech(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/a/x*arcsech(a*x)^2+2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*arcsech
(a*x)-2/a/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.98

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = \frac{2\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)-\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2-2}{x}$$

```
[In] integrate(arcsech(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2
)) + 1)/(a*x)) - log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 2)
/x
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{asech}^2(ax)}{x^2}\,dx$$

```
[In] integrate(asech(a*x)**2/x**2,x)
```

```
[Out] Integral(asech(a*x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = 2\,a\sqrt{\frac{1}{a^2x^2}-1}\,\operatorname{arsech}\left(ax\right) - \frac{\operatorname{arsech}\left(ax\right)^2}{x} - \frac{2}{x}$$

```
[In] integrate(arcsech(a*x)^2/x^2,x, algorithm="maxima")
[Out] 2*a*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x) - arcsech(a*x)^2/x - 2/x
```

Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arsech}\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arcsech(a*x)^2/x^2,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x^2}\,dx$$

```
[In] int(acosh(1/(a*x))^2/x^2,x)
[Out] int(acosh(1/(a*x))^2/x^2, x)
```

3.8 $\int \frac{\text{sech}^{-1}(ax)^2}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 90

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^3} dx = -\frac{(1-ax)(1+ax)}{4x^2} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{2x^2} - \frac{1}{4}a^2\text{sech}^{-1}(ax)^2 - \frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^2}{2x^2}$$

```
[Out] -1/4*(-a*x+1)*(a*x+1)/x^2-1/4*a^2*arcsech(a*x)^2-1/2*(-a*x+1)*(a*x+1)*arcse
ch(a*x)^2/x^2+1/2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6420, 5480, 3391, 30}

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^3} dx = -\frac{1}{4}a^2\text{sech}^{-1}(ax)^2 - \frac{(1-ax)(ax+1)}{4x^2} - \frac{(1-ax)(ax+1)\text{sech}^{-1}(ax)^2}{2x^2} + \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{2x^2}$$

```
[In] Int[ArcSech[a*x]^2/x^3,x]
```

```
[Out] -1/4*((1 - a*x)*(1 + a*x))/x^2 + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSe
ch[a*x])/(2*x^2) - (a^2*ArcSech[a*x]^2)/4 - ((1 - a*x)*(1 + a*x)*ArcSech[a*
x]^2)/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(a^2\text{Subst}\left(\int x^2\cosh(x)\sinh(x)\,dx, x, \text{sech}^{-1}(ax)\right)\right)\\
&= -\frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^2}{2x^2} + a^2\text{Subst}\left(\int x\sinh^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= -\frac{(1-ax)(1+ax)}{4x^2} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{2x^2}\\
&\quad -\frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^2\text{Subst}\left(\int x\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= -\frac{(1-ax)(1+ax)}{4x^2} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{2x^2}\\
&\quad -\frac{1}{4}a^2\text{sech}^{-1}(ax)^2 - \frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^2}{2x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \frac{-1+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)+(-2+a^2x^2)\operatorname{sech}^{-1}(ax)^2}{4x^2}$$

```
[In] Integrate[ArcSech[a*x]^2/x^3,x]
```

```
[Out] (-1 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + (-2 + a^2*x^2)*A
rcSech[a*x]^2)/(4*x^2)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$a^2\left(-\frac{\operatorname{arcsech}(ax)^2}{2a^2x^2}+\frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\operatorname{arcsech}(ax)}{2ax}+\frac{\operatorname{arcsech}(ax)^2}{4}-\frac{1}{4a^2x^2}\right)$	77
default	$a^2\left(-\frac{\operatorname{arcsech}(ax)^2}{2a^2x^2}+\frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\operatorname{arcsech}(ax)}{2ax}+\frac{\operatorname{arcsech}(ax)^2}{4}-\frac{1}{4a^2x^2}\right)$	77

```
[In] int(arcsech(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2/a^2/x^2*arcsech(a*x)^2+1/2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/
2)/a/x*arcsech(a*x)+1/4*arcsech(a*x)^2-1/4/a^2/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \frac{2\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)+(a^2x^2-2)\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2-1}{4\,x^2}$$

```
[In] integrate(arcsech(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2
*x^2)) + 1)/(a*x)) + (a^2*x^2 - 2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))
+ 1)/(a*x))^2 - 1)/x^2
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{asech}^2(ax)}{x^3}\,dx$$

```
[In] integrate(asech(a*x)**2/x**3,x)
[Out] Integral(asech(a*x)**2/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x^3}\,dx$$

```
[In] integrate(arcsech(a*x)^2/x^3,x, algorithm="maxima")
[Out] integrate(arcsech(a*x)^2/x^3, x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x^3}\,dx$$

```
[In] integrate(arcsech(a*x)^2/x^3,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x^3}\,dx$$

```
[In] int(acosh(1/(a*x))^2/x^3,x)
[Out] int(acosh(1/(a*x))^2/x^3, x)
```

3.9 $\int \frac{\text{sech}^{-1}(ax)^2}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 102

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^4} dx = -\frac{2}{27x^3} - \frac{4a^2}{9x} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{9x^3} + \frac{4a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{9x} - \frac{\text{sech}^{-1}(ax)^2}{3x^3}$$

```
[Out] -2/27/x^3-4/9*a^2/x-1/3*arcsech(a*x)^2/x^3+2/9*(a*x+1)*arcsech(a*x)*((-a*x+
1)/(a*x+1))^(1/2)/x^3+4/9*a^2*(a*x+1)*arcsech(a*x)*((-a*x+1)/(a*x+1))^(1/2)
/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6420, 5481, 3391, 3377, 2718}

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^4} dx = -\frac{4a^2}{9x} + \frac{4a^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{9x} - \frac{\text{sech}^{-1}(ax)^2}{3x^3} + \frac{2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)}{9x^3} - \frac{2}{27x^3}$$

```
[In] Int[ArcSech[a*x]^2/x^4,x]
```

```
[Out] -2/(27*x^3) - (4*a^2)/(9*x) + (2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSec
h[a*x])/(9*x^3) + (4*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/
(9*x) - ArcSech[a*x]^2/(3*x^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(a^3\text{Subst}\left(\int x^2\cosh^2(x)\sinh(x)\,dx,x,\operatorname{sech}^{-1}(ax)\right)\right)\\
&= -\frac{\operatorname{sech}^{-1}(ax)^2}{3x^3}+\frac{1}{3}\left(2a^3\right)\text{Subst}\left(\int x\cosh^3(x)\,dx,x,\operatorname{sech}^{-1}(ax)\right)\\
&= -\frac{2}{27x^3}+\frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x^3}-\frac{\operatorname{sech}^{-1}(ax)^2}{3x^3}\\
&\quad+\frac{1}{9}\left(4a^3\right)\text{Subst}\left(\int x\cosh(x)\,dx,x,\operatorname{sech}^{-1}(ax)\right)
\end{aligned}
$$

$$= -\frac{2}{27x^3} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{9x^3} + \frac{4a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{9x} - \frac{\text{sech}^{-1}(ax)^2}{3x^3} - \frac{1}{9}(4a^3)\,\text{Subst}\left(\int \sinh(x)\,dx, x, \text{sech}^{-1}(ax)\right)$$

$$= -\frac{2}{27x^3} - \frac{4a^2}{9x} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{9x^3} + \frac{4a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)}{9x} - \frac{\text{sech}^{-1}(ax)^2}{3x^3}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\text{sech}^{-1}(ax)^2}{x^4}\,dx = \frac{-2(1+6a^2x^2) + 6\sqrt{\frac{1-ax}{1+ax}}(1+ax+2a^2x^2+2a^3x^3)\,\text{sech}^{-1}(ax) - 9\text{sech}^{-1}(ax)^2}{27x^3}$$

```
[In] Integrate[ArcSech[a*x]^2/x^4,x]
```

```
[Out] (-2*(1 + 6*a^2*x^2) + 6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 2*a^2*x^2 + 2*
a^3*x^3)*ArcSech[a*x] - 9*ArcSech[a*x]^2)/(27*x^3)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

method	result
derivativedivides	$a^3\left(-\frac{\text{arcsech}(ax)^2}{3a^3x^3} + \frac{4\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,\text{arcsech}(ax)}{9} + \frac{2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,\text{arcsech}(ax)}{9a^2x^2} - \frac{4}{9ax} - \frac{2}{27x^3a^3}\right)$
default	$a^3\left(-\frac{\text{arcsech}(ax)^2}{3a^3x^3} + \frac{4\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,\text{arcsech}(ax)}{9} + \frac{2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,\text{arcsech}(ax)}{9a^2x^2} - \frac{4}{9ax} - \frac{2}{27x^3a^3}\right)$

```
[In] int(arcsech(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3/a^3/x^3*arcsech(a*x)^2+4/9*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/
2)*arcsech(a*x)+2/9/a^2/x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*arcsec
h(a*x)-4/9/a/x-2/27/x^3/a^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx$$
$$= -\frac{12\,a^2x^2 - 6\,(2\,a^3x^3 + ax)\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) + 9\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 + 2}{27\,x^3}$$

```
[In] integrate(arcsech(a*x)^2/x^4,x, algorithm="fricas")
[Out] -1/27*(12*a^2*x^2 - 6*(2*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log(
(a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) + 9*log((a*x*sqrt(-(a^2*x^2
 - 1)/(a^2*x^2)) + 1)/(a*x))^2 + 2)/x^3
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{asech}^2(ax)}{x^4}\,dx$$

```
[In] integrate(asech(a*x)**2/x**4,x)
[Out] Integral(asech(a*x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x^4}\,dx$$

```
[In] integrate(arcsech(a*x)^2/x^4,x, algorithm="maxima")
[Out] integrate(arcsech(a*x)^2/x^4, x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arsech}(ax)^2}{x^4}\,dx$$

```
[In] integrate(arcsech(a*x)^2/x^4,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^2}{x^4}\,dx$$

```
[In] int(acosh(1/(a*x))^2/x^4,x)
[Out] int(acosh(1/(a*x))^2/x^4, x)
```

3.10 $\int x^4 \text{sech}^{-1}(ax)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 297

$$\begin{aligned}\int x^4\text{sech}^{-1}(ax)^3\,dx =& \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4}-\frac{9x\text{sech}^{-1}(ax)}{20a^4}-\frac{x^3\text{sech}^{-1}(ax)}{10a^2}\\&-\frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4}-\frac{3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{20a^2}\\&+\frac{1}{5}x^5\text{sech}^{-1}(ax)^3-\frac{9\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{20a^5}\\&+\frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{2a^5}+\frac{9i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}\\&-\frac{9i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}\\&-\frac{9i\operatorname{PolyLog}\left(3,-ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}+\frac{9i\operatorname{PolyLog}\left(3,ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}\end{aligned}$$

```
[Out] -9/20*x*arcsech(a*x)/a^4-1/10*x^3*arcsech(a*x)/a^2+1/5*x^5*arcsech(a*x)^3-9
/20*arcsech(a*x)^2*arctan(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/a^5+1/2*ar
ctan((a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/a/x)/a^5+9/20*I*arcsech(a*x)*polylog(
2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5-9/20*I*arcsech(a*x)*polyl
og(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5-9/20*I*polylog(3,-I*(1/
a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^5+9/20*I*polylog(3,I*(1/a/x+(1/a/x-
1)^(1/2)*(1+1/a/x)^(1/2)))/a^5+1/20*x*(a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/a^4-
9/40*x*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^4-3/20*x^3*(a*x+1)
*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6420, 5526, 4271, 3853, 3855, 4265, 2611, 2320, 6724}

$$\int x^4 \operatorname{sech}^{-1}(ax)^3\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}\right)}{2a^5} - \frac{9\operatorname{sech}^{-1}(ax)^2\arctan\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} + \frac{9i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i\operatorname{PolyLog}\left(3,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} + \frac{9i\operatorname{PolyLog}\left(3,ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} + \frac{x\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{20a^4} - \frac{9x\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{40a^4} - \frac{9x\operatorname{sech}^{-1}(ax)}{20a^4} - \frac{3x^3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{20a^2} - \frac{x^3\operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5}x^5\operatorname{sech}^{-1}(ax)^3$$

```
[In] Int[x^4*ArcSech[a*x]^3,x]
```

```
[Out] (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(20*a^4) - (9*x*ArcSech[a*x])/(20*a
^4) - (x^3*ArcSech[a*x])/(10*a^2) - (9*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x
)*ArcSech[a*x]^2)/(40*a^4) - (3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*Arc
Sech[a*x]^2)/(20*a^2) + (x^5*ArcSech[a*x]^3)/5 - (9*ArcSech[a*x]^2*ArcTan[E
^ArcSech[a*x]])/(20*a^5) + ArcTan[(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*
x)]/(2*a^5) + (((9*I)/20)*ArcSech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^5
 - (((9*I)/20)*ArcSech[a*x]*PolyLog[2, I*E^ArcSech[a*x]])/a^5 - (((9*I)/20)
*PolyLog[3, (-I)*E^ArcSech[a*x]])/a^5 + (((9*I)/20)*PolyLog[3, I*E^ArcSech[
a*x]])/a^5
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
```

```
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^3 \text{sech}^5(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^5}$$

$$= \frac{1}{5}x^5\text{sech}^{-1}(ax)^3 - \frac{3\text{Subst}\left(\int x^2\text{sech}^5(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{5a^5}$$

$$= -\frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{20a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^3 + \frac{\text{Subst}\left(\int \text{sech}^3(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{10a^5} - \frac{9\text{Subst}\left(\int x^2\text{sech}^3(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{20a^5}$$

$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{20a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^3 + \frac{\text{Subst}\left(\int \text{sech}(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{20a^5} - \frac{9\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{40a^5} + \frac{9\text{Subst}\left(\int \text{sech}(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{20a^5}$$

$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{20a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^3 - \frac{9\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{20a^5} + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{2a^5} + \frac{(9i)\text{Subst}\left(\int x\log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{20a^5} - \frac{(9i)\text{Subst}\left(\int x\log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{20a^5}$$

$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{20a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^3 - \frac{9\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{20a^5} + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{2a^5} + \frac{9i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{20a^5} - \frac{(9i)\text{Subst}\left(\int \operatorname{PolyLog}(2,-ie^x)\,dx,x,\text{sech}^{-1}(ax)\right)}{20a^5} + \frac{(9i)\text{Subst}\left(\int \operatorname{PolyLog}(2,ie^x)\,dx,x,\text{sech}^{-1}(ax)\right)}{20a^5}$$

$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{20a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^3 - \frac{9\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{20a^5} + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{2a^5} + \frac{9i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{20a^5} - \frac{(9i)\text{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{\text{sech}^{-1}(ax)}\right)}{20a^5} + \frac{(9i)\text{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{\text{sech}^{-1}(ax)}\right)}{20a^5}$$

$$= \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{20a^4} - \frac{9x\text{sech}^{-1}(ax)}{20a^4} - \frac{x^3\text{sech}^{-1}(ax)}{10a^2} - \frac{9x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{20a^2} + \frac{1}{5}x^5\text{sech}^{-1}(ax)^3 - \frac{9\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{20a^5} + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{2a^5} + \frac{9i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i\text{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i\operatorname{PolyLog}\left(3,-ie^{\text{sech}^{-1}(ax)}\right)}{20a^5} + \frac{9i\operatorname{PolyLog}\left(3,ie^{\text{sech}^{-1}(ax)}\right)}{20a^5}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.95

$$\int x^4 \operatorname{sech}^{-1}(ax)^3 \, dx$$
$$= \frac{2ax\sqrt{\frac{1-ax}{1+ax}}(1+ax) - 18ax\operatorname{sech}^{-1}(ax) - 4a^3x^3\operatorname{sech}^{-1}(ax) - 9ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2 - 6a^3x^3\sqrt{\frac{1-ax}{1+ax}}}{}$$

```
[In] Integrate[x^4*ArcSech[a*x]^3,x]
```

```
[Out] (2*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 18*a*x*ArcSech[a*x] - 4*a^3*x^
3*ArcSech[a*x] - 9*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 -
 6*a^3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 + 8*a^5*x^5*A
rcSech[a*x]^3 + 40*ArcTan[Tanh[ArcSech[a*x]/2]] + (9*I)*ArcSech[a*x]^2*Log[
1 - I/E^ArcSech[a*x]] - (9*I)*ArcSech[a*x]^2*Log[1 + I/E^ArcSech[a*x]] + (1
8*I)*ArcSech[a*x]*PolyLog[2, (-I)/E^ArcSech[a*x]] - (18*I)*ArcSech[a*x]*Pol
yLog[2, I/E^ArcSech[a*x]] + (18*I)*PolyLog[3, (-I)/E^ArcSech[a*x]] - (18*I)
*PolyLog[3, I/E^ArcSech[a*x]])/(40*a^5)
```

Maple [F]

$$\int x^4 \operatorname{arcsech}(ax)^3 \, dx$$

```
[In] int(x^4*arcsech(a*x)^3,x)
```

```
[Out] int(x^4*arcsech(a*x)^3,x)
```

Fricas [F]

$$\int x^4 \operatorname{sech}^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arsech}(ax)^3 \, dx$$

```
[In] integrate(x^4*arcsech(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^4*arcsech(a*x)^3, x)
```

Sympy [F]

$$\int x^4 \operatorname{sech}^{-1}(ax)^3\,dx = \int x^4 \operatorname{asech}^3(ax)\,dx$$

```
[In] integrate(x**4*asech(a*x)**3,x)
[Out] Integral(x**4*asech(a*x)**3, x)
```

Maxima [F]

$$\int x^4 \operatorname{sech}^{-1}(ax)^3\,dx = \int x^4 \operatorname{arsech}(ax)^3\,dx$$

```
[In] integrate(x^4*arcsech(a*x)^3,x, algorithm="maxima")
[Out] integrate(x^4*arcsech(a*x)^3, x)
```

Giac [F]

$$\int x^4 \operatorname{sech}^{-1}(ax)^3\,dx = \int x^4 \operatorname{arsech}(ax)^3\,dx$$

```
[In] integrate(x^4*arcsech(a*x)^3,x, algorithm="giac")
[Out] integrate(x^4*arcsech(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{sech}^{-1}(ax)^3\,dx = \int x^4 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^3\,dx$$

```
[In] int(x^4*acosh(1/(a*x))^3,x)
[Out] int(x^4*acosh(1/(a*x))^3, x)
```

3.11 $\int x^3\text{sech}^{-1}(ax)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 184

$$\begin{aligned}\int x^3\text{sech}^{-1}(ax)^3\,dx=&\ \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{4a^4}-\frac{x^2\text{sech}^{-1}(ax)}{4a^2}\\&-\frac{\text{sech}^{-1}(ax)^2}{2a^4}-\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^4}\\&-\frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4a^2}+\frac{1}{4}x^4\text{sech}^{-1}(ax)^3\\&+\frac{\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^4}+\frac{\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)}{2a^4}\end{aligned}$$

```
[Out] -1/4*x^2*arcsech(a*x)/a^2-1/2*arcsech(a*x)^2/a^4+1/4*x^4*arcsech(a*x)^3+arc
sech(a*x)*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)/a^4+1/2*polylog(2
,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)/a^4+1/4*(a*x+1)*((-a*x+1)/(a*x
+1))^(1/2)/a^4-1/2*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^4-1/4*
x^2*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules

used = {6420, 5526, 4271, 3852, 8, 4269, 3799, 2221, 2317, 2438}

$$\int x^3\text{sech}^{-1}(ax)^3\,dx = \frac{\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)}{2a^4}+\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{4a^4}-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^4}-\frac{\text{sech}^{-1}(ax)^2}{2a^4}+\frac{\text{sech}^{-1}(ax)\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)}{a^4}-\frac{x^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{4a^2}-\frac{x^2\text{sech}^{-1}(ax)}{4a^2}+\frac{1}{4}x^4\text{sech}^{-1}(ax)^3$$

```
[In] Int[x^3*ArcSech[a*x]^3,x]
```

```
[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(4*a^4) - (x^2*ArcSech[a*x])/(4*a^2)
- ArcSech[a*x]^2/(2*a^4) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x
]^2)/(2*a^4) - (x^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(4*
a^2) + (x^4*ArcSech[a*x]^3)/4 + (ArcSech[a*x]*Log[1 + E^(2*ArcSech[a*x])])/
a^4 + PolyLog[2, -E^(2*ArcSech[a*x])]/(2*a^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^3\text{sech}^4(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^4}$$

$$= \frac{1}{4}x^4\text{sech}^{-1}(ax)^3 - \frac{3\text{Subst}\left(\int x^2\text{sech}^4(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{4a^4}$$

$$= -\frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^3 + \frac{\text{Subst}\left(\int \text{sech}^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{4a^4} - \frac{\text{Subst}\left(\int x^2\text{sech}^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{2a^4}$$

$$= -\frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^3 + \frac{i\text{Subst}\left(\int 1\,dx, x, -i\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{4a^4} + \frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^4}$$

$$= \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{4a^4} - \frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\text{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^3 + \frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(ax)\right)}{a^4}$$

$$= \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{4a^4} - \frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\text{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^3 + \frac{\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^4} - \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^4}$$

$$= \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{4a^4} - \frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\text{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^3 + \frac{\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^4} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{sech}^{-1}(ax)}\right)}{2a^4}$$

$$= \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{4a^4} - \frac{x^2\text{sech}^{-1}(ax)}{4a^2} - \frac{\text{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4}x^4\text{sech}^{-1}(ax)^3 + \frac{\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^4} + \frac{\text{PolyLog}\left(2, -e^{2\text{sech}^{-1}(ax)}\right)}{2a^4}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02

$$\int x^3 \operatorname{sech}^{-1}(ax)^3 \, dx$$
$$= \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax) - \left(-2+2\sqrt{\frac{1-ax}{1+ax}}+2ax\sqrt{\frac{1-ax}{1+ax}}+a^2x^2\sqrt{\frac{1-ax}{1+ax}}+a^3x^3\sqrt{\frac{1-ax}{1+ax}}\right)\operatorname{sech}^{-1}(ax)^2 + a^4x^4\operatorname{sech}^{-1}(a}{4a^4}$$

[In] Integrate[x^3*ArcSech[a*x]^3,x]

[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - (-2 + 2*Sqrt[(1 - a*x)/(1 + a*x)] + 2*a*x*Sqrt[(1 - a*x)/(1 + a*x)] + a^2*x^2*Sqrt[(1 - a*x)/(1 + a*x)] + a^3*x^3*Sqrt[(1 - a*x)/(1 + a*x)])*ArcSech[a*x]^2 + a^4*x^4*ArcSech[a*x]^3 + ArcSech[a*x]*(-(a^2*x^2) + 4*Log[1 + E^(-2*ArcSech[a*x])]) - 2*PolyLog[2, -E^(-2*ArcSech[a*x])])/(4*a^4)

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{a^4x^4 \operatorname{arcsech}(ax)^3}{4} - \frac{\operatorname{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,a^3x^3}{4} - \frac{\operatorname{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,ax}{2} - \frac{\operatorname{arcsech}(ax)a^2x^2}{4} + \frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+}{ax}}}{4}}{a^4}$
default	$\frac{\frac{a^4x^4 \operatorname{arcsech}(ax)^3}{4} - \frac{\operatorname{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,a^3x^3}{4} - \frac{\operatorname{arcsech}(ax)^2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\,ax}{2} - \frac{\operatorname{arcsech}(ax)a^2x^2}{4} + \frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+}{ax}}}{4}}{a^4}$

[In] int(x^3*arcsech(a*x)^3,x,method=_RETURNVERBOSE)

[Out] 1/a^4*(1/4*a^4*x^4*arcsech(a*x)^3-1/4*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a^3*x^3-1/2*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a*x-1/4*arcsech(a*x)*a^2*x^2+1/4*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a*x-1/2*arcsech(a*x)^2-1/4+arcsech(a*x)*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)+1/2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2))

Fricas [F]

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arsech}(ax)^3 \, dx$$

```
[In] integrate(x^3*arcsech(a*x)^3,x, algorithm="fricas")
[Out] integral(x^3*arcsech(a*x)^3, x)
```

Sympy [F]

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \operatorname{asech}^3(ax) \, dx$$

```
[In] integrate(x**3*asech(a*x)**3,x)
[Out] Integral(x**3*asech(a*x)**3, x)
```

Maxima [F]

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arsech}(ax)^3 \, dx$$

```
[In] integrate(x^3*arcsech(a*x)^3,x, algorithm="maxima")
[Out] integrate(x^3*arcsech(a*x)^3, x)
```

Giac [F]

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arsech}(ax)^3 \, dx$$

```
[In] integrate(x^3*arcsech(a*x)^3,x, algorithm="giac")
[Out] integrate(x^3*arcsech(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{sech}^{-1}(ax)^3 \, dx = \int x^3 \, \text{acosh}\left(\frac{1}{a\,x}\right)^3 \, dx$$

```
[In] int(x^3*acosh(1/(a*x))^3,x)
[Out] int(x^3*acosh(1/(a*x))^3, x)
```

3.12 $\int x^2 \text{sech}^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 198

$$\begin{aligned}\int x^2 \text{sech}^{-1}(ax)^3 \, dx = & -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} \\ & + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{sech}^{-1}(ax)^2 \arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a^3} \\ & + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{a^3} + \frac{i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3} \\ & - \frac{i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{a^3} \\ & - \frac{i\,\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3} + \frac{i\,\text{PolyLog}\left(3,ie^{\text{sech}^{-1}(ax)}\right)}{a^3}\end{aligned}$$

```
[Out] -x*arcsech(a*x)/a^2+1/3*x^3*arcsech(a*x)^3-arcsech(a*x)^2*arctan(1/a/x+(1/a
/x-1)^(1/2)*(1+1/a/x)^(1/2))/a^3+arctan((a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/a/
x)/a^3+I*arcsech(a*x)*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))
/a^3-I*arcsech(a*x)*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^
3-I*polylog(3,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^3+I*polylog(3,I
*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a^3-1/2*x*(a*x+1)*arcsech(a*x)^2*
((-a*x+1)/(a*x+1))^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6420, 5526, 4271, 3855, 4265, 2611, 2320, 6724}

$$\int x^2 \text{sech}^{-1}(ax)^3 \, dx = \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{ax}\right)}{a^3} - \frac{\text{sech}^{-1}(ax)^2 \arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a^3} + \frac{i\text{sech}^{-1}(ax) \text{PolyLog}\left(2, -ie^{\text{sech}^{-1}(ax)}\right)}{a^3} - \frac{i\text{sech}^{-1}(ax) \text{PolyLog}\left(2, ie^{\text{sech}^{-1}(ax)}\right)}{a^3} - \frac{i \text{PolyLog}\left(3, -ie^{\text{sech}^{-1}(ax)}\right)}{a^3} + \frac{i \text{PolyLog}\left(3, ie^{\text{sech}^{-1}(ax)}\right)}{a^3} - \frac{x\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^2} - \frac{x\text{sech}^{-1}(ax)}{a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3$$

```
[In] Int[x^2*ArcSech[a*x]^3,x]
```

```
[Out] -((x*ArcSech[a*x])/a^2) - (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*
x]^2)/(2*a^2) + (x^3*ArcSech[a*x]^3)/3 - (ArcSech[a*x]^2*ArcTan[E^ArcSech[a
*x]])/a^3 + ArcTan[(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x)]/a^3 + (I*Ar
cSech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^3 - (I*ArcSech[a*x]*PolyLog[2
, I*E^ArcSech[a*x]])/a^3 - (I*PolyLog[3, (-I)*E^ArcSech[a*x]])/a^3 + (I*Pol
yLog[3, I*E^ArcSech[a*x]])/a^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^3 \text{sech}^3(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}\\
&= \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{Subst}\left(\int x^2 \text{sech}^3(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}\\
&= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3\\
&\quad - \frac{\text{Subst}\left(\int x^2 \text{sech}(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{2a^3} + \frac{\text{Subst}\left(\int \text{sech}(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}\\
&= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3\\
&\quad - \frac{\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a^3} + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{a^3}\\
&\quad + \frac{i\text{Subst}\left(\int x\log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}\\
&\quad - \frac{i\text{Subst}\left(\int x\log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}\\
&= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3\\
&\quad - \frac{\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a^3} + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{a^3}\\
&\quad + \frac{i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3} - \frac{i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{a^3}\\
&\quad - \frac{i\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}\\
&\quad + \frac{i\text{Subst}\left(\int \text{PolyLog}\left(2,ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^3}
\end{aligned}
$$

$$= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a^3} + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{a^3} + \frac{i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3} - \frac{i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{a^3} - \frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{\text{sech}^{-1}(ax)}\right)}{a^3} + \frac{i\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{\text{sech}^{-1}(ax)}\right)}{a^3}$$

$$= -\frac{x\text{sech}^{-1}(ax)}{a^2} - \frac{x\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3}x^3\text{sech}^{-1}(ax)^3 - \frac{\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a^3} + \frac{\arctan\left(\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)}{a^3} + \frac{i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3} - \frac{i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{a^3} - \frac{i\,\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(ax)}\right)}{a^3} + \frac{i\,\text{PolyLog}\left(3,ie^{\text{sech}^{-1}(ax)}\right)}{a^3}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01

$$\int x^2\text{sech}^{-1}(ax)^3\,dx$$

$$= \frac{-6ax\text{sech}^{-1}(ax) - 3ax\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2 + 2a^3x^3\text{sech}^{-1}(ax)^3 + 3i\Big(-4i\arctan\left(\tanh\left(\tfrac{1}{2}\text{sech}^{-1}(\right.\right.}{}$$

```
[In] Integrate[x^2*ArcSech[a*x]^3,x]
```

```
[Out] (-6*a*x*ArcSech[a*x] - 3*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*
x]^2 + 2*a^3*x^3*ArcSech[a*x]^3 + (3*I)*((-4*I)*ArcTan[Tanh[ArcSech[a*x]/2]
] + ArcSech[a*x]^2*Log[1 - I/E^ArcSech[a*x]] - ArcSech[a*x]^2*Log[1 + I/E^A
rcSech[a*x]] + 2*ArcSech[a*x]*PolyLog[2, (-I)/E^ArcSech[a*x]] - 2*ArcSech[a
*x]*PolyLog[2, I/E^ArcSech[a*x]] + 2*PolyLog[3, (-I)/E^ArcSech[a*x]] - 2*Po
lyLog[3, I/E^ArcSech[a*x]]))/(6*a^3)
```

Maple [F]

$$\int x^2 \operatorname{arcsech}(ax)^3 \, dx$$

[In] int(x^2*arcsech(a*x)^3,x)

[Out] int(x^2*arcsech(a*x)^3,x)

Fricas [F]

$$\int x^2 \operatorname{sech}^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arsech}(ax)^3 \, dx$$

[In] integrate(x^2*arcsech(a*x)^3,x, algorithm="fricas")

[Out] integral(x^2*arcsech(a*x)^3, x)

Sympy [F]

$$\int x^2 \operatorname{sech}^{-1}(ax)^3 \, dx = \int x^2 \operatorname{asech}^3(ax) \, dx$$

[In] integrate(x**2*asech(a*x)**3,x)

[Out] Integral(x**2*asech(a*x)**3, x)

Maxima [F]

$$\int x^2 \operatorname{sech}^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arsech}(ax)^3 \, dx$$

[In] integrate(x^2*arcsech(a*x)^3,x, algorithm="maxima")

[Out] integrate(x^2*arcsech(a*x)^3, x)

Giac [F]

$$\int x^2 \operatorname{sech}^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arsech}(ax)^3 \, dx$$

```
[In] integrate(x^2*arcsech(a*x)^3,x, algorithm="giac")
[Out] integrate(x^2*arcsech(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{sech}^{-1}(ax)^3 \, dx = \int x^2 \operatorname{acosh}\left(\frac{1}{a\,x}\right)^3 \, dx$$

```
[In] int(x^2*acosh(1/(a*x))^3,x)
[Out] int(x^2*acosh(1/(a*x))^3, x)
```

3.13 $\int x\text{sech}^{-1}(ax)^3\,dx$

Optimal result

Integrand size = 8, antiderivative size = 102

$$\int x\text{sech}^{-1}(ax)^3\,dx = -\frac{3\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{3\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^2} + \frac{3\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)}{2a^2}$$

```
[Out] -3/2*arcsech(a*x)^2/a^2+1/2*x^2*arcsech(a*x)^3+3*arcsech(a*x)*ln(1+(1/a/x+(
1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)/a^2+3/2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2
)*(1+1/a/x)^(1/2))^2)/a^2-3/2*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/
2)/a^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6420, 5526, 4269, 3799, 2221, 2317, 2438}

$$\int x\text{sech}^{-1}(ax)^3\,dx = \frac{3\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)}{2a^2} - \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\text{sech}^{-1}(ax)^2}{2a^2} + \frac{3\text{sech}^{-1}(ax)\log\left(e^{2\text{sech}^{-1}(ax)}+1\right)}{a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3$$

```
[In] Int[x*ArcSech[a*x]^3,x]
```

```
[Out] (-3*ArcSech[a*x]^2)/(2*a^2) - (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSec
h[a*x]^2)/(2*a^2) + (x^2*ArcSech[a*x]^3)/2 + (3*ArcSech[a*x]*Log[1 + E^(2*A
rcSech[a*x])])/a^2 + (3*PolyLog[2, -E^(2*ArcSech[a*x])])/(2*a^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^3\text{sech}^2(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}$$

$$= \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 - \frac{3\text{Subst}\left(\int x^2\text{sech}^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{2a^2}$$

$$= -\frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{3\text{Subst}\left(\int x\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}$$

$$= -\frac{3\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{6\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}$$

$$= -\frac{3\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{3\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^2} - \frac{3\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a^2}$$

$$= -\frac{3\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{3\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^2} - \frac{3\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{sech}^{-1}(ax)}\right)}{2a^2}$$

$$= -\frac{3\text{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\text{sech}^{-1}(ax)^3 + \frac{3\text{sech}^{-1}(ax)\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)}{a^2} + \frac{3\,\text{PolyLog}\left(2, -e^{2\text{sech}^{-1}(ax)}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int x\text{sech}^{-1}(ax)^3\,dx$$

$$= \frac{\text{sech}^{-1}(ax)\left(-3\left(-1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)\text{sech}^{-1}(ax)+a^2x^2\text{sech}^{-1}(ax)^2+6\log\left(1+e^{-2\text{sech}^{-1}(ax)}\right)\right) - 3}{2a^2}$$

```
[In] Integrate[x*ArcSech[a*x]^3,x]
[Out] (ArcSech[a*x]*(-3*(-1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 +
 a*x)])*ArcSech[a*x] + a^2*x^2*ArcSech[a*x]^2 + 6*Log[1 + E^(-2*ArcSech[a*x
])]) - 3*PolyLog[2, -E^(-2*ArcSech[a*x])])/(2*a^2)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsech}(ax)^2\left(-3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax+\operatorname{arcsech}(ax)a^2x^2+3\right)}{2}-3\operatorname{arcsech}(ax)^2+3\operatorname{arcsech}(ax)\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right.\right.}{a^2}$
default	$\frac{\frac{\operatorname{arcsech}(ax)^2\left(-3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,ax+\operatorname{arcsech}(ax)a^2x^2+3\right)}{2}-3\operatorname{arcsech}(ax)^2+3\operatorname{arcsech}(ax)\ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right.\right.}{a^2}$

```
[In] int(x*arcsech(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*arcsech(a*x)^2*(-3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*a*x+
arcsech(a*x)*a^2*x^2+3)-3*arcsech(a*x)^2+3*arcsech(a*x)*ln(1+(1/a/x+(1/a/x-
1)^(1/2)*(1+1/a/x)^(1/2))^2)+3/2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x
)^(1/2))^2))
```

Fricas [F]

$$\int x\text{sech}^{-1}(ax)^3\,dx=\int x\,\text{arsech}\left(ax\right)^3\,dx$$

```
[In] integrate(x*arcsech(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arcsech(a*x)^3, x)
```

Sympy [F]

$$\int x\text{sech}^{-1}(ax)^3\,dx=\int x\,\text{asech}^3\left(ax\right)dx$$

```
[In] integrate(x*asech(a*x)**3,x)
```

```
[Out] Integral(x*asech(a*x)**3, x)
```

Maxima [F]

$$\int x\text{sech}^{-1}(ax)^3\,dx = \int x\,\text{arsech}\,(ax)^3\;dx$$

```
[In] integrate(x*arcsech(a*x)^3,x, algorithm="maxima")
[Out] integrate(x*arcsech(a*x)^3, x)
```

Giac [F]

$$\int x\text{sech}^{-1}(ax)^3\,dx = \int x\,\text{arsech}\,(ax)^3\;dx$$

```
[In] integrate(x*arcsech(a*x)^3,x, algorithm="giac")
[Out] integrate(x*arcsech(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{-1}(ax)^3\,dx = \int x\,\text{acosh}\left(\frac{1}{a\,x}\right)^3\,dx$$

```
[In] int(x*acosh(1/(a*x))^3,x)
[Out] int(x*acosh(1/(a*x))^3, x)
```

3.14 $\int \text{sech}^{-1}(ax)^3\,dx$

Optimal result

Integrand size = 6, antiderivative size = 111

$$\int \text{sech}^{-1}(ax)^3\,dx = x\text{sech}^{-1}(ax)^3 - \frac{6\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a} + \frac{6i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{a} - \frac{6i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{a} - \frac{6i\,\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(ax)}\right)}{a} + \frac{6i\,\text{PolyLog}\left(3,ie^{\text{sech}^{-1}(ax)}\right)}{a}$$

```
[Out] x*arcsech(a*x)^3-6*arcsech(a*x)^2*arctan(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1
/2))/a+6*I*arcsech(a*x)*polylog(2,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)
))/a-6*I*arcsech(a*x)*polylog(2,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/
a-6*I*polylog(3,-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a+6*I*polylog(3
,I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used

= {6414, 5526, 4265, 2611, 2320, 6724}

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = -\frac{6\operatorname{sech}^{-1}(ax)^2 \arctan\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{PolyLog}\left(3,-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{PolyLog}\left(3,ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} + x\operatorname{sech}^{-1}(ax)^3$$

```
[In] Int[ArcSech[a*x]^3,x]
```

```
[Out] x*ArcSech[a*x]^3 - (6*ArcSech[a*x]^2*ArcTan[E^ArcSech[a*x]])/a + ((6*I)*Arc
Sech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]])/a - ((6*I)*ArcSech[a*x]*PolyLog[
2, I*E^ArcSech[a*x]])/a - ((6*I)*PolyLog[3, (-I)*E^ArcSech[a*x]])/a + ((6*I
)*PolyLog[3, I*E^ArcSech[a*x]])/a
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6414

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^3\text{sech}(x)\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}\\
&= x\text{sech}^{-1}(ax)^3 - \frac{3\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}\\
&= x\text{sech}^{-1}(ax)^3 - \frac{6\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a}\\
&\quad + \frac{(6i)\text{Subst}\left(\int x\log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}\\
&\quad - \frac{(6i)\text{Subst}\left(\int x\log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}\\
&= x\text{sech}^{-1}(ax)^3 - \frac{6\text{sech}^{-1}(ax)^2\arctan\left(e^{\text{sech}^{-1}(ax)}\right)}{a}\\
&\quad + \frac{6i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(ax)}\right)}{a} - \frac{6i\text{sech}^{-1}(ax)\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(ax)}\right)}{a}\\
&\quad - \frac{(6i)\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}\\
&\quad + \frac{(6i)\text{Subst}\left(\int \text{PolyLog}\left(2,ie^x\right)\,dx, x, \text{sech}^{-1}(ax)\right)}{a}
\end{aligned}
$$

$$= x\operatorname{sech}^{-1}(ax)^3 - \frac{6\operatorname{sech}^{-1}(ax)^2 \arctan\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{(6i)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{(6i)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

$$= x\operatorname{sech}^{-1}(ax)^3 - \frac{6\operatorname{sech}^{-1}(ax)^2 \arctan\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i\operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i\operatorname{PolyLog}\left(3, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = x\operatorname{sech}^{-1}(ax)^3 - \frac{3i\left(-\operatorname{sech}^{-1}(ax)^2\left(\log\left(1-ie^{-\operatorname{sech}^{-1}(ax)}\right) - \log\left(1+ie^{-\operatorname{sech}^{-1}(ax)}\right)\right) - 2\operatorname{sech}^{-1}(ax)\left(\operatorname{PolyLog}\left(2, -ie^{-\operatorname{sec}\right.\right.}{a}$$

```
[In] Integrate[ArcSech[a*x]^3,x]
```

```
[Out] x*ArcSech[a*x]^3 - ((3*I)*(-(ArcSech[a*x]^2*(Log[1 - I/E^ArcSech[a*x]] - Lo
g[1 + I/E^ArcSech[a*x]])) - 2*ArcSech[a*x]*(PolyLog[2, (-I)/E^ArcSech[a*x]]
 - PolyLog[2, I/E^ArcSech[a*x]]) - 2*(PolyLog[3, (-I)/E^ArcSech[a*x]] - Pol
yLog[3, I/E^ArcSech[a*x]])))/a
```

Maple [F]

$$\int \operatorname{arcsech}(ax)^3\,dx$$

```
[In] int(arcsech(a*x)^3,x)
```

```
[Out] int(arcsech(a*x)^3,x)
```

Fricas [F]

$$\int \operatorname{sech}^{-1}(ax)^3 \, dx = \int \operatorname{arsech}\left(ax\right)^3 \, dx$$

[In] integrate(arcsech(a*x)^3,x, algorithm="fricas")

[Out] integral(arcsech(a*x)^3, x)

Sympy [F]

$$\int \operatorname{sech}^{-1}(ax)^3 \, dx = \int \operatorname{asech}^3\left(ax\right) dx$$

[In] integrate(asech(a*x)**3,x)

[Out] Integral(asech(a*x)**3, x)

Maxima [F]

$$\int \operatorname{sech}^{-1}(ax)^3 \, dx = \int \operatorname{arsech}\left(ax\right)^3 \, dx$$

[In] integrate(arcsech(a*x)^3,x, algorithm="maxima")

[Out] x*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^3 - integrate((a^2*x^2*log(a)^3 + (a^2*x^2 - 1)*log(x)^3 + 3*(a^2*x^2*log(a) + (a^2*x^2*(log(a) + 1) + (a^2*x^2 - 1)*log(x) - log(a))*sqrt(a*x + 1)*sqrt(-a*x + 1) + (a^2*x^2 - 1)*log(x) - log(a))*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^2 - log(a)^3 + 3*(a^2*x^2*log(a) - log(a))*log(x)^2 + (a^2*x^2*log(a)^3 + (a^2*x^2 - 1)*log(x)^3 - log(a)^3 + 3*(a^2*x^2*log(a) - log(a))*log(x)^2 + 3*(a^2*x^2*log(a)^2 - log(a)^2)*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - 3*(a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 + (a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 - log(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - log(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1) + 3*(a^2*x^2*log(a)^2 - log(a)^2)*log(x))/(a^2*x^2 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 1), x)

Giac [F]

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = \int \operatorname{arsech}\left(ax\right)^3\,dx$$

```
[In] integrate(arcsech(a*x)^3,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{-1}(ax)^3\,dx = \int \operatorname{acosh}\left(\frac{1}{a\,x}\right)^3\,dx$$

```
[In] int(acosh(1/(a*x))^3,x)
[Out] int(acosh(1/(a*x))^3, x)
```

3.15 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 88

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x} dx = \frac{1}{4}\operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3 \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{2}\operatorname{sech}^{-1}(ax)^2 \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2}\operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{4} \operatorname{PolyLog}\left(4, -e^{2\operatorname{sech}^{-1}(ax)}\right)$$

```
[Out] 1/4*arcsech(a*x)^4-arcsech(a*x)^3*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)-3/2*arcsech(a*x)^2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)
)^2)+3/2*arcsech(a*x)*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
-3/4*polylog(4,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6420, 3799, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x} dx = -\frac{3}{2}\operatorname{sech}^{-1}(ax)^2 \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2}\operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{4} \operatorname{PolyLog}\left(4, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{4}\operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3 \log\left(e^{2\operatorname{sech}^{-1}(ax)} + 1\right)$$

```
[In] Int[ArcSech[a*x]^3/x,x]
```

```
[Out] ArcSech[a*x]^4/4 - ArcSech[a*x]^3*Log[1 + E^(2*ArcSech[a*x])] - (3*ArcSech[
a*x]^2*PolyLog[2, -E^(2*ArcSech[a*x])])/2 + (3*ArcSech[a*x]*PolyLog[3, -E^(
2*ArcSech[a*x])])/2 - (3*PolyLog[4, -E^(2*ArcSech[a*x])])/4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^3\tanh(x)\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= \frac{1}{4}\text{sech}^{-1}(ax)^4 - 2\text{Subst}\left(\int \frac{e^{2x}x^3}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= \frac{1}{4}\text{sech}^{-1}(ax)^4 - \text{sech}^{-1}(ax)^3\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)\\
&\quad + 3\text{Subst}\left(\int x^2\log\left(1+e^{2x}\right)\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= \frac{1}{4}\text{sech}^{-1}(ax)^4 - \text{sech}^{-1}(ax)^3\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)\\
&\quad - \frac{3}{2}\text{sech}^{-1}(ax)^2\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)\\
&\quad + 3\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-e^{2x}\right)\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= \frac{1}{4}\text{sech}^{-1}(ax)^4 - \text{sech}^{-1}(ax)^3\log\left(1+e^{2\text{sech}^{-1}(ax)}\right)\\
&\quad - \frac{3}{2}\text{sech}^{-1}(ax)^2\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)\\
&\quad + \frac{3}{2}\text{sech}^{-1}(ax)\,\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(ax)}\right)\\
&\quad - \frac{3}{2}\text{Subst}\left(\int \text{PolyLog}\left(3,-e^{2x}\right)\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= \frac{1}{4}\text{sech}^{-1}(ax)^4 - \text{sech}^{-1}(ax)^3\log\left(1+e^{2\text{sech}^{-1}(ax)}\right) - \frac{3}{2}\text{sech}^{-1}(ax)^2\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)\\
&\quad + \frac{3}{2}\text{sech}^{-1}(ax)\,\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(ax)}\right) - \frac{3}{4}\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx, x, e^{2\text{sech}^{-1}(ax)}\right)\\
&= \frac{1}{4}\text{sech}^{-1}(ax)^4 - \text{sech}^{-1}(ax)^3\log\left(1+e^{2\text{sech}^{-1}(ax)}\right) - \frac{3}{2}\text{sech}^{-1}(ax)^2\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax)}\right)\\
&\quad + \frac{3}{2}\text{sech}^{-1}(ax)\,\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(ax)}\right) - \frac{3}{4}\text{PolyLog}\left(4,-e^{2\text{sech}^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \frac{1}{4}\Big(-\operatorname{sech}^{-1}(ax)^4 - 4\operatorname{sech}^{-1}(ax)^3\log\left(1+e^{-2\operatorname{sech}^{-1}(ax)}\right) + 6\operatorname{sech}^{-1}(ax)^2\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(ax)}\right) + 6\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(3,-e^{-2\operatorname{sech}^{-1}(ax)}\right) + 3\operatorname{PolyLog}\left(4,-e^{-2\operatorname{sech}^{-1}(ax)}\right)\Big)$$

```
[In] Integrate[ArcSech[a*x]^3/x,x]
```

```
[Out] (-ArcSech[a*x]^4 - 4*ArcSech[a*x]^3*Log[1 + E^(-2*ArcSech[a*x])] + 6*ArcSec
h[a*x]^2*PolyLog[2, -E^(-2*ArcSech[a*x])] + 6*ArcSech[a*x]*PolyLog[3, -E^(-
2*ArcSech[a*x])] + 3*PolyLog[4, -E^(-2*ArcSech[a*x])])/4
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.06

method	result
derivativedivides	$\frac{\operatorname{arcsech}(ax)^4}{4} - \operatorname{arcsech}(ax)^3 \ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^2\right) - \frac{3\operatorname{arcsech}(ax)^2 \operatorname{polylog}\left(2,-\left(\right.\right.}{2}$
default	$\frac{\operatorname{arcsech}(ax)^4}{4} - \operatorname{arcsech}(ax)^3 \ln\left(1+\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)^2\right) - \frac{3\operatorname{arcsech}(ax)^2 \operatorname{polylog}\left(2,-\left(\right.\right.}{2}$

```
[In] int(arcsech(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arcsech(a*x)^4-arcsech(a*x)^3*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/
2))^2)-3/2*arcsech(a*x)^2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)
)^2)+3/2*arcsech(a*x)*polylog(3,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
-3/4*polylog(4,-(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsech}\left(ax\right)^3}{x}\,dx$$

```
[In] integrate(arcsech(a*x)^3/x,x, algorithm="fricas")
[Out] integral(arcsech(a*x)^3/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{asech}^{3}\left(ax\right)}{x}\,dx$$

```
[In] integrate(asech(a*x)**3/x,x)
[Out] Integral(asech(a*x)**3/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsech}\left(ax\right)^3}{x}\,dx$$

```
[In] integrate(arcsech(a*x)^3/x,x, algorithm="maxima")
[Out] integrate(arcsech(a*x)^3/x, x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsech}\left(ax\right)^3}{x}\,dx$$

```
[In] integrate(arcsech(a*x)^3/x,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x}\,dx$$

```
[In] int(acosh(1/(a*x))^3/x,x)
[Out] int(acosh(1/(a*x))^3/x, x)
```

3.16 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 83

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \frac{6\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{x} - \frac{6\operatorname{sech}^{-1}(ax)}{x} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{x} - \frac{\operatorname{sech}^{-1}(ax)^3}{x}$$

```
[Out] -6*arcsech(a*x)/x-arcsech(a*x)^3/x+6*(a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/x+3*(
a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6420, 3377, 2717}

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \frac{6\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{x} - \frac{\operatorname{sech}^{-1}(ax)^3}{x} + \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{x} - \frac{6\operatorname{sech}^{-1}(ax)}{x}$$

```
[In] Int[ArcSech[a*x]^3/x^2,x]
```

```
[Out] (6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/x - (6*ArcSech[a*x])/x + (3*Sqrt[(1
 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/x - ArcSech[a*x]^3/x
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\left(a\text{Subst}\left(\int x^3\sinh(x)\,dx, x, \text{sech}^{-1}(ax)\right)\right)$$

$$= -\frac{\text{sech}^{-1}(ax)^3}{x} + (3a)\text{Subst}\left(\int x^2\cosh(x)\,dx, x, \text{sech}^{-1}(ax)\right)$$

$$= \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{x} - \frac{\text{sech}^{-1}(ax)^3}{x} - (6a)\text{Subst}\left(\int x\sinh(x)\,dx, x, \text{sech}^{-1}(ax)\right)$$

$$= -\frac{6\text{sech}^{-1}(ax)}{x} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{x} - \frac{\text{sech}^{-1}(ax)^3}{x} + (6a)\text{Subst}\left(\int \cosh(x)\,dx, x, \text{sech}^{-1}(ax)\right)$$

$$= \frac{6\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{x} - \frac{6\text{sech}^{-1}(ax)}{x} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{x} - \frac{\text{sech}^{-1}(ax)^3}{x}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\text{sech}^{-1}(ax)^3}{x^2}\,dx = \frac{6\sqrt{\frac{1-ax}{1+ax}}(1+ax) - 6\text{sech}^{-1}(ax) + 3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2 - \text{sech}^{-1}(ax)^3}{x}$$

```
[In] Integrate[ArcSech[a*x]^3/x^2,x]
```

```
[Out] (6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 6*ArcSech[a*x] + 3*Sqrt[(1 - a*x)/
(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 - ArcSech[a*x]^3)/x
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

method	result	s
derivativedivides	$a\left(-\frac{\operatorname{arcsech}(ax)^3}{ax}+3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^2-\frac{6\operatorname{arcsech}(ax)}{ax}+6\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\right)$	9
default	$a\left(-\frac{\operatorname{arcsech}(ax)^3}{ax}+3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^2-\frac{6\operatorname{arcsech}(ax)}{ax}+6\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\right)$	9

```
[In] int(arcsech(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/a/x*arcsech(a*x)^3+3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*arcsech
(a*x)^2-6/a/x*arcsech(a*x)+6*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \frac{3\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2-\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3+6\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}-6\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)}{x}$$

```
[In] integrate(arcsech(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] (3*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2
)) + 1)/(a*x))^2 - log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^3 +
6*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) - 6*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*
x^2)) + 1)/(a*x)))/x
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{asech}^3(ax)}{x^2}\,dx$$

```
[In] integrate(asech(a*x)**3/x**2,x)
[Out] Integral(asech(a*x)**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = 3\,a\sqrt{\frac{1}{a^2x^2}-1}\,\operatorname{arsech}(ax)^2 - \frac{\operatorname{arsech}(ax)^3}{x} + 6\,a\sqrt{\frac{1}{a^2x^2}-1} - \frac{6\,\operatorname{arsech}(ax)}{x}$$

```
[In] integrate(arcsech(a*x)^3/x^2,x, algorithm="maxima")
[Out] 3*a*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x)^2 - arcsech(a*x)^3/x + 6*a*sqrt(1/(a
^2*x^2) - 1) - 6*arcsech(a*x)/x
```

Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arsech}(ax)^3}{x^2}\,dx$$

```
[In] integrate(arcsech(a*x)^3/x^2,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x^2}\,dx$$

```
[In] int(acosh(1/(a*x))^3/x^2,x)
[Out] int(acosh(1/(a*x))^3/x^2, x)
```

3.17 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 137

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{8x^2} - \frac{3}{8}a^2\operatorname{sech}^{-1}(ax) - \frac{3(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{4x^2} - \frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^3 - \frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)^3}{2x^2}$$

```
[Out] -3/8*a^2*arcsech(a*x)-3/4*(-a*x+1)*(a*x+1)*arcsech(a*x)/x^2-1/4*a^2*arcsech
(a*x)^3-1/2*(-a*x+1)*(a*x+1)*arcsech(a*x)^3/x^2+3/8*(a*x+1)*((-a*x+1)/(a*x+
1))^(1/2)/x^2+3/4*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6420, 5480, 3392, 30, 2715, 8}

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = -\frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^3 - \frac{3}{8}a^2\operatorname{sech}^{-1}(ax) + \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{8x^2} - \frac{(1-ax)(ax+1)\operatorname{sech}^{-1}(ax)^3}{2x^2} + \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{4x^2} - \frac{3(1-ax)(ax+1)\operatorname{sech}^{-1}(ax)}{4x^2}$$

```
[In] Int[ArcSech[a*x]^3/x^3,x]
```

```
[Out] (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(8*x^2) - (3*a^2*ArcSech[a*x])/8 -
(3*(1 - a*x)*(1 + a*x)*ArcSech[a*x])/(4*x^2) + (3*Sqrt[(1 - a*x)/(1 + a*x)]
*(1 + a*x)*ArcSech[a*x]^2)/(4*x^2) - (a^2*ArcSech[a*x]^3)/4 - ((1 - a*x)*(1
 + a*x)*ArcSech[a*x]^3)/(2*x^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(a^2\text{Subst}\left(\int x^3\cosh(x)\sinh(x)\,dx, x, \text{sech}^{-1}(ax)\right)\right)\\
&= -\frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^3}{2x^2} + \frac{1}{2}\left(3a^2\right)\text{Subst}\left(\int x^2\sinh^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= -\frac{3(1-ax)(1+ax)\text{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4x^2}\\
&\quad - \frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^3}{2x^2} - \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int x^2\,dx, x, \text{sech}^{-1}(ax)\right)\\
&\quad + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sinh^2(x)\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{8x^2} - \frac{3(1-ax)(1+ax)\text{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4x^2}\\
&\quad - \frac{1}{4}a^2\text{sech}^{-1}(ax)^3 - \frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^3}{2x^2} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int 1\,dx, x, \text{sech}^{-1}(ax)\right)\\
&= \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{8x^2} - \frac{3}{8}a^2\text{sech}^{-1}(ax) - \frac{3(1-ax)(1+ax)\text{sech}^{-1}(ax)}{4x^2}\\
&\quad + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{4x^2} - \frac{1}{4}a^2\text{sech}^{-1}(ax)^3 - \frac{(1-ax)(1+ax)\text{sech}^{-1}(ax)^3}{2x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$
\int \frac{\text{sech}^{-1}(ax)^3}{x^3}\,dx = \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax) - 6\text{sech}^{-1}(ax) + 6\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2 + 2\left(-2+a^2x^2\right)\text{sech}^{-1}(ax)^3 - 3a^2x^2\log(\ldots}{8x^2}
$$

```
[In] Integrate[ArcSech[a*x]^3/x^3,x]
```

```
[Out] (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 6*ArcSech[a*x] + 6*Sqrt[(1 - a*x)/
(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 + 2*(-2 + a^2*x^2)*ArcSech[a*x]^3 - 3*a
^2*x^2*Log[x] + 3*a^2*x^2*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 -
 a*x)/(1 + a*x)]])/(8*x^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arcsech}(ax)^3}{2a^2x^2}+\frac{3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^2}{4ax}+\frac{\operatorname{arcsech}(ax)^3}{4}-\frac{3\operatorname{arcsech}(ax)}{4a^2x^2}+\frac{3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{8ax}\right.$
default	$a^2\left(-\frac{\operatorname{arcsech}(ax)^3}{2a^2x^2}+\frac{3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}\,\operatorname{arcsech}(ax)^2}{4ax}+\frac{\operatorname{arcsech}(ax)^3}{4}-\frac{3\operatorname{arcsech}(ax)}{4a^2x^2}+\frac{3\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{8ax}\right.$

```
[In] int(arcsech(a*x)^3/x^3,x,method=_RETURNVERBOSE)
[Out] a^2*(-1/2/a^2/x^2*arcsech(a*x)^3+3/4*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/
2)/a/x*arcsech(a*x)^2+1/4*arcsech(a*x)^3-3/4/a^2/x^2*arcsech(a*x)+3/8*(-(a*
x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)/a/x+3/8*arcsech(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.27

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx$$
$$=\frac{6\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2+2\,(a^2x^2-2)\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3+3\,ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+3\,(a^2x^2-2)\,l}{8\,x^2}$$

```
[In] integrate(arcsech(a*x)^3/x^3,x, algorithm="fricas")
[Out] 1/8*(6*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2
*x^2)) + 1)/(a*x))^2 + 2*(a^2*x^2 - 2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^
2)) + 1)/(a*x))^3 + 3*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 3*(a^2*x^2 - 2)*
log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)))/x^2
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx=\int \frac{\operatorname{asech}^3(ax)}{x^3}\,dx$$

```
[In] integrate(asech(a*x)**3/x**3,x)
[Out] Integral(asech(a*x)**3/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arsech}(ax)^3}{x^3}\,dx$$

```
[In] integrate(arcsech(a*x)^3/x^3,x, algorithm="maxima")
[Out] integrate(arcsech(a*x)^3/x^3, x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arsech}(ax)^3}{x^3}\,dx$$

```
[In] integrate(arcsech(a*x)^3/x^3,x, algorithm="giac")
[Out] integrate(arcsech(a*x)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x^3}\,dx$$

```
[In] int(acosh(1/(a*x))^3/x^3,x)
[Out] int(acosh(1/(a*x))^3/x^3, x)
```

3.18 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 179

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4} dx = \frac{14a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{9x} + \frac{2\left(\frac{1-ax}{1+ax}\right)^{3/2}(1+ax)^3}{27x^3} - \frac{2\operatorname{sech}^{-1}(ax)}{9x^3} - \frac{4a^2\operatorname{sech}^{-1}(ax)}{3x} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x^3} + \frac{2a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x} - \frac{\operatorname{sech}^{-1}(ax)^3}{3x^3}$$

```
[Out] 2/27*((-a*x+1)/(a*x+1))^(3/2)*(a*x+1)^3/x^3-2/9*arcsech(a*x)/x^3-4/3*a^2*ar
csech(a*x)/x-1/3*arcsech(a*x)^3/x^3+14/9*a^2*(a*x+1)*((-a*x+1)/(a*x+1))^(1/
2)/x+1/3*(a*x+1)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x^3+2/3*a^2*(a*x+1
)*arcsech(a*x)^2*((-a*x+1)/(a*x+1))^(1/2)/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6420, 5481, 3392, 3377, 2717, 2713}

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4} dx = \frac{14a^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{9x} + \frac{2a^2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{3x} - \frac{4a^2\operatorname{sech}^{-1}(ax)}{3x} + \frac{2\left(\frac{1-ax}{ax+1}\right)^{3/2}(ax+1)^3}{27x^3} + \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{3x^3} - \frac{\operatorname{sech}^{-1}(ax)^3}{3x^3} - \frac{2\operatorname{sech}^{-1}(ax)}{9x^3}$$

```
[In] Int[ArcSech[a*x]^3/x^4,x]
```

```
[Out] (14*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(9*x) + (2*((1 - a*x)/(1 + a*x
))^(3/2)*(1 + a*x)^3)/(27*x^3) - (2*ArcSech[a*x])/(9*x^3) - (4*a^2*ArcSech[
a*x])/(3*x) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(3*x^3)
+ (2*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(3*x) - ArcSec
h[a*x]^3/(3*x^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(a^3\text{Subst}\left(\int x^3\cosh^2(x)\sinh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\right)\\
&= -\frac{\text{sech}^{-1}(ax)^3}{3x^3}+a^3\text{Subst}\left(\int x^2\cosh^3(x)\,dx,x,\text{sech}^{-1}(ax)\right)\\
&= -\frac{2\text{sech}^{-1}(ax)}{9x^3}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{3x^3}-\frac{\text{sech}^{-1}(ax)^3}{3x^3}\\
&\quad+\frac{1}{9}\left(2a^3\right)\text{Subst}\left(\int\cosh^3(x)\,dx,x,\text{sech}^{-1}(ax)\right)\\
&\quad+\frac{1}{3}\left(2a^3\right)\text{Subst}\left(\int x^2\cosh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\\
&= -\frac{2\text{sech}^{-1}(ax)}{9x^3}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{3x^3}+\frac{2a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{3x}\\
&\quad-\frac{\text{sech}^{-1}(ax)^3}{3x^3}+\frac{1}{9}\left(2ia^3\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\frac{i\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{ax}\right)\\
&\quad-\frac{1}{3}\left(4a^3\right)\text{Subst}\left(\int x\sinh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\\
&= \frac{2a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{9x}+\frac{2\left(\frac{1-ax}{1+ax}\right)^{3/2}(1+ax)^3}{27x^3}-\frac{2\text{sech}^{-1}(ax)}{9x^3}-\frac{4a^2\text{sech}^{-1}(ax)}{3x}\\
&\quad+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{3x^3}+\frac{2a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{3x}\\
&\quad-\frac{\text{sech}^{-1}(ax)^3}{3x^3}+\frac{1}{3}\left(4a^3\right)\text{Subst}\left(\int\cosh(x)\,dx,x,\text{sech}^{-1}(ax)\right)\\
&= \frac{14a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{9x}+\frac{2\left(\frac{1-ax}{1+ax}\right)^{3/2}(1+ax)^3}{27x^3}-\frac{2\text{sech}^{-1}(ax)}{9x^3}-\frac{4a^2\text{sech}^{-1}(ax)}{3x}\\
&\quad+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{3x^3}+\frac{2a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\text{sech}^{-1}(ax)^2}{3x}-\frac{\text{sech}^{-1}(ax)^3}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.67

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4}\,dx = \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax+20a^2x^2+20a^3x^3)-6(1+6a^2x^2)\operatorname{sech}^{-1}(ax)+9\sqrt{\frac{1-ax}{1+ax}}(1+ax+2a^2x^2+2a^3x^3)\operatorname{sech}^{-1}}{27x^3}$$

```
[In] Integrate[ArcSech[a*x]^3/x^4,x]
```

```
[Out] (2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 20*a^2*x^2 + 20*a^3*x^3) - 6*(1 + 6
*a^2*x^2)*ArcSech[a*x] + 9*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 2*a^2*x^2 +
 2*a^3*x^3)*ArcSech[a*x]^2 - 9*ArcSech[a*x]^3)/(27*x^3)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.07

method	result
derivativedivides	$a^3\left(-\frac{\operatorname{arcsech}(ax)^3}{3a^3x^3}+\frac{2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\operatorname{arcsech}(ax)^2}{3}+\frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\operatorname{arcsech}(ax)^2}{3a^2x^2}-\frac{4\operatorname{arcsech}(ax)}{3ax}+\right.$
default	$a^3\left(-\frac{\operatorname{arcsech}(ax)^3}{3a^3x^3}+\frac{2\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\operatorname{arcsech}(ax)^2}{3}+\frac{\sqrt{-\frac{ax-1}{ax}}\sqrt{\frac{ax+1}{ax}}\operatorname{arcsech}(ax)^2}{3a^2x^2}-\frac{4\operatorname{arcsech}(ax)}{3ax}+\right.$

```
[In] int(arcsech(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3/a^3/x^3*arcsech(a*x)^3+2/3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/
2)*arcsech(a*x)^2+1/3/a^2/x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*arcs
ech(a*x)^2-4/3/a/x*arcsech(a*x)+40/27*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1
/2)-2/9*arcsech(a*x)/a^3/x^3+2/27*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)/
a^2/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4}\,dx = \frac{9\,(2\,a^3x^3+ax)\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2-9\,\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3-6\,(6\,a^2x^2+1)\log\left(\frac{ax\sqrt{-\frac{a^2x^2-}{a^2x^2}}}{ax}\right.}{27\,x^3}$$

[In] integrate(arcsech(a*x)^3/x^4,x, algorithm="fricas")

[Out] 1/27*(9*(2*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 9*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^3 - 6*(6*a^2*x^2 + 1)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) + 2*(20*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)))/x^3

Sympy [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{asech}^3(ax)}{x^4}\,dx$$

[In] integrate(asech(a*x)**3/x**4,x)

[Out] Integral(asech(a*x)**3/x**4, x)

Maxima [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arsech}(ax)^3}{x^4}\,dx$$

[In] integrate(arcsech(a*x)^3/x^4,x, algorithm="maxima")

[Out] integrate(arcsech(a*x)^3/x^4, x)

Giac [F]

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arsech}(ax)^3}{x^4}\,dx$$

[In] integrate(arcsech(a*x)^3/x^4,x, algorithm="giac")

[Out] integrate(arcsech(a*x)^3/x^4, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{acosh}\left(\frac{1}{a\,x}\right)^3}{x^4}\,dx$$

```
[In] int(acosh(1/(a*x))^3/x^4,x)
[Out] int(acosh(1/(a*x))^3/x^4, x)
```

3.19 $\int x^6 (a + b\text{sech}^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int x^6 (a + b\text{sech}^{-1}(cx))\,dx = -\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{1+cx}}} - \frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{7}x^7(a + b\text{sech}^{-1}(cx)) + \frac{5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{112c^7}$$

```
[Out] 1/7*x^7*(a+b*arcsech(c*x))-5/112*b*x*(-c*x+1)^(1/2)/c^6/(1/(c*x+1))^(1/2)-5
/168*b*x^3*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/42*b*x^5*(-c*x+1)^(1/2)/c
^2/(1/(c*x+1))^(1/2)+5/112*b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^
7
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6418, 102, 12, 92, 41, 222}

$$\int x^6 (a + b\text{sech}^{-1}(cx))\,dx = \frac{1}{7}x^7(a + b\text{sech}^{-1}(cx)) + \frac{5b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{112c^7} - \frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{cx+1}}} - \frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{cx+1}}} - \frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[x^6*(a + b*ArcSech[c*x]),x]
```

```
[Out] (-5*b*x*Sqrt[1 - c*x])/(112*c^6*Sqrt[(1 + c*x)^(-1)]) - (5*b*x^3*Sqrt[1 - c
*x])/(168*c^4*Sqrt[(1 + c*x)^(-1)]) - (b*x^5*Sqrt[1 - c*x])/(42*c^2*Sqrt[(1
```

```
 + c*x)^(-1)]) + (x^7*(a + b*ArcSech[c*x]))/7 + (5*b*Sqrt[(1 + c*x)^(-1)]*S
qrt[1 + c*x]*ArcSin[c*x])/(112*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{7}x^7\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{1}{7}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^6}{\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\,\text{sech}^{-1}(cx)\right)-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{5x^4}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{42c^2}$$

$$= -\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{\left(5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^4}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{42c^2}$$

$$= -\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\,\text{sech}^{-1}(cx)\right) - \frac{\left(5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{3x^2}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{168c^4}$$

$$= -\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{7}x^7\left(a+b\,\text{sech}^{-1}(cx)\right) + \frac{\left(5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{56c^4}$$

$$= -\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{1+cx}}}-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{7}x^7\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{\left(5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{112c^6}$$

$$= -\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{1+cx}}}-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{7}x^7\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{\left(5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx}{112c^6}$$

$$= -\frac{5bx\sqrt{1-cx}}{112c^6\sqrt{\frac{1}{1+cx}}}-\frac{5bx^3\sqrt{1-cx}}{168c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^5\sqrt{1-cx}}{42c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{7}x^7\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{5b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\arcsin(cx)}{112c^7}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{ax^7}{7}+b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{5x}{112c^6}-\frac{5x^2}{112c^5}-\frac{5x^3}{168c^4}-\frac{5x^4}{168c^3}-\frac{x^5}{42c^2}-\frac{x^6}{42c}\right)+\frac{1}{7}bx^7\operatorname{sech}^{-1}(cx)+\frac{5ib\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{112c^7}$$

```
[In] Integrate[x^6*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^7)/7 + b*Sqrt[(1 - c*x)/(1 + c*x)]*((-5*x)/(112*c^6) - (5*x^2)/(112*c^
5) - (5*x^3)/(168*c^4) - (5*x^4)/(168*c^3) - x^5/(42*c^2) - x^6/(42*c)) + (
b*x^7*ArcSech[c*x])/7 + (((5*I)/112)*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x)])/c^7
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

method	result
parts	$\frac{a\,x^{7}}{7}+\frac{b\left(\frac{c^{7}x^{7}\operatorname{arcsech}(cx)}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(8\sqrt{-c^{2}x^{2}+1}\,c^{5}x^{5}+10c^{3}x^{3}\sqrt{-c^{2}x^{2}+1}+15cx\sqrt{-c^{2}x^{2}+1}-15\arcsin(cx)\right)}{336\sqrt{-c^{2}x^{2}+1}}\right)}{c^{7}}$
derivativedivides	$\frac{\frac{a\,c^{7}x^{7}}{7}+b\left(\frac{c^{7}x^{7}\operatorname{arcsech}(cx)}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(8\sqrt{-c^{2}x^{2}+1}\,c^{5}x^{5}+10c^{3}x^{3}\sqrt{-c^{2}x^{2}+1}+15cx\sqrt{-c^{2}x^{2}+1}-15\arcsin(cx)\right)}{336\sqrt{-c^{2}x^{2}+1}}\right)}{c^{7}}$
default	$\frac{\frac{a\,c^{7}x^{7}}{7}+b\left(\frac{c^{7}x^{7}\operatorname{arcsech}(cx)}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(8\sqrt{-c^{2}x^{2}+1}\,c^{5}x^{5}+10c^{3}x^{3}\sqrt{-c^{2}x^{2}+1}+15cx\sqrt{-c^{2}x^{2}+1}-15\arcsin(cx)\right)}{336\sqrt{-c^{2}x^{2}+1}}\right)}{c^{7}}$

```
[In] int(x^6*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a*x^7+b/c^7*(1/7*c^7*x^7*arcsech(c*x)-1/336*(-(c*x-1)/c/x)^(1/2)*c*x*((
c*x+1)/c/x)^(1/2)*(8*(-c^2*x^2+1)^(1/2)*c^5*x^5+10*c^3*x^3*(-c^2*x^2+1)^(1/
2)+15*c*x*(-c^2*x^2+1)^(1/2)-15*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.29

$$\int x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{48\,ac^7x^7-48\,bc^7\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)-30\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)+48\,(bc^7x^7-bc^7)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{336\,c^7}$$

```
[In] integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] 1/336*(48*a*c^7*x^7 - 48*b*c^7*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)
/x) - 30*b*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 48*(b*c
^7*x^7 - b*c^7)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*
c^6*x^6 + 10*b*c^4*x^4 + 15*b*c^2*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

Sympy [F]

$$\int x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int x^6(a+b\operatorname{asech}(cx))\,dx$$

```
[In] integrate(x**6*(a+b*asech(c*x)),x)
[Out] Integral(x**6*(a + b*asech(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{1}{7}\,ax^7 + \frac{1}{336}\left(48\,x^7\operatorname{arsech}(cx)-\frac{\frac{15\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+40\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}-1}}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^6}}{c}\right)b$$

```
[In] integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] 1/7*a*x^7 + 1/336*(48*x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40*
(1/(c^2*x^2) - 1)^(3/2) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1)^
3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arctan(
sqrt(1/(c^2*x^2) - 1))/c^6)/c)*b
```

Giac [F]

$$\int x^6 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int (b \,\text{arsech}\,(cx) + a)x^6 \, dx$$

```
[In] integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int x^6 \left(a + b \,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^6*(a + b*acosh(1/(c*x))),x)
[Out] int(x^6*(a + b*acosh(1/(c*x))), x)
```

3.20 $\int x^5 \left(a + b\text{sech}^{-1}(cx)\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int x^5 \left(a + b\text{sech}^{-1}(cx)\right) dx = -\frac{4b\sqrt{1-cx}}{45c^6\sqrt{\frac{1}{1+cx}}} - \frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{6}x^6\left(a + b\text{sech}^{-1}(cx)\right)$$

```
[Out] 1/6*x^6*(a+b*arcsech(c*x))-4/45*b*(-c*x+1)^(1/2)/c^6/(1/(c*x+1))^(1/2)-2/45
*b*x^2*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/30*b*x^4*(-c*x+1)^(1/2)/c^2/(
1/(c*x+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6418, 102, 12, 75}

$$\int x^5 \left(a+b\text{sech}^{-1}(cx)\right) dx = \frac{1}{6}x^6\left(a+b\text{sech}^{-1}(cx)\right) - \frac{4b\sqrt{1-cx}}{45c^6\sqrt{\frac{1}{cx+1}}} - \frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{cx+1}}} - \frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[x^5*(a + b*ArcSech[c*x]),x]
```

```
[Out] (-4*b*Sqrt[1 - c*x])/(45*c^6*Sqrt[(1 + c*x)^(-1)]) - (2*b*x^2*Sqrt[1 - c*x]
)/(45*c^4*Sqrt[(1 + c*x)^(-1)]) - (b*x^4*Sqrt[1 - c*x])/(30*c^2*Sqrt[(1 + c
*x)^(-1)]) + (x^6*(a + b*ArcSech[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{6}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^5}{\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&= -\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{6}x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{4x^3}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{30c^2}\\
&= -\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{6}x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{15c^2}\\
&= -\frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{6}x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad-\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{2x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{45c^4}
\end{aligned}$$

$$= -\frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{6}x^6\left(a + b\text{sech}^{-1}(cx)\right) + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{45c^4}$$

$$= -\frac{4b\sqrt{1-cx}}{45c^6\sqrt{\frac{1}{1+cx}}} - \frac{2bx^2\sqrt{1-cx}}{45c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^4\sqrt{1-cx}}{30c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{6}x^6\left(a + b\text{sech}^{-1}(cx)\right)$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int x^5\left(a + b\text{sech}^{-1}(cx)\right)\,dx = \frac{ax^6}{6} + b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{4}{45c^6} - \frac{4x}{45c^5} - \frac{2x^2}{45c^4} - \frac{2x^3}{45c^3} - \frac{x^4}{30c^2} - \frac{x^5}{30c}\right) + \frac{1}{6}bx^6\text{sech}^{-1}(cx)$$

```
[In] Integrate[x^5*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^6)/6 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-4/(45*c^6) - (4*x)/(45*c^5) - (2*
x^2)/(45*c^4) - (2*x^3)/(45*c^3) - x^4/(30*c^2) - x^5/(30*c)) + (b*x^6*ArcS
ech[c*x])/6
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

method	result	size
parts	$\frac{a\,x^{6}}{6}+\frac{b\left(\frac{c^{6}x^{6}\,\mathrm{arcsech}(cx)}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(3c^{4}x^{4}+4c^{2}x^{2}+8\right)}{90}\right)}{c^{6}}$	77
derivativedivides	$\frac{\frac{a\,c^{6}x^{6}}{6}+b\left(\frac{c^{6}x^{6}\,\mathrm{arcsech}(cx)}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(3c^{4}x^{4}+4c^{2}x^{2}+8\right)}{90}\right)}{c^{6}}$	81
default	$\frac{\frac{a\,c^{6}x^{6}}{6}+b\left(\frac{c^{6}x^{6}\,\mathrm{arcsech}(cx)}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(3c^{4}x^{4}+4c^{2}x^{2}+8\right)}{90}\right)}{c^{6}}$	81

```
[In] int(x^5*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*x^6+b/c^6*(1/6*c^6*x^6*arcsech(c*x)-1/90*(-(c*x-1)/c/x)^(1/2)*c*x*((c
*x+1)/c/x)^(1/2)*(3*c^4*x^4+4*c^2*x^2+8))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int x^5\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{15\,bc^5x^6\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+15\,ac^5x^6-\left(3\,bc^4x^5+4\,bc^2x^3+8\,bx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{90\,c^5}$$

```
[In] integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*b*c^5*x^6*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 15
*a*c^5*x^6 - (3*b*c^4*x^5 + 4*b*c^2*x^3 + 8*b*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int x^5\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \begin{cases} \frac{ax^6}{6}+\frac{bx^6\,\text{asech}\,(cx)}{6}-\frac{bx^4\sqrt{-c^2x^2+1}}{30c^2}-\frac{2bx^2\sqrt{-c^2x^2+1}}{45c^4}-\frac{4b\sqrt{-c^2x^2+1}}{45c^6} & \text{for}\ c\neq 0 \\ \frac{x^6(a+\infty b)}{6} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*asech(c*x)/6 - b*x**4*sqrt(-c**2*x**2 + 1)/(30
*c**2) - 2*b*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 4*b*sqrt(-c**2*x**2 + 1)
/(45*c**6), Ne(c, 0)), (x**6*(a + oo*b)/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int x^5\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{1}{6}\,ax^6 + \frac{1}{90}\left(15\,x^6\,\text{arsech}\,(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)b$$

```
[In] integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2)
- 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b
```

Giac [F]

$$\int x^5 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int (b \,\text{arsech}\,(cx) + a)x^5 \, dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int x^5 \left(a + b \,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^5*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^5*(a + b*acosh(1/(c*x))), x)
```

3.21 $\int x^4(a+b\text{sech}^{-1}(cx))\ dx$

Optimal result

Integrand size = 12, antiderivative size = 110

$$\int x^4(a+b\text{sech}^{-1}(cx))\ dx=-\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{5}x^5(a+b\text{sech}^{-1}(cx))+\frac{3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{40c^5}$$

```
[Out] 1/5*x^5*(a+b*arcsech(c*x))-3/40*b*x*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/
20*b*x^3*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)+3/40*b*arcsin(c*x)*(1/(c*x+1)
)^(1/2)*(c*x+1)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6418, 102, 12, 92, 41, 222}

$$\int x^4(a+b\text{sech}^{-1}(cx))\ dx=\frac{1}{5}x^5(a+b\text{sech}^{-1}(cx))+\frac{3b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{40c^5}-\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{cx+1}}}-\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[x^4*(a + b*ArcSech[c*x]),x]
```

```
[Out] (-3*b*x*Sqrt[1 - c*x])/(40*c^4*Sqrt[(1 + c*x)^(-1)]) - (b*x^3*Sqrt[1 - c*x]
)/(20*c^2*Sqrt[(1 + c*x)^(-1)]) + (x^5*(a + b*ArcSech[c*x]))/5 + (3*b*Sqrt[
(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(40*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{5}x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^4}{\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5}x^5\left(a + b\operatorname{sech}^{-1}(cx)\right) - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{3x^2}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{20c^2}$$

$$= -\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5}x^5\left(a + b\operatorname{sech}^{-1}(cx)\right) + \frac{\left(3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{20c^2}$$

$$= -\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5}x^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{\left(3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{40c^4}$$

$$= -\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5}x^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{\left(3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx}{40c^4}$$

$$= -\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5}x^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{3b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{40c^5}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int x^4\left(a + b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{ax^5}{5} + b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{3x}{40c^4} - \frac{3x^2}{40c^3} - \frac{x^3}{20c^2} - \frac{x^4}{20c}\right) + \frac{1}{5}bx^5\operatorname{sech}^{-1}(cx) + \frac{3ib\log\left(-2icx + 2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{40c^5}$$

```
[In] Integrate[x^4*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^5)/5 + b*Sqrt[(1 - c*x)/(1 + c*x)]*((-3*x)/(40*c^4) - (3*x^2)/(40*c^3)
 - x^3/(20*c^2) - x^4/(20*c)) + (b*x^5*ArcSech[c*x])/5 + (((3*I)/40)*b*Log[
(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^5
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04

method	result	size
parts	$\frac{a\,x^{5}}{5}+\frac{b\left(\frac{c^{5} x^{5} \operatorname{arcsech}\left(c x\right)}{5}+\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(-2 c^{3} x^{3} \sqrt{-c^{2} x^{2}+1}-3 c x \sqrt{-c^{2} x^{2}+1}+3 \arcsin \left(c x \right)\right)}{40 \sqrt{-c^{2} x^{2}+1}}\right)}{c^{5}}$	114
derivativedivides	$\frac{\frac{a\,c^{5} x^{5}}{5}+b\left(\frac{c^{5} x^{5} \operatorname{arcsech}\left(c x\right)}{5}+\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(-2 c^{3} x^{3} \sqrt{-c^{2} x^{2}+1}-3 c x \sqrt{-c^{2} x^{2}+1}+3 \arcsin \left(c x \right)\right)}{40 \sqrt{-c^{2} x^{2}+1}}\right)}{c^{5}}$	118
default	$\frac{\frac{a\,c^{5} x^{5}}{5}+b\left(\frac{c^{5} x^{5} \operatorname{arcsech}\left(c x\right)}{5}+\frac{\sqrt{-\frac{c x -1}{c x}}\, c x \sqrt{\frac{c x +1}{c x}}\, \left(-2 c^{3} x^{3} \sqrt{-c^{2} x^{2}+1}-3 c x \sqrt{-c^{2} x^{2}+1}+3 \arcsin \left(c x \right)\right)}{40 \sqrt{-c^{2} x^{2}+1}}\right)}{c^{5}}$	118

```
[In] int(x^4*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+b/c^5*(1/5*c^5*x^5*arcsech(c*x)+1/40*(-(c*x-1)/c/x)^(1/2)*c*x*((c
*x+1)/c/x)^(1/2)*(-2*c^3*x^3*(-c^2*x^2+1)^(1/2)-3*c*x*(-c^2*x^2+1)^(1/2)+3*
arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.58

$$\int x^4\big(a+b\text{sech}^{-1}(cx)\big)\;dx$$

$$= \frac{8\,ac^5x^5 - 8\,bc^5\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 6\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + 8\,(bc^5x^5-bc^5)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - (2}{40\,c^5}$$

```
[In] integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*x^5 - 8*b*c^5*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x)
 - 6*b*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 8*(b*c^5*x^
5 - b*c^5)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*b*c^4*x
^4 + 3*b*c^2*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^5
```

Sympy [F]

$$\int x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int x^4(a+b\operatorname{asech}(cx))\,dx$$

```
[In] integrate(x**4*(a+b*asech(c*x)),x)
[Out] Integral(x**4*(a + b*asech(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{1}{5}\,ax^5+\frac{1}{40}\left(8\,x^5\operatorname{arsech}(cx)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}+\frac{3\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)b$$

```
[In] integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] 1/5*a*x^5 + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqrt
(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4
) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b
```

Giac [F]

$$\int x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int (b\operatorname{arsech}(cx)+a)x^4\,dx$$

```
[In] integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int x^4 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^4*(a + b*acosh(1/(c*x))),x)
[Out] int(x^4*(a + b*acosh(1/(c*x))), x)
```

3.22 $\int x^3 (a + b\text{sech}^{-1}(cx)) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int x^3 (a + b\text{sech}^{-1}(cx)) \, dx = -\frac{b\sqrt{1-cx}}{6c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{4}x^4 (a + b\text{sech}^{-1}(cx))$$

```
[Out] 1/4*x^4*(a+b*arcsech(c*x))-1/6*b*(-c*x+1)^(1/2)/c^4/(1/(c*x+1))^(1/2)-1/12*
b*x^2*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6418, 102, 12, 75}

$$\int x^3 (a + b\text{sech}^{-1}(cx)) \, dx = \frac{1}{4}x^4 (a + b\text{sech}^{-1}(cx)) - \frac{b\sqrt{1-cx}}{6c^4\sqrt{\frac{1}{cx+1}}} - \frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[x^3*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/6*(b*Sqrt[1 - c*x])/(c^4*Sqrt[(1 + c*x)^(-1)]) - (b*x^2*Sqrt[1 - c*x])/(
12*c^2*Sqrt[(1 + c*x)^(-1)]) + (x^4*(a + b*ArcSech[c*x]))/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3}{\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&= -\frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{2x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{12c^2}\\
&= -\frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{6c^2}\\
&= -\frac{b\sqrt{1-cx}}{6c^4\sqrt{\frac{1}{1+cx}}}-\frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int x^3(a+b\text{sech}^{-1}(cx))\,dx=\frac{ax^4}{4}+b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{1}{6c^4}-\frac{x}{6c^3}-\frac{x^2}{12c^2}-\frac{x^3}{12c}\right)+\frac{1}{4}bx^4\text{sech}^{-1}(cx)$$

```
[In] Integrate[x^3*(a + b*ArcSech[c*x]),x]
[Out] (a*x^4)/4 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/6*1/c^4 - x/(6*c^3) - x^2/(12*c
^2) - x^3/(12*c)) + (b*x^4*ArcSech[c*x])/4
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{a\,x^4}{4}+\frac{b\left(\frac{c^4x^4\,\text{arcsech}(cx)}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(c^2x^2+2)}{12}\right)}{c^4}$	68
derivativedivides	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\,\text{arcsech}(cx)}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(c^2x^2+2)}{12}\right)}{c^4}$	72
default	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\,\text{arcsech}(cx)}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(c^2x^2+2)}{12}\right)}{c^4}$	72

```
[In] int(x^3*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/4*a*x^4+b/c^4*(1/4*c^4*x^4*arcsech(c*x)-1/12*(-(c*x-1)/c/x)^(1/2)*c*x*((c
*x+1)/c/x)^(1/2)*(c^2*x^2+2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int x^3(a+b\text{sech}^{-1}(cx))\,dx=\frac{3\,bc^3x^4\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+3\,ac^3x^4-(bc^2x^3+2\,bx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{12\,c^3}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] 1/12*(3*b*c^3*x^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 3*a
*c^3*x^4 - (b*c^2*x^3 + 2*b*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\begin{cases}\frac{ax^4}{4}+\frac{bx^4\operatorname{asech}(cx)}{4}-\frac{bx^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{b\sqrt{-c^2x^2+1}}{6c^4} & \text{for } c\neq 0\\ \frac{x^4(a+\infty b)}{4} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3*(a+b*asech(c*x)),x)
[Out] Piecewise((a*x**4/4 + b*x**4*asech(c*x)/4 - b*x**2*sqrt(-c**2*x**2 + 1)/(12
*c**2) - b*sqrt(-c**2*x**2 + 1)/(6*c**4), Ne(c, 0)), (x**4*(a + oo*b)/4, Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\operatorname{arsech}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)b$$

```
[In] integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3
*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b
```

Giac [F]

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int (b\operatorname{arsech}(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\operatorname{sech}^{-1}(cx)\right) \, dx = \int x^3 \left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^3*(a + b*acosh(1/(c*x))),x)
[Out] int(x^3*(a + b*acosh(1/(c*x))), x)
```

3.23 $\int x^2(a+b\text{sech}^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int x^2(a+b\text{sech}^{-1}(cx))\,dx=-\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{3}x^3(a+b\text{sech}^{-1}(cx))+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3}$$

```
[Out] 1/3*x^3*(a+b*arcsech(c*x))-1/6*b*x*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)+1/6
*b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6418, 92, 41, 222}

$$\int x^2(a+b\text{sech}^{-1}(cx))\,dx=\frac{1}{3}x^3(a+b\text{sech}^{-1}(cx))+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{6c^3}-\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[x^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/6*(b*x*Sqrt[1 - c*x])/(c^2*Sqrt[(1 + c*x)^(-1)]) + (x^3*(a + b*ArcSech[c
*x]))/3 + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

```
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2}{\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&= -\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{3}x^3\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{6c^2}\\
&= -\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{3}x^3\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx}{6c^2}\\
&= -\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{3}x^3\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.32

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{ax^3}{3}+b\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{x}{6c^2}-\frac{x^2}{6c}\right)+\frac{1}{3}bx^3\text{sech}^{-1}(cx)+\frac{ib\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{6c^3}$$

```
[In] Integrate[x^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^3)/3 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/6*x/c^2 - x^2/(6*c)) + (b*x^3*A
rcSech[c*x])/3 + ((I/6)*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)])/c^3
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

method	result	size
parts	$\frac{a\,x^3}{3}+\frac{b\left(\frac{c^3x^3\,\text{arcsech}(cx)}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(-cx\sqrt{-c^2x^2+1}+\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^3}$	92
derivativedivides	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\text{arcsech}(cx)}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(-cx\sqrt{-c^2x^2+1}+\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^3}$	96
default	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\text{arcsech}(cx)}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(-cx\sqrt{-c^2x^2+1}+\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^3}$	96

```
[In] int(x^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+b/c^3*(1/3*c^3*x^3*arcsech(c*x)+1/6*(-(c*x-1)/c/x)^(1/2)*c*x*((c*
x+1)/c/x)^(1/2)*(-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.08

$$\int x^2 \left(a + b\mathrm{sech}^{-1}(cx)\right)\, dx$$
$$= \frac{2\, ac^3x^3 - bc^2x^2\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2\, bc^3 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\, b \arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + 2\left(bc^3x^3 - bc^3\right)\log(}{6\, c^3}$$

```
[In] integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3 - b*c^2*x^2*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*b*c^3*log((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*arctan((c*x*sqrt(-(c^2*x^2
 - 1)/(c^2*x^2)) - 1)/(c*x)) + 2*(b*c^3*x^3 - b*c^3)*log((c*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c^3
```

Sympy [F]

$$\int x^2 \left(a + b\mathrm{sech}^{-1}(cx)\right)\, dx = \int x^2 \left(a + b \operatorname{asech}\left(cx\right)\right)\, dx$$

```
[In] integrate(x**2*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int x^2 \left(a+b\mathrm{sech}^{-1}(cx)\right)\, dx = \frac{1}{3}\, ax^3 + \frac{1}{6}\left(2\, x^3\, \operatorname{arsech}\left(cx\right) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right) b$$

```
[In] integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x
^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b
```

Giac [F]

$$\int x^2 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int (b \operatorname{arsech}(cx) + a)x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^2*(a + b*acosh(1/(c*x))),x)
[Out] int(x^2*(a + b*acosh(1/(c*x))), x)
```

3.24 $\int x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx=-\frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{2}x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)$$

```
[Out] 1/2*x^2*(a+b*arcsech(c*x))-1/2*b*(-c*x+1)^(1/2)/c^2/(1/(c*x+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6418, 75}

$$\int x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx=\frac{1}{2}x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)-\frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[x*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/2*(b*Sqrt[1 - c*x])/(c^2*Sqrt[(1 + c*x)^(-1)]) + (x^2*(a + b*ArcSech[c*x
]))/2
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\left(a+b\,\text{sech}^{-1}(cx)\right)+\frac{1}{2}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx\\ &= -\frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{1+cx}}}+\frac{1}{2}x^2\left(a+b\,\text{sech}^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int x\left(a+b\,\text{sech}^{-1}(cx)\right)\,dx=\frac{ax^2}{2}+b\left(-\frac{1}{2c^2}-\frac{x}{2c}\right)\sqrt{\frac{1-cx}{1+cx}}+\frac{1}{2}bx^2\text{sech}^{-1}(cx)$$

```
[In] Integrate[x*(a + b*ArcSech[c*x]),x]
[Out] (a*x^2)/2 + b*(-1/2*1/c^2 - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)] + (b*x^2*Arc
Sech[c*x])/2
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

method	result	size
parts	$\frac{a\,x^2}{2}+\frac{b\left(\frac{c^2x^2\,\text{arcsech}(cx)}{2}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{2}\right)}{c^2}$	59
derivativedivides	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\text{arcsech}(cx)}{2}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{2}\right)}{c^2}$	63
default	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\text{arcsech}(cx)}{2}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{2}\right)}{c^2}$	63

```
[In] int(x*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arcsech(c*x)-1/2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*
x+1)/c/x)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int x\left(a + b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{bcx^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + acx^2 - bx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{2\,c}$$

```
[In] integrate(x*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] 1/2*(b*c*x^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + a*c*x^2
- b*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x\left(a + b\operatorname{sech}^{-1}(cx)\right)\,dx = \begin{cases} \frac{ax^2}{2} + \frac{bx^2\operatorname{asech}(cx)}{2} - \frac{b\sqrt{-c^2x^2+1}}{2c^2} & \text{for}\ c \neq 0 \\ \frac{x^2(a+\infty b)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(a+b*asech(c*x)),x)
[Out] Piecewise((a*x**2/2 + b*x**2*asech(c*x)/2 - b*sqrt(-c**2*x**2 + 1)/(2*c**2)
, Ne(c, 0)), (x**2*(a + oo*b)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int x\left(a + b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{1}{2}\,ax^2 + \frac{1}{2}\left(x^2\operatorname{arsech}(cx) - \frac{x\sqrt{\frac{1}{c^2x^2} - 1}}{c}\right)b$$

```
[In] integrate(x*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*b
```

Giac [F]

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (b\,\text{arsech}\,(cx)+a)x\,dx$$

[In] integrate(x*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x, x)

Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,x^2\,\text{acosh}\left(\frac{1}{c\,x}\right)}{2}-\frac{b\,x\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{2\,c}$$

[In] int(x*(a + b*acosh(1/(c*x))),x)

[Out] (a*x^2)/2 + (b*x^2*acosh(1/(c*x)))/2 - (b*x*(1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))/(2*c)

3.25 $\int (a + b\text{sech}^{-1}(cx))\ dx$

Optimal result

Integrand size = 8, antiderivative size = 40

$$\int (a + b\text{sech}^{-1}(cx))\ dx = ax + bx\text{sech}^{-1}(cx) + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}$$

```
[Out] a*x+b*x*arcsech(c*x)+b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6412, 222}

$$\int (a + b\text{sech}^{-1}(cx))\ dx = ax + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{c} + bx\text{sech}^{-1}(cx)$$

```
[In] Int[a + b*ArcSech[c*x],x]
```

```
[Out] a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])
/c
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6412

```
Int[ArcSech[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSech[c*x], x] + Dist[Sqrt[1
 + c*x]*Sqrt[1/(1 + c*x)], Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= ax + b\int \operatorname{sech}^{-1}(cx)\,dx \\
&= ax + bx\operatorname{sech}^{-1}(cx) + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx \\
&= ax + bx\operatorname{sech}^{-1}(cx) + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$
\int \left(a + b\operatorname{sech}^{-1}(cx)\right)\,dx = ax + bx\operatorname{sech}^{-1}(cx) + \frac{2b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x}
$$

```
[In] Integrate[a + b*ArcSech[c*x],x]
[Out] a*x + b*x*ArcSech[c*x] + (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*A
rcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^2*x)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

method	result	size
default	$ax + bx\ \operatorname{arcsech}(cx) - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)}{c}$	42
parts	$ax + bx\ \operatorname{arcsech}(cx) - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)}{c}$	42
derivativedivides	$\frac{acx+b\left(cx\ \operatorname{arcsech}(cx)-\arctan\left(\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{c}$	46

```
[In] int(a+b*arcsech(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b*x*arcsech(c*x)-b/c*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.98

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{acx - bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + (bcx - bc)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{c}$$

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="fricas")
[Out] (a*c*x - b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*arctan((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + (b*c*x - b*c)*log((c*x*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

Sympy [F]

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int (a + b\operatorname{asech}(cx))\,dx$$

```
[In] integrate(a+b*asech(c*x),x)
[Out] Integral(a + b*asech(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = ax + \frac{\left(cx\operatorname{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)b}{c}$$

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="maxima")
[Out] a*x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b/c
```

Giac [F]

$$\int \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int b \operatorname{arsech}(cx) + a \, dx$$

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="giac")
[Out] integrate(b*arcsech(c*x) + a, x)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a + b\text{sech}^{-1}(cx)\right) \, dx = a\,x + b\,x\,\mathrm{acosh}\left(\frac{1}{c\,x}\right) + \frac{b\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}$$

```
[In] int(a + b*acosh(1/(c*x)),x)
[Out] a*x + b*x*acosh(1/(c*x)) + (b*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/
2))))/c
```

3.26 $\int \frac{a+b\text{sech}^{-1}(cx)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x} dx = -\frac{(a+b\text{sech}^{-1}(cx))^2}{2b} - (a+b\text{sech}^{-1}(cx))\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) + \frac{1}{2}b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)$$

```
[Out] -1/2*(a+b*arcsech(c*x))^2/b-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2)+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6416, 5882, 3799, 2221, 2317, 2438}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x} dx = -\frac{(a+b\text{sech}^{-1}(cx))^2}{2b} - \log\left(e^{-2\text{sech}^{-1}(cx)}+1\right)(a+b\text{sech}^{-1}(cx)) + \frac{1}{2}b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)$$

```
[In] Int[(a + b*ArcSech[c*x])/x,x]
```

```
[Out] -1/2*(a + b*ArcSech[c*x])^2/b - (a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[
c*x])] + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6416

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\text{Subst}\left(\int x\tanh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{sech}^{-1}(cx)\right)}{b}\\
&= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2b}+\frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{sech}^{-1}(cx)\right)}{b}
\end{aligned}
$$

$$= -\frac{(a+b\text{sech}^{-1}(cx))^2}{2b} - (a+b\text{sech}^{-1}(cx))\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) + \text{Subst}\left(\int \log\left(1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{sech}^{-1}(cx)\right)$$

$$= -\frac{(a+b\text{sech}^{-1}(cx))^2}{2b} - (a+b\text{sech}^{-1}(cx))\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) - \frac{1}{2}b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{sech}^{-1}(cx)}{b}\right)}\right)$$

$$= -\frac{(a+b\text{sech}^{-1}(cx))^2}{2b} - (a+b\text{sech}^{-1}(cx))\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) + \frac{1}{2}b\,\text{PolyLog}\left(2, -e^{2\left(\frac{a}{b}-\frac{a+b\text{sech}^{-1}(cx)}{b}\right)}\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x}\,dx = a\log(x) + \frac{1}{2}b\left(-\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx)+2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2, -e^{-2\text{sech}^{-1}(cx)}\right)\right)$$

```
[In] Integrate[(a + b*ArcSech[c*x])/x,x]
[Out] a*Log[x] + (b*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])
])) + PolyLog[2, -E^(-2*ArcSech[c*x])]))/2
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.75

method	result
parts	$a\ln(x) + b\left(\frac{\text{arcsech}(cx)^2}{2} - \text{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right) - \frac{\text{polylog}(}{}\right.$
derivativedivides	$a\ln(cx) + b\left(\frac{\text{arcsech}(cx)^2}{2} - \text{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right) - \frac{\text{polylog}}{}\right.$
default	$a\ln(cx) + b\left(\frac{\text{arcsech}(cx)^2}{2} - \text{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right) - \frac{\text{polylog}}{}\right.$

```
[In] int((a+b*arcsech(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(x)+b*(1/2*arcsech(c*x)^2-arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))^2)-1/2*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)
)
```

Fricas [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x}\,dx = \int \frac{b\operatorname{arsech}(cx) + a}{x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x}\,dx = \int \frac{a + b\operatorname{asech}(cx)}{x}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x,x)
```

```
[Out] Integral((a + b*asech(c*x))/x, x)
```

Maxima [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x}\,dx = \int \frac{b\operatorname{arsech}(cx) + a}{x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x) + a*lo
g(x)
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x} \, dx = \int \frac{b \operatorname{arsech}(cx) + a}{x} \, dx$$

```
[In] integrate((a+b*arcsech(c*x))/x,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x} \, dx = \int \frac{a + b \operatorname{acosh}\left(\frac{1}{cx}\right)}{x} \, dx$$

```
[In] int((a + b*acosh(1/(c*x)))/x,x)
[Out] int((a + b*acosh(1/(c*x)))/x, x)
```

3.27 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2}\,dx=\frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{1+cx}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{x}$$

```
[Out] (-a-b*arcsech(c*x))/x+b*(-c*x+1)^(1/2)/x/(1/(c*x+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6418, 97}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2}\,dx=\frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{cx+1}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{x}$$

```
[In] Int[(a + b*ArcSech[c*x])/x^2,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/x
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{x}-\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx\\ &= \frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{1+cx}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int\frac{a+b\operatorname{sech}^{-1}(cx)}{x^2}\,dx=-\frac{a}{x}+b\left(c+\frac{1}{x}\right)\sqrt{\frac{1-cx}{1+cx}}-\frac{b\operatorname{sech}^{-1}(cx)}{x}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/x^2,x]
[Out] -(a/x) + b*(c + x^(-1))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/x
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\right)$	54
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\right)\right)$	58
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsech}(cx)}{cx}+\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\right)\right)$	58

```
[In] int((a+b*arcsech(c*x))/x^2,x,method=_RETURNVERBOSE)
[Out] -a/x+b*c*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2} \, dx = \frac{bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - b\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - a}{x}$$

```
[In] integrate((a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
[Out] (b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2
*x^2)) + 1)/(c*x)) - a)/x
```

Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2} \, dx = \int \frac{a + b \operatorname{asech}(cx)}{x^2} \, dx$$

```
[In] integrate((a+b*asech(c*x))/x**2,x)
[Out] Integral((a + b*asech(c*x))/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2} \, dx = \left(c\sqrt{\frac{1}{c^2x^2} - 1} - \frac{\operatorname{arsech}(cx)}{x}\right)b - \frac{a}{x}$$

```
[In] integrate((a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b - a/x
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{x^2}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2}\,dx = b\,c\,\sqrt{\frac{1}{c\,x} - 1}\,\sqrt{\frac{1}{c\,x} + 1} - \frac{b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x} - \frac{a}{x}$$

```
[In] int((a + b*acosh(1/(c*x)))/x^2,x)
```

```
[Out] b*c*(1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) - (b*acosh(1/(c*x)))/x - a/x
```

3.28 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 94

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3} \, dx = \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{2x^2} + \frac{1}{4}bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)$$

```
[Out] 1/2*(-a-b*arcsech(c*x))/x^2+1/4*b*(-c*x+1)^(1/2)/x^2/(1/(c*x+1))^(1/2)+1/4*
b*c^2*arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6418, 105, 12, 94, 214}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3} \, dx = -\frac{a+b\text{sech}^{-1}(cx)}{2x^2} + \frac{1}{4}bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\sqrt{1-cx}\sqrt{cx+1}\right) + \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[(a + b*ArcSech[c*x])/x^3,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(4*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(2*x^
2) + (b*c^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1
 + c*x]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\,\text{sech}^{-1}(cx)}{2x^2}-\frac{1}{2}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^3\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&=\frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}}-\frac{a+b\,\text{sech}^{-1}(cx)}{2x^2}-\frac{1}{4}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{c^2}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&=\frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}}-\frac{a+b\,\text{sech}^{-1}(cx)}{2x^2}-\frac{1}{4}\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx
\end{aligned}$$

$$= \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{2x^2} + \frac{1}{4}\left(bc^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{c-cx^2}\,dx, x, \sqrt{1-cx}\sqrt{1+cx}\right)$$

$$= \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{2x^2} + \frac{1}{4}bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3}\,dx = -\frac{a}{2x^2} + b\left(\frac{1}{4x^2}+\frac{c}{4x}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{b\text{sech}^{-1}(cx)}{2x^2} - \frac{1}{4}bc^2\log(x) + \frac{1}{4}bc^2\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)$$

```
[In] Integrate[(a + b*ArcSech[c*x])/x^3,x]
```

```
[Out] -1/2*a/x^2 + b*(1/(4*x^2) + c/(4*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech
[c*x])/(2*x^2) - (b*c^2*Log[x])/4 + (b*c^2*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)
] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/4
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

method	result	size
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\text{arcsech}(cx)}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2x^2+\sqrt{-c^2x^2+1}\right)}{4cx\sqrt{-c^2x^2+1}}\right)$	108
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\text{arcsech}(cx)}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2x^2+\sqrt{-c^2x^2+1}\right)}{4cx\sqrt{-c^2x^2+1}}\right)\right)$	112
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\text{arcsech}(cx)}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2x^2+\sqrt{-c^2x^2+1}\right)}{4cx\sqrt{-c^2x^2+1}}\right)\right)$	112

```
[In] int((a+b*arcsech(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c/x*((
c*x+1)/c/x)^(1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/2)
```

```
)/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^3}\,dx = \frac{bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + (bc^2x^2 - 2\,b)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - 2\,a}{4\,x^2}$$

```
[In] integrate((a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
[Out] 1/4*(b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + (b*c^2*x^2 - 2*b)*log((c*x*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*a)/x^2
```

Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^3}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{x^3}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**3,x)
[Out] Integral((a + b*asech(c*x))/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^3}\,dx = -\frac{1}{8}\,b\left(\frac{\frac{2\,c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) + c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right)}{c} + \frac{4\,\text{arsech}\,(cx)}{x^2}\right) - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
[Out] -1/8*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^
3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) -
1))/c + 4*arcsech(c*x)/x^2) - 1/2*a/x^2
```

Giac [F]

$$\int \frac{a + b\mathrm{sech}^{-1}(cx)}{x^3}\, dx = \int \frac{b\,\mathrm{arsech}\,(cx) + a}{x^3}\, dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{a + b\mathrm{sech}^{-1}(cx)}{x^3}\, dx = \frac{b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\,\left(\frac{c^2\,x}{4} - \frac{1}{2\,x}\right)}{x} - \frac{a}{2\,x^2} + \frac{b\,c\,\sqrt{\frac{1}{c\,x} - 1}\,\sqrt{\frac{1}{c\,x} + 1}}{4\,x}$$

```
[In] int((a + b*acosh(1/(c*x)))/x^3,x)
[Out] (b*acosh(1/(c*x))*((c^2*x)/4 - 1/(2*x)))/x - a/(2*x^2) + (b*c*(1/(c*x) - 1)
^(1/2)*(1/(c*x) + 1)^(1/2))/(4*x)
```

3.29 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4}\,dx=\frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{1+cx}}}+\frac{2bc^2\sqrt{1-cx}}{9x\sqrt{\frac{1}{1+cx}}}-\frac{a+b\text{sech}^{-1}(cx)}{3x^3}$$

```
[Out] 1/3*(-a-b*arcsech(c*x))/x^3+1/9*b*(-c*x+1)^(1/2)/x^3/(1/(c*x+1))^(1/2)+2/9*
b*c^2*(-c*x+1)^(1/2)/x/(1/(c*x+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6418, 105, 12, 97}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4}\,dx=-\frac{a+b\text{sech}^{-1}(cx)}{3x^3}+\frac{2bc^2\sqrt{1-cx}}{9x\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[(a + b*ArcSech[c*x])/x^4,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(9*x^3*Sqrt[(1 + c*x)^(-1)]) + (2*b*c^2*Sqrt[1 - c*x])/(9
*x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3}-\frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^4\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&=\frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{1+cx}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3}+\frac{1}{9}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{2c^2}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&=\frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{1+cx}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3}-\frac{1}{9}\left(2bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&=\frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{1+cx}}}+\frac{2bc^2\sqrt{1-cx}}{9x\sqrt{\frac{1}{1+cx}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4}\,dx=-\frac{a}{3x^3}+b\left(\frac{2c^3}{9}+\frac{1}{9x^3}+\frac{c}{9x^2}+\frac{2c^2}{9x}\right)\sqrt{\frac{1-cx}{1+cx}}-\frac{b\text{sech}^{-1}(cx)}{3x^3}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 + b*((2*c^3)/9 + 1/(9*x^3) + c/(9*x^2) + (2*c^2)/(9*x))*Sqrt[(1
- c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(3*x^3)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\text{arcsech}(cx)}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(2c^2x^2+1)}{9c^2x^2}\right)$	73
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\text{arcsech}(cx)}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(2c^2x^2+1)}{9c^2x^2}\right)\right)$	77
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\text{arcsech}(cx)}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(2c^2x^2+1)}{9c^2x^2}\right)\right)$	77

```
[In] int((a+b*arcsech(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arcsech(c*x)+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^
2*((c*x+1)/c/x)^(1/2)*(2*c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4}\,dx=-\frac{3\,b\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(2\,bc^3x^3+bcx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+3\,a}{9\,x^3}$$

```
[In] integrate((a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*b*c^3*x^
3 + b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 3*a)/x^3
```

Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{x^4}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**4,x)
[Out] Integral((a + b*asech(c*x))/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4}\,dx = \frac{1}{9}b\left(\frac{c^4\left(\frac{1}{c^2x^2} - 1\right)^{\frac{3}{2}} + 3\,c^4\sqrt{\frac{1}{c^2x^2} - 1}}{c} - \frac{3\ \text{arsech}\,(cx)}{x^3}\right) - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
[Out] 1/9*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*ar
csech(c*x)/x^3) - 1/3*a/x^3
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{x^4}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^4,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/x^4,x)
[Out] int((a + b*acosh(1/(c*x)))/x^4, x)
```

3.30 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 126

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx=\frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}}+\frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}}-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)$$

```
[Out] 1/4*(-a-b*arcsech(c*x))/x^4+1/16*b*(-c*x+1)^(1/2)/x^4/(1/(c*x+1))^(1/2)+3/3
2*b*c^2*(-c*x+1)^(1/2)/x^2/(1/(c*x+1))^(1/2)+3/32*b*c^4*arctanh((-c*x+1)^(1
/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6418, 105, 12, 94, 214}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx=-\frac{a+b\text{sech}^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{arctanh}\left(\sqrt{1-cx}\sqrt{cx+1}\right)+\frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[(a + b*ArcSech[c*x])/x^5,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(16*x^4*Sqrt[(1 + c*x)^(-1)]) + (3*b*c^2*Sqrt[1 - c*x])/(
32*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(4*x^4) + (3*b*c^4*Sqrt
[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\,\text{sech}^{-1}(cx)}{4x^4} - \frac{1}{4}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^5\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} - \frac{a+b\,\text{sech}^{-1}(cx)}{4x^4} + \frac{1}{16}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{3c^2}{x^3\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$
\begin{aligned}
&= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} - \frac{1}{16}\left(3bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^3\sqrt{1-cx}\sqrt{1+cx}}\,dx \\
&= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} \\
&\quad - \frac{1}{32}\left(3bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{c^2}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx \\
&= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} \\
&\quad - \frac{1}{32}\left(3bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx \\
&= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} \\
&\quad + \frac{1}{32}\left(3bc^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{c-cx^2}\,dx, x, \sqrt{1-cx}\sqrt{1+cx}\right) \\
&= \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{1+cx}}} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{4x^4} \\
&\quad + \frac{3}{32}bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09

$$
\begin{aligned}
\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx = &-\frac{a}{4x^4} + b\left(\frac{1}{16x^4} + \frac{c}{16x^3} + \frac{3c^2}{32x^2} + \frac{3c^3}{32x}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{b\text{sech}^{-1}(cx)}{4x^4} \\
&- \frac{3}{32}bc^4\log(x) + \frac{3}{32}bc^4\log\left(1 + \sqrt{\frac{1-cx}{1+cx}} + cx\sqrt{\frac{1-cx}{1+cx}}\right)
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSech[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 + b*(1/(16*x^4) + c/(16*x^3) + (3*c^2)/(32*x^2) + (3*c^3)/(32*x)
)*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(4*x^4) - (3*b*c^4*Log[x])/3
2 + (3*b*c^4*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*
x)]])/32
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

method	result
parts	$-\frac{a}{4x^4}+b\,c^4\left(-\frac{\operatorname{arcsech}(cx)}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^4x^4+3\sqrt{-c^2x^2+1}\,c^2x^2+2\sqrt{-c^2x^2+1}\right)}{32c^3x^3\sqrt{-c^2x^2+1}}\right.$
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arcsech}(cx)}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^4x^4+3\sqrt{-c^2x^2+1}\,c^2x^2+2\sqrt{-c^2x^2}\right.}{32c^3x^3\sqrt{-c^2x^2+1}}\right.\right.$
default	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arcsech}(cx)}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^4x^4+3\sqrt{-c^2x^2+1}\,c^2x^2+2\sqrt{-c^2x^2}\right.}{32c^3x^3\sqrt{-c^2x^2+1}}\right.\right.$

```
[In] int((a+b*arcsech(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4+b*c^4*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/c^3/x
^3*((c*x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*(-c^2*x^2
+1)^(1/2)*c^2*x^2+2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx$$
$$=\frac{(3\,bc^4x^4-8\,b)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+(3\,bc^3x^3+2\,bcx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-8\,a}{32\,x^4}$$

```
[In] integrate((a+b*arcsech(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*((3*b*c^4*x^4 - 8*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x
)) + (3*b*c^3*x^3 + 2*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 8*a)/x^4
```

Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{x^5}\,dx$$

[In] integrate((a+b*asech(c*x))/x**5,x)

[Out] Integral((a + b*asech(c*x))/x**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.17

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5}\,dx = \frac{1}{64}\,b\left(\frac{3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) - 3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right) - \frac{2\left(3\,c^8x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} - 5\,c^6x\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2 - 2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c} - \frac{16\,a}{} \right) - \frac{a}{4\,x^4}$$

[In] integrate((a+b*arcsech(c*x))/x^5,x, algorithm="maxima")

[Out] 1/64*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt(1/(c^2*x^2) - 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 16*arcsech(c*x)/x^4) - 1/4*a/x^4

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{x^5}\,dx$$

[In] integrate((a+b*arcsech(c*x))/x^5,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/x^5, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5}\,dx=\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/x^5,x)
[Out] int((a + b*acosh(1/(c*x)))/x^5, x)
```

3.31 $\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^6}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^6}\,dx=\frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}}+\frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{1+cx}}}+\frac{8bc^4\sqrt{1-cx}}{75x\sqrt{\frac{1}{1+cx}}}-\frac{a+b\,\text{sech}^{-1}(cx)}{5x^5}$$

```
[Out] 1/5*(-a-b*arcsech(c*x))/x^5+1/25*b*(-c*x+1)^(1/2)/x^5/(1/(c*x+1))^(1/2)+4/7
5*b*c^2*(-c*x+1)^(1/2)/x^3/(1/(c*x+1))^(1/2)+8/75*b*c^4*(-c*x+1)^(1/2)/x/(1
/(c*x+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6418, 105, 12, 97}

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{x^6}\,dx=-\frac{a+b\,\text{sech}^{-1}(cx)}{5x^5}+\frac{8bc^4\sqrt{1-cx}}{75x\sqrt{\frac{1}{cx+1}}}+\frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[(a + b*ArcSech[c*x])/x^6,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(25*x^5*Sqrt[(1 + c*x)^(-1)]) + (4*b*c^2*Sqrt[1 - c*x])/(
75*x^3*Sqrt[(1 + c*x)^(-1)]) + (8*b*c^4*Sqrt[1 - c*x])/(75*x*Sqrt[(1 + c*x)
^(-1)]) - (a + b*ArcSech[c*x])/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{5x^5}-\frac{1}{5}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^6\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&=\frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{5x^5}+\frac{1}{25}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{4c^2}{x^4\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&=\frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{5x^5}-\frac{1}{25}\left(4bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^4\sqrt{1-cx}\sqrt{1+cx}}\,dx\\
&=\frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}}+\frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{1+cx}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{5x^5}\\
&\quad+\frac{1}{75}\left(4bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{2c^2}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx
\end{aligned}$$

$$= \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}} + \frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{5x^5} - \frac{1}{75}\left(8bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{1+cx}}} + \frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{1+cx}}} + \frac{8bc^4\sqrt{1-cx}}{75x\sqrt{\frac{1}{1+cx}}} - \frac{a+b\text{sech}^{-1}(cx)}{5x^5}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5} + b\left(\frac{8c^5}{75} + \frac{1}{25x^5} + \frac{c}{25x^4} + \frac{4c^2}{75x^3} + \frac{4c^3}{75x^2} + \frac{8c^4}{75x}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{b\text{sech}^{-1}(cx)}{5x^5}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 + b*((8*c^5)/75 + 1/(25*x^5) + c/(25*x^4) + (4*c^2)/(75*x^3) + (
4*c^3)/(75*x^2) + (8*c^4)/(75*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*
x])/(5*x^5)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

method	result	size
parts	$-\frac{a}{5x^5} + b\,c^5\left(-\frac{\text{arcsech}(cx)}{5c^5x^5} + \frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(8c^4x^4+4c^2x^2+3)}{75c^4x^4}\right)$	81
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5} + b\left(-\frac{\text{arcsech}(cx)}{5c^5x^5} + \frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(8c^4x^4+4c^2x^2+3)}{75c^4x^4}\right)\right)$	85
default	$c^5\left(-\frac{a}{5c^5x^5} + b\left(-\frac{\text{arcsech}(cx)}{5c^5x^5} + \frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(8c^4x^4+4c^2x^2+3)}{75c^4x^4}\right)\right)$	85

```
[In] int((a+b*arcsech(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arcsech(c*x)+1/75*(-(c*x-1)/c/x)^(1/2)/c^4/x
^4*((c*x+1)/c/x)^(1/2)*(8*c^4*x^4+4*c^2*x^2+3))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^6}\,dx = -\frac{15\,b\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(8\,bc^5x^5+4\,bc^3x^3+3\,bcx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+15\,a}{75\,x^5}$$

```
[In] integrate((a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
[Out] -1/75*(15*b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*c^5*
x^5 + 4*b*c^3*x^3 + 3*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 15*a)/x^5
```

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^6}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{x^6}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**6,x)
[Out] Integral((a + b*asech(c*x))/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^6}\,dx = \frac{1}{75}\,b\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\,\text{arsech}\,(cx)}{x^5}\right)-\frac{a}{5\,x^5}$$

```
[In] integrate((a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
[Out] 1/75*b*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) + 1
5*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) - 1/5*a/x^5
```

Giac [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^6} \, dx = \int \frac{b \operatorname{arsech}(cx) + a}{x^6} \, dx$$

[In] integrate((a+b*arcsech(c*x))/x^6,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/x^6, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^6} \, dx = \int \frac{a + b \operatorname{acosh}\left(\frac{1}{c x}\right)}{x^6} \, dx$$

[In] int((a + b*acosh(1/(c*x)))/x^6,x)

[Out] int((a + b*acosh(1/(c*x)))/x^6, x)

3.32 $\int \frac{a+b\mathbf{sech}^{-1}(cx)}{x^7} dx$

Optimal result

Integrand size = 12, antiderivative size = 158

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^7}\,dx=\frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}}+\frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}}+\frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6}+\frac{5}{96}bc^6\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)$$

```
[Out] 1/6*(-a-b*arcsech(c*x))/x^6+1/36*b*(-c*x+1)^(1/2)/x^6/(1/(c*x+1))^(1/2)+5/1
44*b*c^2*(-c*x+1)^(1/2)/x^4/(1/(c*x+1))^(1/2)+5/96*b*c^4*(-c*x+1)^(1/2)/x^2
/(1/(c*x+1))^(1/2)+5/96*b*c^6*arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6418, 105, 12, 94, 214}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^7}\,dx=-\frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6}+\frac{5}{96}bc^6\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\operatorname{arctanh}\left(\sqrt{1-cx}\sqrt{cx+1}\right)+\frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{cx+1}}}+\frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{cx+1}}}+\frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{cx+1}}}$$

```
[In] Int[(a + b*ArcSech[c*x])/x^7,x]
```

```
[Out] (b*Sqrt[1 - c*x])/(36*x^6*Sqrt[(1 + c*x)^(-1)]) + (5*b*c^2*Sqrt[1 - c*x])/(
144*x^4*Sqrt[(1 + c*x)^(-1)]) + (5*b*c^4*Sqrt[1 - c*x])/(96*x^2*Sqrt[(1 + c
*x)^(-1)]) - (a + b*ArcSech[c*x])/(6*x^6) + (5*b*c^6*Sqrt[(1 + c*x)^(-1)]*S
qrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/96
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\text{sech}^{-1}(cx)}{6x^6}-\frac{1}{6}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^7\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$=\frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}}-\frac{a+b\text{sech}^{-1}(cx)}{6x^6}+\frac{1}{36}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int-\frac{5c^2}{x^5\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6} - \frac{1}{36}\left(5bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^5\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6} + \frac{1}{144}\left(5bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{3c^2}{x^3\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6} - \frac{1}{48}\left(5bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^3\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6} - \frac{1}{96}\left(5bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{c^2}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6} - \frac{1}{96}\left(5bc^6\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6} + \frac{1}{96}\left(5bc^7\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{c-cx^2}\,dx, x, \sqrt{1-cx}\sqrt{1+cx}\right)$$

$$= \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{1+cx}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{1+cx}}} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6} + \frac{5}{96}bc^6\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-cx}\sqrt{1+cx}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6}+b\left(\frac{1}{36x^6}+\frac{c}{36x^5}+\frac{5c^2}{144x^4}+\frac{5c^3}{144x^3}+\frac{5c^4}{96x^2}+\frac{5c^5}{96x}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{b\operatorname{sech}^{-1}(cx)}{6x^6}-\frac{5}{96}bc^6\log(x) + \frac{5}{96}bc^6\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)$$

```
[In] Integrate[(a + b*ArcSech[c*x])/x^7,x]
```

```
[Out] -1/6*a/x^6 + b*(1/(36*x^6) + c/(36*x^5) + (5*c^2)/(144*x^4) + (5*c^3)/(144*
x^3) + (5*c^4)/(96*x^2) + (5*c^5)/(96*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*Ar
cSech[c*x])/(6*x^6) - (5*b*c^6*Log[x])/96 + (5*b*c^6*Log[1 + Sqrt[(1 - c*x)
/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/96
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

method	result
parts	$-\frac{a}{6x^6}+b\,c^6\left(-\frac{\operatorname{arcsech}(cx)}{6c^6x^6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(15\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^6x^6+15\sqrt{-c^2x^2+1}\,c^4x^4+10\sqrt{-c^2x}\right.}{288c^5x^5\sqrt{-c^2x^2+1}}\right.$
derivativedivides	$c^6\left(-\frac{a}{6c^6x^6}+b\left(-\frac{\operatorname{arcsech}(cx)}{6c^6x^6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(15\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^6x^6+15\sqrt{-c^2x^2+1}\,c^4x^4+10\sqrt{-}\right.}{288c^5x^5\sqrt{-c^2x^2+1}}\right.\right.$
default	$c^6\left(-\frac{a}{6c^6x^6}+b\left(-\frac{\operatorname{arcsech}(cx)}{6c^6x^6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\left(15\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^6x^6+15\sqrt{-c^2x^2+1}\,c^4x^4+10\sqrt{-}\right.}{288c^5x^5\sqrt{-c^2x^2+1}}\right.\right.$

```
[In] int((a+b*arcsech(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6+b*c^6*(-1/6/c^6/x^6*arcsech(c*x)+1/288*(-(c*x-1)/c/x)^(1/2)/c^5/
x^5*((c*x+1)/c/x)^(1/2)*(15*arctanh(1/(-c^2*x^2+1)^(1/2))*c^6*x^6+15*(-c^2*
x^2+1)^(1/2)*c^4*x^4+10*(-c^2*x^2+1)^(1/2)*c^2*x^2+8*(-c^2*x^2+1)^(1/2))/(-
c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^7}\,dx = \frac{3\,(5\,bc^6x^6 - 16\,b)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + (15\,bc^5x^5 + 10\,bc^3x^3 + 8\,bcx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 48\,a}{288\,x^6}$$

```
[In] integrate((a+b*arcsech(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/288*(3*(5*b*c^6*x^6 - 16*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x)) + (15*b*c^5*x^5 + 10*b*c^3*x^3 + 8*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) - 48*a)/x^6
```

Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^7}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{x^7}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**7,x)
```

```
[Out] Integral((a + b*asech(c*x))/x**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.17

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^7}\,dx = \frac{1}{576}\,b\left(\frac{15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) - 15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right) - \frac{2\left(15\,c^{12}x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}} - 40\,c^{10}x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 3\right.}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3 - 3\,c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2 + 3\,c^2x^2}}{c}\right. - \frac{a}{6\,x^6}$$

```
[In] integrate((a+b*arcsech(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] 1/576*b*((15*c^7*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 15*c^7*log(c*x*sqrt(1
/(c^2*x^2) - 1) - 1) - 2*(15*c^12*x^5*(1/(c^2*x^2) - 1)^(5/2) - 40*c^10*x^3
*(1/(c^2*x^2) - 1)^(3/2) + 33*c^8*x*sqrt(1/(c^2*x^2) - 1))/(c^6*x^6*(1/(c^2
*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) - 1)^2 + 3*c^2*x^2*(1/(c^2*x^2) - 1)
- 1))/c - 96*arcsech(c*x)/x^6) - 1/6*a/x^6
```

Giac [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^7} \, dx = \int \frac{b \operatorname{arsech}(cx) + a}{x^7} \, dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^7,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^7} \, dx = \int \frac{a + b \operatorname{acosh}\left(\frac{1}{c x}\right)}{x^7} \, dx$$

```
[In] int((a + b*acosh(1/(c*x)))/x^7,x)
[Out] int((a + b*acosh(1/(c*x)))/x^7, x)
```

3.33 $\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 124

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = -\frac{b^2x^2}{12c^2}-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^4} -\frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{6c^2} +\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{b^2\log(x)}{3c^4}$$

```
[Out] -1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arcsech(c*x))^2-1/3*b^2*ln(x)/c^4-1/3*b*(c*x
+1)*(a+b*arcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/c^4-1/6*b*x^2*(c*x+1)*(a+b*
arcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6420, 5559, 4270, 4269, 3556}

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = -\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^4} -\frac{bx^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{6c^2} +\frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{b^2\log(x)}{3c^4}-\frac{b^2x^2}{12c^2}$$

```
[In] Int[x^3*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] -1/12*(b^2*x^2)/c^2 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech
[c*x]))/(3*c^4) - (b*x^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech
[c*x]))/(6*c^2) + (x^4*(a + b*ArcSech[c*x])^2)/4 - (b^2*Log[x])/(3*c^4)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2\text{sech}^4(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^4}\\
&= \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{b\text{Subst}\left(\int (a+bx)\text{sech}^4(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{2c^4}\\
&= -\frac{b^2x^2}{12c^2} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{6c^2} + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^2\\
&\quad - \frac{b\text{Subst}\left(\int (a+bx)\text{sech}^2(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{3c^4}
\end{aligned}$$

$$= -\frac{b^2x^2}{12c^2} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6c^2} + \frac{1}{4}x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 + \frac{b^2\text{Subst}\left(\int\tanh(x)\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{3c^4}$$

$$= -\frac{b^2x^2}{12c^2} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6c^2} + \frac{1}{4}x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 - \frac{b^2\log(x)}{3c^4}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.71

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx = -\frac{b^2c^2x^2-3a^2c^4x^4+4ab\sqrt{\frac{1-cx}{1+cx}}+4abcx\sqrt{\frac{1-cx}{1+cx}}+2abc^2x^2\sqrt{\frac{1-cx}{1+cx}}+2abc^3x^3\sqrt{\frac{1-cx}{1+cx}}+2b\Big(-3ac^4x^4+b\sqrt{\frac{1-}{1+}}}{12c^4}$$

```
[In] Integrate[x^3*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] -1/12*(b^2*c^2*x^2 - 3*a^2*c^4*x^4 + 4*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*a*
b*c*x*Sqrt[(1 - c*x)/(1 + c*x)] + 2*a*b*c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)] +
 2*a*b*c^3*x^3*Sqrt[(1 - c*x)/(1 + c*x)] + 2*b*(-3*a*c^4*x^4 + b*Sqrt[(1 -
c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^3))*ArcSech[c*x] - 3*b^2*c^4*x
^4*ArcSech[c*x]^2 + 4*b^2*Log[x])/c^4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.72 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.81

method	result
parts	$\frac{x^4a^2}{4}+\frac{b^2\left(-\frac{\operatorname{arcsech}(cx)}{3}+\frac{\operatorname{arcsech}(cx)^2c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}c^3x^3}{6}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}cx}{3}-\frac{c^2x^2}{12}+\right.}{c^4}$
derivativedivides	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(-\frac{\operatorname{arcsech}(cx)}{3}+\frac{\operatorname{arcsech}(cx)^2c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}c^3x^3}{6}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}cx}{3}-\frac{c^2x^2}{12}+\right.}{c^4}$
default	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(-\frac{\operatorname{arcsech}(cx)}{3}+\frac{\operatorname{arcsech}(cx)^2c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}c^3x^3}{6}-\frac{\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}cx}{3}-\frac{c^2x^2}{12}+\right.}{c^4}$

```
[In] int(x^3*(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*a^2+b^2/c^4*(-1/3*arcsech(c*x)+1/4*arcsech(c*x)^2*c^4*x^4-1/6*arcse
ch(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^3*x^3-1/3*arcsech(c*x)*(
-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1/12*c^2*x^2+1/3*ln(1+(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+2*a*b/c^4*(1/4*c^4*x^4*arcsech(c*x)-1/1
2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(c^2*x^2+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(110) = 220$.

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.97

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx$$
$$=\frac{3\,b^2c^4x^4\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2+3\,a^2c^4x^4-6\,abc^4\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)-b^2c^2x^2-4\,b^2\log\left(x\right)+2\left(3\,abc^4x\right.}{12\,c^4}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 +
 3*a^2*c^4*x^4 - 6*a*b*c^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x)
- b^2*c^2*x^2 - 4*b^2*log(x) + 2*(3*a*b*c^4*x^4 - 3*a*b*c^4 - (b^2*c^3*x^3
+ 2*b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 1)/(c*x)) - 2*(a*b*c^3*x^3 + 2*a*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)))/c^4
```

Sympy [F]

$$\int x^3 \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx = \int x^3 (a + b \operatorname{asech}(cx))^2 \, dx$$

```
[In] integrate(x**3*(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))**2, x)
```

Maxima [F]

$$\int x^3 \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arsech}(cx) + a)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 + 1/6*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*a*b + b^2*integrate(x^3*log(sqrt(1/(c*x) +
1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)
```

Giac [F]

$$\int x^3 \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arsech}(cx) + a)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx = \int x^3 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^2 \, dx$$

```
[In] int(x^3*(a + b*acosh(1/(c*x)))^2,x)
```

```
[Out] int(x^3*(a + b*acosh(1/(c*x)))^2, x)
```

3.34 $\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 140

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = -\frac{b^2x}{3c^2}-\frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{2b\left(a+b\text{sech}^{-1}(cx)\right)\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{3c^3}-\frac{ib^2\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{3c^3}$$

```
[Out] -1/3*b^2*x/c^2+1/3*x^3*(a+b*arcsech(c*x))^2-2/3*b*(a+b*arcsech(c*x))*arctan
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c^3+1/3*I*b^2*polylog(2,-I*(1/c/x+
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-1/3*I*b^2*polylog(2,I*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-1/3*b*x*(c*x+1)*(a+b*arcsech(c*x))*((-c*x+1
)/(c*x+1))^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

= {6420, 5559, 4270, 4265, 2317, 2438}

$$\int x^2 \left(a + b\text{sech}^{-1}(cx)\right)^2 dx = -\frac{2b \arctan\left(e^{\text{sech}^{-1}(cx)}\right)\left(a + b\text{sech}^{-1}(cx)\right)}{3c^3} - \frac{bx\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a + b\text{sech}^{-1}(cx)\right)}{3c^2} + \frac{1}{3}x^3\left(a + b\text{sech}^{-1}(cx)\right)^2 + \frac{ib^2 \operatorname{PolyLog}\left(2, -ie^{\text{sech}^{-1}(cx)}\right)}{3c^3} - \frac{ib^2 \operatorname{PolyLog}\left(2, ie^{\text{sech}^{-1}(cx)}\right)}{3c^3} - \frac{b^2 x}{3c^2}$$

```
[In] Int[x^2*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] -1/3*(b^2*x)/c^2 - (b*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[
c*x]))/(3*c^2) + (x^3*(a + b*ArcSech[c*x])^2)/3 - (2*b*(a + b*ArcSech[c*x])
*ArcTan[E^ArcSech[c*x]])/(3*c^3) + ((I/3)*b^2*PolyLog[2, (-I)*E^ArcSech[c*x
]])/c^3 - ((I/3)*b^2*PolyLog[2, I*E^ArcSech[c*x]])/c^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
```

```
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2 \text{sech}^3(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3} \\
&= \frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{(2b)\text{Subst}\left(\int (a+bx)\text{sech}^3(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{3c^3} \\
&= -\frac{b^2x}{3c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2} + \frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2 \\
&\quad - \frac{b\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{3c^3} \\
&= -\frac{b^2x}{3c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{3c^2} \\
&\quad + \frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{2b\left(a+b\text{sech}^{-1}(cx)\right)\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{3c^3} \\
&\quad + \frac{(ib^2)\,\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{3c^3} \\
&\quad - \frac{(ib^2)\,\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{3c^3}
\end{aligned}
$$

$$= -\frac{b^2x}{3c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3c^2} + \frac{1}{3}x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 - \frac{2b\left(a+b\operatorname{sech}^{-1}(cx)\right)\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3} + \frac{(ib^2)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3} - \frac{(ib^2)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3}$$

$$= -\frac{b^2x}{3c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3c^2} + \frac{1}{3}x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 - \frac{2b\left(a+b\operatorname{sech}^{-1}(cx)\right)\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3} + \frac{ib^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3} - \frac{ib^2\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3}$$

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.72

$$\int x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx$$

$$= \frac{1}{3}\left(a^2x^3 + ab\left(2x^3\operatorname{sech}^{-1}(cx) + \frac{\sqrt{\frac{1-cx}{1+cx}}\left(cx-c^3x^3+2\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c^3(-1+cx)}\right) + \frac{b^2\left(-cx-cx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\operatorname{sech}^{-1}(cx)+c^3x^3\operatorname{sech}^{-1}(cx)^2+i\operatorname{sech}^{-1}(cx)\log\left(1-ie^{-\operatorname{sech}^{-1}(cx)}\right)-i\operatorname{sech}^{-}\right.}{c^3}$$

```
[In] Integrate[x^2*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] (a^2*x^3 + a*b*(2*x^3*ArcSech[c*x] + (Sqrt[(1 - c*x)/(1 + c*x)]*(c*x - c^3*
x^3 + 2*Sqrt[1 - c^2*x^2]*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))/(c^3*(-1 +
c*x))) + (b^2*(-(c*x) - c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*ArcSech[c*x
] + c^3*x^3*ArcSech[c*x]^2 + I*ArcSech[c*x]*Log[1 - I/E^ArcSech[c*x]] - I*A
rcSech[c*x]*Log[1 + I/E^ArcSech[c*x]] + I*PolyLog[2, (-I)/E^ArcSech[c*x]] -
 I*PolyLog[2, I/E^ArcSech[c*x]]))/c^3)/3
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.34

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{\left(c^2x^2\,\mathrm{arcsech}(cx)^2-\mathrm{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,cx-1\right)cx}{3}+\frac{i\,\mathrm{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{3}-\frac{i\,\mathrm{ar}\ldots}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\,\mathrm{arcsech}(cx)^2-\mathrm{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,cx-1\right)cx}{3}+\frac{i\,\mathrm{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{3}-\frac{i\,\mathrm{arc}\ldots}{}\right.}{}$
default	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\,\mathrm{arcsech}(cx)^2-\mathrm{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,cx-1\right)cx}{3}+\frac{i\,\mathrm{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{3}-\frac{i\,\mathrm{arc}\ldots}{}\right.}{}$

```
[In] int(x^2*(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+b^2/c^3*(1/3*(c^2*x^2*arcsech(c*x)^2-arcsech(c*x)*(-(c*x-1)/c/x
)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1)*c*x+1/3*I*arcsech(c*x)*ln(1+I*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/3*I*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2)))+1/3*I*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2)))-1/3*I*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))))+2*a*b/c^
3*(1/3*c^3*x^3*arcsech(c*x)+1/6*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2
)*(-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int x^2\left(a+b\mathrm{sech}^{-1}(cx)\right)^2\,dx=\int\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arcsech(c*x)^2 + 2*a*b*x^2*arcsech(c*x) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\mathrm{sech}^{-1}(cx)\right)^2\,dx=\int x^2\left(a+b\,\mathrm{asech}\left(cx\right)\right)^2\,dx$$

```
[In] integrate(x**2*(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))**2, x)
```

Maxima [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (b\,\text{arsech}\,(cx)+a)^2x^2\,dx$$

[In] integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="maxima")

[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*a*b + b^2*integrate(x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)

Giac [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (b\,\text{arsech}\,(cx)+a)^2x^2\,dx$$

[In] integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^2*x^2, x)

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int x^2\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

[In] int(x^2*(a + b*acosh(1/(c*x)))^2,x)

[Out] int(x^2*(a + b*acosh(1/(c*x)))^2, x)

3.35 $\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{c^2}+\frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{b^2\log(x)}{c^2}$$

```
[Out] 1/2*x^2*(a+b*arcsech(c*x))^2-b^2*ln(x)/c^2-b*(c*x+1)*(a+b*arcsech(c*x))*((-
c*x+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6420, 5559, 4269, 3556}

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=-\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{c^2}+\frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^2-\frac{b^2\log(x)}{c^2}$$

```
[In] Int[x*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] -((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/c^2) + (x^2*
(a + b*ArcSech[c*x])^2)/2 - (b^2*Log[x])/c^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2\text{sech}^2(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^2}\\
&= \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{b\text{Subst}\left(\int (a+bx)\text{sech}^2(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^2}\\
&= -\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{c^2} + \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^2\\
&\quad + \frac{b^2\text{Subst}\left(\int \tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^2}\\
&= -\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{c^2} + \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{b^2\log(x)}{c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx$$
$$=\frac{a\left(ac^2x^2-2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)-2b\left(-ac^2x^2+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\operatorname{sech}^{-1}(cx)+b^2c^2x^2\operatorname{sech}^{-1}(cx)^2-2b^2\log\cdots}{2c^2}$$

```
[In] Integrate[x*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] (a*(a*c^2*x^2 - 2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) - 2*b*(-(a*c^2*x^2
) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] + b^2*c^2*x^2*ArcSe
ch[c*x]^2 - 2*b^2*Log[c*x])/(2*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(61) = 122.

Time = 0.64 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.54

method	result
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(-2\operatorname{arcsech}(cx)+\frac{\operatorname{arcsech}(cx)\left(c^2x^2\operatorname{arcsech}(cx)-2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+2\right)}{2}+\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2\cdots\right)\cdots\right)}{c^2}$
derivativedivides	$\frac{\frac{a^2c^2x^2}{2}+b^2\left(-2\operatorname{arcsech}(cx)+\frac{\operatorname{arcsech}(cx)\left(c^2x^2\operatorname{arcsech}(cx)-2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+2\right)}{2}+\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2\cdots\right)\cdots\right)}{c^2}$
default	$\frac{\frac{a^2c^2x^2}{2}+b^2\left(-2\operatorname{arcsech}(cx)+\frac{\operatorname{arcsech}(cx)\left(c^2x^2\operatorname{arcsech}(cx)-2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+2\right)}{2}+\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^2\cdots\right)\cdots\right)}{c^2}$

```
[In] int(x*(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2+b^2/c^2*(-2*arcsech(c*x)+1/2*arcsech(c*x)*(c^2*x^2*arcsech(c*x)
-2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+2)+ln(1+(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2))+2*a*b/c^2*(1/2*c^2*x^2*arcsech(c*x)-1/2*(-(c*x-1)
/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(61) = 122$.

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.15

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx = \frac{b^2c^2x^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 + a^2c^2x^2 - 2\,abc^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2\,b^2\log(x) + 2\,\Big(a}{2\,c^2}$$

[In] integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="fricas")

[Out] 1/2*(b^2*c^2*x^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + a^2*c^2*x^2 - 2*a*b*c^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*b^2*log(x) + 2*(a*b*c^2*x^2 - b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a*b*c^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c^2

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx = \begin{cases} \frac{a^2x^2}{2} + abx^2\operatorname{asech}(cx) - \frac{ab\sqrt{-c^2x^2+1}}{c^2} + \frac{b^2x^2\operatorname{asech}^2(cx)}{2} - \frac{b^2\sqrt{-c^2x^2+1}\operatorname{asech}(cx)}{c^2} - \frac{b^2\log(x)}{c^2} & \text{for } c \neq 0 \\ \frac{x^2(a+\infty b)^2}{2} & \text{otherwise} \end{cases}$$

[In] integrate(x*(a+b*asech(c*x))**2,x)

[Out] Piecewise((a**2*x**2/2 + a*b*x**2*asech(c*x) - a*b*sqrt(-c**2*x**2 + 1)/c**2 + b**2*x**2*asech(c*x)**2/2 - b**2*sqrt(-c**2*x**2 + 1)*asech(c*x)/c**2 - b**2*log(x)/c**2, Ne(c, 0)), (x**2*(a + oo*b)**2/2, True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\frac{1}{2}\,b^2x^2\,\text{arsech}\,(cx)^2+\frac{1}{2}\,a^2x^2+\left(x^2\,\text{arsech}\,(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)ab-\left(\frac{x\sqrt{\frac{1}{c^2x^2}-1}\,\text{arsech}\,(cx)}{c}+\frac{\log{(x)}}{c^2}\right)b^2$$

```
[In] integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arcsech(c*x)^2 + 1/2*a^2*x^2 + (x^2*arcsech(c*x) - x*sqrt(1/(c^
2*x^2) - 1)/c)*a*b - (x*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x)/c + log(x)/c^2)*
b^2
```

Giac [F]

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int\left(b\,\text{arsech}\,(cx)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

```
[In] int(x*(a + b*acosh(1/(c*x)))^2,x)
```

```
[Out] int(x*(a + b*acosh(1/(c*x)))^2, x)
```

3.36 $\int \left(a+b\,\text{sech}^{-1}(cx)\right)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \left(a+b\,\text{sech}^{-1}(cx)\right)^2\,dx=x\left(a+b\,\text{sech}^{-1}(cx)\right)^2-\frac{4b\left(a+b\,\text{sech}^{-1}(cx)\right)\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{c}+\frac{2ib^2\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{c}-\frac{2ib^2\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arcsech(c*x))^2-4*b*(a+b*arcsech(c*x))*arctan(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))/c+2*I*b^2*polylog(2,-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2)))/c-2*I*b^2*polylog(2,I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6414, 5559, 4265, 2317, 2438}

$$\int \left(a+b\,\text{sech}^{-1}(cx)\right)^2\,dx=-\frac{4b\arctan\left(e^{\text{sech}^{-1}(cx)}\right)\left(a+b\,\text{sech}^{-1}(cx)\right)}{c}+x\left(a+b\,\text{sech}^{-1}(cx)\right)^2+\frac{2ib^2\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{c}-\frac{2ib^2\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{c}$$

```
[In] Int[(a + b*ArcSech[c*x])^2,x]
```

```
[Out] x*(a + b*ArcSech[c*x])^2 - (4*b*(a + b*ArcSech[c*x])*ArcTan[E^ArcSech[c*x]]
)/c + ((2*I)*b^2*PolyLog[2, (-I)*E^ArcSech[c*x]])/c - ((2*I)*b^2*PolyLog[2,
 I*E^ArcSech[c*x]])/c
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6414

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}\\
&= x\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{(2b)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}\\
&= x\left(a+b\text{sech}^{-1}(cx)\right)^2 - \frac{4b\left(a+b\text{sech}^{-1}(cx)\right)\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{c}\\
&\quad+\frac{(2ib^2)\,\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}\\
&\quad-\frac{(2ib^2)\,\text{Subst}\left(\int \log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}
\end{aligned}
$$

$$
\begin{aligned}
&= x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 - \frac{4b\left(a+b\operatorname{sech}^{-1}(cx)\right)\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c} \\
&\quad + \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{c} \\
&\quad - \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{c} \\
&= x\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 - \frac{4b\left(a+b\operatorname{sech}^{-1}(cx)\right)\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c} \\
&\quad + \frac{2ib^2\,\text{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(cx)}\right)}{c} - \frac{2ib^2\,\text{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(cx)}\right)}{c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.62

$$
\int \left(a+b\operatorname{sech}^{-1}(cx)\right)^2\,dx = a^2x + \frac{2ab\left(cx\operatorname{sech}^{-1}(cx) - 2\arctan\left(\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(cx)\right)\right)\right)}{c} + \frac{ib^2\left(\operatorname{sech}^{-1}(cx)\left(-icx\operatorname{sech}^{-1}(cx) + 2\log\left(1-ie^{-\operatorname{sech}^{-1}(cx)}\right) - 2\log\left(1+ie^{-\operatorname{sech}^{-1}(cx)}\right)\right) + 2\,\text{PolyLog}\left(2, \ldots\right.}{c}
$$

```
[In] Integrate[(a + b*ArcSech[c*x])^2,x]
```

```
[Out] a^2*x + (2*a*b*(c*x*ArcSech[c*x] - 2*ArcTan[Tanh[ArcSech[c*x]/2]]))/c + (I*
b^2*(ArcSech[c*x]*((-I)*c*x*ArcSech[c*x] + 2*Log[1 - I/E^ArcSech[c*x]] - 2*
Log[1 + I/E^ArcSech[c*x]]) + 2*PolyLog[2, (-I)/E^ArcSech[c*x]] - 2*PolyLog[
2, I/E^ArcSech[c*x]]))/c
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

method	result
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arcsech}(cx)^2cx+2i\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\right)-2i\operatorname{arcsech}(cx)\ln\left(1-i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\ldots}\right.\right.\right.}{\ldots}$
default	$\frac{a^2cx+b^2\left(\operatorname{arcsech}(cx)^2cx+2i\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\right)-2i\operatorname{arcsech}(cx)\ln\left(1-i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\ldots}\right.\right.\right.}{\ldots}$
parts	$a^2x+\frac{b^2\left(\operatorname{arcsech}(cx)^2cx+2i\operatorname{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\right)-2i\operatorname{arcsech}(cx)\ln\left(1-i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1\ldots}\right.\right.\right.}{c}$

```
[In] int((a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2*c*x+b^2*(arcsech(c*x)^2*c*x+2*I*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2)))-2*I*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2)))+2*I*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-
2*I*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))))+2*a*b*(c*x*arcsech
(c*x)-arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))))
```

Fricas [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (b\,\text{arsech}\,(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (a+b\,\text{asech}\,(cx))^2\,dx$$

```
[In] integrate((a+b*asech(c*x))**2,x)
[Out] Integral((a + b*asech(c*x))**2, x)
```

Maxima [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (b\,\text{arsech}\,(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^2,x, algorithm="maxima")
[Out] (x*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 - integrate(-(c^2*x^2*log(c)^2 +
 (c^2*x^2 - 1)*log(x)^2 + (c^2*x^2*log(c)^2 + (c^2*x^2 - 1)*log(x)^2 - log(
c)^2 + 2*(c^2*x^2*log(c) - log(c))*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2
*(c^2*x^2*log(c) + (c^2*x^2*(log(c) + 1) + (c^2*x^2 - 1)*log(x) - log(c))*s
qrt(c*x + 1)*sqrt(-c*x + 1) + (c^2*x^2 - 1)*log(x) - log(c))*log(sqrt(c*x +
 1)*sqrt(-c*x + 1) + 1) - log(c)^2 + 2*(c^2*x^2*log(c) - log(c))*log(x))/(c
^2*x^2 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 1), x))*b^2 + a^2*x +
 2*(c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*a*b/c
```

Giac [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int (b\,\text{arsech}\,(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^2\,dx=\int \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^2,x)
[Out] int((a + b*acosh(1/(c*x)))^2, x)
```

3.37 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}\,dx=\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{3b}-\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\mathrm{sech}^{-1}(cx)}\right)-b\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-e^{2\mathrm{sech}^{-1}(cx)}\right)+\frac{1}{2}b^2\,\mathrm{PolyLog}\left(3,-e^{2\mathrm{sech}^{-1}(cx)}\right)$$

```
[Out] 1/3*(a+b*arcsech(c*x))^3/b-(a+b*arcsech(c*x))^2*ln(1+(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2)-b*(a+b*arcsech(c*x))*polylog(2,-(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2)+1/2*b^2*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6420, 3799, 2221, 2611, 2320, 6724}

$$\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}\,dx=-b\,\mathrm{PolyLog}\left(2,-e^{2\mathrm{sech}^{-1}(cx)}\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{3b}-\log\left(e^{2\mathrm{sech}^{-1}(cx)}+1\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2+\frac{1}{2}b^2\,\mathrm{PolyLog}\left(3,-e^{2\mathrm{sech}^{-1}(cx)}\right)$$

```
[In] Int[(a + b*ArcSech[c*x])^2/x,x]
```

```
[Out] (a + b*ArcSech[c*x])^3/(3*b) - (a + b*ArcSech[c*x])^2*Log[1 + E^(2*ArcSech[
c*x])] - b*(a + b*ArcSech[c*x])*PolyLog[2, -E^(2*ArcSech[c*x])] + (b^2*Poly
Log[3, -E^(2*ArcSech[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3b} - 2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)\\
&\quad + (2b)\text{Subst}\left(\int (a+bx)\log\left(1+e^{2x}\right)\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)\\
&\quad - b\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)\\
&\quad + b^2\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2x}\right)\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)\\
&\quad - b\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)\\
&\quad + \frac{1}{2}b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2\text{sech}^{-1}(cx)}\right)\\
&= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{sech}^{-1}(cx)\right)^2\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)\\
&\quad - b\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right) + \frac{1}{2}b^2\,\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.40

$$
\begin{aligned}
\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x}\,dx &= a^2\log(cx)\\
&\quad + ab\Big(-\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx)+2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\right)\\
&\quad + \text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)\Big)\\
&\quad + b^2\Big(-\frac{1}{3}\text{sech}^{-1}(cx)^3 - \text{sech}^{-1}(cx)^2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\\
&\quad + \text{sech}^{-1}(cx)\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)\\
&\quad + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\text{sech}^{-1}(cx)}\right)\Big)
\end{aligned}
$$

[In] Integrate[(a + b*ArcSech[c*x])^2/x,x]

[Out] a^2*Log[c*x] + a*b*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])])) + PolyLog[2, -E^(-2*ArcSech[c*x])]) + b^2*(-1/3*ArcSech[c*x]^3 - ArcSech[c*x]^2*Log[1 + E^(-2*ArcSech[c*x])] + ArcSech[c*x]*PolyLog[2, -E^(-2*ArcSech[c*x])] + PolyLog[3, -E^(-2*ArcSech[c*x])]/2)

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.90

method	result
parts	$a^2 \ln(x) + b^2 \left(\frac{\operatorname{arcsech}(cx)^3}{3} - \operatorname{arcsech}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right)^2\right) - \operatorname{arcse}\right.$
derivativedivides	$a^2 \ln(cx) + b^2 \left(\frac{\operatorname{arcsech}(cx)^3}{3} - \operatorname{arcsech}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right)^2\right) - \operatorname{arcs}\right.$
default	$a^2 \ln(cx) + b^2 \left(\frac{\operatorname{arcsech}(cx)^3}{3} - \operatorname{arcsech}(cx)^2 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right)^2\right) - \operatorname{arcs}\right.$

[In] int((a+b*arcsech(c*x))^2/x,x,method=_RETURNVERBOSE)

[Out] a^2*ln(x)+b^2*(1/3*arcsech(c*x)^3-arcsech(c*x)^2*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-arcsech(c*x)*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+1/2*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+2*a*b*(1/2*arcsech(c*x)^2-arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))

Fricas [F]

$$\int \frac{(a + b\text{sech}^{-1}(cx))^2}{x}\, dx = \int \frac{(b \operatorname{arsech}(cx) + a)^2}{x}\, dx$$

[In] integrate((a+b*arcsech(c*x))^2/x,x, algorithm="fricas")

[Out] integral((b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2)/x, x)

Sympy [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{\left(a + b\,\text{asech}\,(cx)\right)^2}{x}\,dx$$

```
[In] integrate((a+b*asech(c*x))**2/x,x)
[Out] Integral((a + b*asech(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{\left(b\,\text{arsech}\,(cx) + a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^2/x,x, algorithm="maxima")
[Out] a^2*log(x) + integrate(b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x
))^2/x + 2*a*b*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

Giac [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{\left(b\,\text{arsech}\,(cx) + a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^2/x,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x}\,dx = \int \frac{\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^2/x,x)
[Out] int((a + b*acosh(1/(c*x)))^2/x, x)
```

3.38 $\int \frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx=-\frac{2b^2}{x}+\frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}-\frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{x}$$

```
[Out] -2*b^2/x-(a+b*arcsech(c*x))^2/x+2*b*(c*x+1)*(a+b*arcsech(c*x))*((-c*x+1)/(c
*x+1))^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6420, 3377, 2718}

$$\int \frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx=\frac{2b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x}-\frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{x}-\frac{2b^2}{x}$$

```
[In] Int[(a + b*ArcSech[c*x])^2/x^2,x]
```

```
[Out] (-2*b^2)/x + (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))
/x - (a + b*ArcSech[c*x])^2/x
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int (a+bx)^2\sinh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x} + (2bc)\text{Subst}\left(\int (a+bx)\cosh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{x} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x}\\
&\quad -\left(2b^2c\right)\text{Subst}\left(\int \sinh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= -\frac{2b^2}{x} + \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{x} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{a^2+2b^2-2ab\sqrt{\frac{1-cx}{1+cx}}(1+cx)-2b\left(-a+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\text{sech}^{-1}(cx)+b^2\text{sech}^{-1}(cx)^2}{x}$$

```
[In] Integrate[(a + b*ArcSech[c*x])^2/x^2,x]
[Out] -((a^2 + 2*b^2 - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 2*b*(-a + b*Sq
rt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] + b^2*ArcSech[c*x]^2)/x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{a^2}{x} + b^2c\left(-\frac{\operatorname{arcsech}(cx)^2}{cx} + 2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}} - \frac{2}{cx}\right) + 2abc\left(-\frac{\operatorname{arcsech}(cx)}{cx} + \sqrt{\right.$
derivativedivides	$c\left(-\frac{a^2}{cx} + b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{cx} + 2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}} - \frac{2}{cx}\right) + 2ab\left(-\frac{\operatorname{arcsech}(cx)}{cx} + \sqrt{\right.\right.$
default	$c\left(-\frac{a^2}{cx} + b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{cx} + 2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}} - \frac{2}{cx}\right) + 2ab\left(-\frac{\operatorname{arcsech}(cx)}{cx} + \sqrt{\right.\right.$

```
[In] int((a+b*arcsech(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/x+b^2*c*(-1/c/x*arcsech(c*x)^2+2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c
*x+1)/c/x)^(1/2)-2/c/x)+2*a*b*c*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1/2)*(
(c*x+1)/c/x)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.34

$$\int \frac{(a + b\operatorname{sech}^{-1}(cx))^2}{x^2}\,dx = \frac{2\,abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - b^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 - a^2 - 2\,b^2 + 2\left(b^2cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - ab\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{x}$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - b^2*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x))^2 - a^2 - 2*b^2 + 2*(b^2*c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) - a*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

Sympy [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{(a + b\,\text{asech}\,(cx))^2}{x^2}\,dx$$

[In] integrate((a+b*asech(c*x))**2/x**2,x)

[Out] Integral((a + b*asech(c*x))**2/x**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx = 2\left(c\sqrt{\frac{1}{c^2x^2} - 1} - \frac{\text{arsech}\,(cx)}{x}\right)ab + 2\left(c\sqrt{\frac{1}{c^2x^2} - 1}\,\text{arsech}\,(cx) - \frac{1}{x}\right)b^2 - \frac{b^2\,\text{arsech}\,(cx)^2}{x} - \frac{a^2}{x}$$

[In] integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="maxima")

[Out] 2*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*a*b + 2*(c*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x) - 1/x)*b^2 - b^2*arcsech(c*x)^2/x - a^2/x

Giac [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)^2}{x^2}\,dx$$

[In] integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^2/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^2}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^2/x^2,x)
[Out] int((a + b*acosh(1/(c*x)))^2/x^2, x)
```

3.39 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 118

$$\begin{aligned}\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^3}\,dx = &-\frac{b^2(1-cx)(1+cx)}{4x^2}-\frac{1}{2}abc^2\,\mathrm{sech}^{-1}(cx)\\ &-\frac{1}{4}b^2c^2\,\mathrm{sech}^{-1}(cx)^2+\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{2x^2}\\ &-\frac{(1-cx)(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{2x^2}\end{aligned}$$

```
[Out] -1/4*b^2*(-c*x+1)*(c*x+1)/x^2-1/2*a*b*c^2*arcsech(c*x)-1/4*b^2*c^2*arcsech(
c*x)^2-1/2*(-c*x+1)*(c*x+1)*(a+b*arcsech(c*x))^2/x^2+1/2*b*(c*x+1)*(a+b*arc
sech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6420, 5554, 3391}

$$\begin{aligned}\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x^3}\,dx = &-\frac{1}{2}abc^2\,\mathrm{sech}^{-1}(cx)+\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{2x^2}\\ &-\frac{(1-cx)(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{2x^2}\\ &-\frac{1}{4}b^2c^2\,\mathrm{sech}^{-1}(cx)^2-\frac{b^2(1-cx)(cx+1)}{4x^2}\end{aligned}$$

```
[In] Int[(a + b*ArcSech[c*x])^2/x^3,x]
```

```
[Out] -1/4*(b^2*(1 - c*x)*(1 + c*x))/x^2 - (a*b*c^2*ArcSech[c*x])/2 - (b^2*c^2*Ar
cSech[c*x]^2)/4 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x
]))/(2*x^2) - ((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*x^2)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^2\text{Subst}\left(\int (a+bx)^2\cosh(x)\sinh(x)\,dx,x,\text{sech}^{-1}(cx)\right)\right)\\
&= -\frac{(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2x^2}+\left(bc^2\right)\text{Subst}\left(\int (a+bx)\sinh^2(x)\,dx,x,\text{sech}^{-1}(cx)\right)\\
&= -\frac{b^2(1-cx)(1+cx)}{4x^2}+\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2}\\
&\quad -\frac{(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2x^2}\\
&\quad -\frac{1}{2}\left(bc^2\right)\text{Subst}\left(\int (a+bx)\,dx,x,\text{sech}^{-1}(cx)\right)\\
&= -\frac{b^2(1-cx)(1+cx)}{4x^2}-\frac{1}{2}abc^2\text{sech}^{-1}(cx)-\frac{1}{4}b^2c^2\text{sech}^{-1}(cx)^2\\
&\quad +\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2}-\frac{(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.55

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^3}\,dx = \frac{-2a^2-b^2+2ab\sqrt{\frac{1-cx}{1+cx}}+2abcx\sqrt{\frac{1-cx}{1+cx}}+2b\left(-2a+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\text{sech}^{-1}(cx)+b^2(-2+c^2x^2)\,\text{sech}^{-1}}{4x^2}$$

```
[In] Integrate[(a + b*ArcSech[c*x])^2/x^3,x]
```

```
[Out] (-2*a^2 - b^2 + 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 2*a*b*c*x*Sqrt[(1 - c*x)/
(1 + c*x)] + 2*b*(-2*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x
] + b^2*(-2 + c^2*x^2)*ArcSech[c*x]^2 - 2*a*b*c^2*x^2*Log[x] + 2*a*b*c^2*x^
2*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(4*x^
2)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.62

method	result
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(-\frac{\text{arcsech}(cx)^2}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,\text{arcsech}(cx)}{2cx}+\frac{\text{arcsech}(cx)^2}{4}-\frac{1}{4c^2x^2}\right)+2ab\,c^2\Bigg(-\frac{\text{ar}}{}$
derivativedivides	$c^2\Bigg(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\text{arcsech}(cx)^2}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,\text{arcsech}(cx)}{2cx}+\frac{\text{arcsech}(cx)^2}{4}-\frac{1}{4c^2x^2}\right)+2ab\Bigg(-$
default	$c^2\Bigg(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\text{arcsech}(cx)^2}{2c^2x^2}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,\text{arcsech}(cx)}{2cx}+\frac{\text{arcsech}(cx)^2}{4}-\frac{1}{4c^2x^2}\right)+2ab\Bigg(-$

```
[In] int((a+b*arcsech(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/x^2+b^2*c^2*(-1/2/c^2/x^2*arcsech(c*x)^2+1/2*(-(c*x-1)/c/x)^(1/2)*
((c*x+1)/c/x)^(1/2)/c/x*arcsech(c*x)+1/4*arcsech(c*x)^2-1/4/c^2/x^2)+2*a*b*
c^2*(-1/2/c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c/x*((c*x+1)/c/x)^(
1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^3}\,dx$$
$$=\frac{2\,abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+(b^2c^2x^2-2\,b^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2-2\,a^2-b^2+2\left(abc^2x^2+b^2cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-2\,ab\right)\log}{4\,x^2}$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + (b^2*c^2*x^2 - 2*b^2)*log((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 - 2*a^2 - b^2 + 2*(a*b*c^2
*x^2 + b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*a*b)*log((c*x*sqrt(-(c^2*
x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

Sympy [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\text{asech}\,(cx)\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*asech(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*asech(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^3}\,dx=\int \frac{\left(b\,\text{arsech}\,(cx)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1)
- 1))/c + 4*arcsech(c*x)/x^2) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/
(c*x) - 1) + 1/(c*x))^2/x^3, x) - 1/2*a^2/x^2
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^3}\,dx=\int \frac{\left(b\operatorname{arsech}\left(cx\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^3}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^2/x^3,x)
[Out] int((a + b*acosh(1/(c*x)))^2/x^3, x)
```

3.40 $\int \frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 122

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^4}\,dx=-\frac{2b^2}{27x^3}-\frac{4b^2c^2}{9x}+\frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3}+\frac{4bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{9x}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3}$$

```
[Out] -2/27*b^2/x^3-4/9*b^2*c^2/x-1/3*(a+b*arcsech(c*x))^2/x^3+2/9*b*(c*x+1)*(a+b
*arcsech(c*x))*((-c*x+1)/(c*x+1))^(1/2)/x^3+4/9*b*c^2*(c*x+1)*(a+b*arcsech(
c*x))*((-c*x+1)/(c*x+1))^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6420, 5555, 3391, 3377, 2718}

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^4}\,dx=\frac{4bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{9x}+\frac{2b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3}-\frac{4b^2c^2}{9x}-\frac{2b^2}{27x^3}$$

```
[In] Int[(a + b*ArcSech[c*x])^2/x^4,x]
```

```
[Out] (-2*b^2)/(27*x^3) - (4*b^2*c^2)/(9*x) + (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)*(a + b*ArcSech[c*x]))/(9*x^3) + (4*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1
 + c*x)*(a + b*ArcSech[c*x]))/(9*x) - (a + b*ArcSech[c*x])^2/(3*x^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(c^3\text{Subst}\left(\int (a+bx)^2\cosh^2(x)\sinh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\ &= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int (a+bx)\cosh^3(x)\,dx, x, \text{sech}^{-1}(cx)\right)\end{aligned}$$

$$= -\frac{2b^2}{27x^3} + \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{9x^3} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x^3} + \frac{1}{9}\left(4bc^3\right)\operatorname{Subst}\left(\int(a+bx)\cosh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)$$

$$= -\frac{2b^2}{27x^3} + \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{9x^3} + \frac{4bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{9x} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x^3} - \frac{1}{9}\left(4b^2c^3\right)\operatorname{Subst}\left(\int\sinh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)$$

$$= -\frac{2b^2}{27x^3} - \frac{4b^2c^2}{9x} + \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{9x^3} + \frac{4bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{9x} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x^3}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10

$$\int\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^4}\,dx = \frac{-9a^2-2b^2(1+6c^2x^2)+6ab\sqrt{\frac{1-cx}{1+cx}}(1+cx+2c^2x^2+2c^3x^3)+6b\left(-3a+b\sqrt{\frac{1-cx}{1+cx}}(1+cx+2c^2x^2+2c^3x^3}{27x^3}$$

```
[In] Integrate[(a + b*ArcSech[c*x])^2/x^4,x]
```

```
[Out] (-9*a^2 - 2*b^2*(1 + 6*c^2*x^2) + 6*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x
+ 2*c^2*x^2 + 2*c^3*x^3) + 6*b*(-3*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x
 + 2*c^2*x^2 + 2*c^3*x^3))*ArcSech[c*x] - 9*b^2*ArcSech[c*x]^2)/(27*x^3)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.57

method	result
parts	$-\frac{a^2}{3x^3}+b^2c^3\left(-\frac{\operatorname{arcsech}(cx)^2}{3c^3x^3}+\frac{4\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{9}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{9c^2x^2}-\frac{4}{9cx}-\right.$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{3c^3x^3}+\frac{4\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{9}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{9c^2x^2}-\frac{4}{9cx}\right.\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{3c^3x^3}+\frac{4\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{9}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{9c^2x^2}-\frac{4}{9cx}\right.\right.$

```
[In] int((a+b*arcsech(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arcsech(c*x)^2+4/9*arcsech(c*x)*(-(c*x-1
)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+2/9/c^2/x^2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/
c/x)^(1/2)*arcsech(c*x)-4/9/c/x-2/27/c^3/x^3)+2*a*b*c^3*(-1/3/c^3/x^3*arcse
ch(c*x)+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^2*((c*x+1)/c/x)^(1/2)*(2*c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.48

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^4}\,dx =$$
$$-\frac{12\,b^2c^2x^2+9\,b^2\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2+9\,a^2+2\,b^2+6\left(3\,ab-\left(2\,b^2c^3x^3+b^2cx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{-\right.}{}\right.}{27\,x^3}$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(12*b^2*c^2*x^2 + 9*b^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x))^2 + 9*a^2 + 2*b^2 + 6*(3*a*b - (2*b^2*c^3*x^3 + b^2*c*x)*sqrt(-(c^2*
x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) -
6*(2*a*b*c^3*x^3 + a*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^3
```

Sympy [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{\left(a+b\operatorname{asech}(cx)\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*asech(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x^4}\, dx = \int \frac{\left(b\,\text{arsech}\,(cx) + a\right)^2}{x^4}\, dx$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*a*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*
arcsech(c*x)/x^3) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) +
 1/(c*x))^2/x^4, x) - 1/3*a^2/x^3
```

Giac [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x^4}\, dx = \int \frac{\left(b\,\text{arsech}\,(cx) + a\right)^2}{x^4}\, dx$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^2}{x^4}\, dx = \int \frac{\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^4}\, dx$$

```
[In] int((a + b*acosh(1/(c*x)))^2/x^4,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^2/x^4, x)
```

3.41 $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5} dx$

Optimal result

Integrand size = 14, antiderivative size = 151

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5} dx = -\frac{b^2}{32x^4}-\frac{3b^2c^2}{32x^2}+\frac{3}{16}abc^4\text{sech}^{-1}(cx)+\frac{3}{32}b^2c^4\text{sech}^{-1}(cx)^2 + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{8x^4} + \frac{3bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{16x^2}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^4}$$

```
[Out] -1/32*b^2/x^4-3/32*b^2*c^2/x^2+3/16*a*b*c^4*arcsech(c*x)+3/32*b^2*c^4*arcse
ch(c*x)^2-1/4*(a+b*arcsech(c*x))^2/x^4+1/8*b*(c*x+1)*(a+b*arcsech(c*x))*((-
c*x+1)/(c*x+1))^(1/2)/x^4+3/16*b*c^2*(c*x+1)*(a+b*arcsech(c*x))*((-c*x+1)/(
c*x+1))^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6420, 5555, 3391}

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5} dx = \frac{3}{16}abc^4\text{sech}^{-1}(cx)+\frac{3bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{16x^2} -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^4}+\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{8x^4} +\frac{3}{32}b^2c^4\text{sech}^{-1}(cx)^2-\frac{3b^2c^2}{32x^2}-\frac{b^2}{32x^4}$$

```
[In] Int[(a + b*ArcSech[c*x])^2/x^5,x]
```

```
[Out] -1/32*b^2/x^4 - (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcSech[c*x])/16 + (3*b^2
*c^4*ArcSech[c*x]^2)/32 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*Arc
Sech[c*x]))/(8*x^4) + (3*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*A
rcSech[c*x]))/(16*x^2) - (a + b*ArcSech[c*x])^2/(4*x^4)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^4 \text{Subst}\left(\int (a+bx)^2 \cosh^3(x) \sinh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bc^4\right) \text{Subst}\left(\int (a+bx)\cosh^4(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= -\frac{b^2}{32x^4} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{8x^4} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{4x^4}\\
&\quad + \frac{1}{8}\left(3bc^4\right) \text{Subst}\left(\int (a+bx)\cosh^2(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)
\end{aligned}$$

$$= -\frac{b^2}{32x^4} - \frac{3b^2c^2}{32x^2} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{8x^4} + \frac{3bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16x^2} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{4x^4} + \frac{1}{16}\left(3bc^4\right)\text{Subst}\left(\int(a+bx)\,dx,x,\operatorname{sech}^{-1}(cx)\right)$$

$$= -\frac{b^2}{32x^4} - \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\operatorname{sech}^{-1}(cx) + \frac{3}{32}b^2c^4\operatorname{sech}^{-1}(cx)^2 + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{8x^4} + \frac{3bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16x^2} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{4x^4}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.77

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^5}\,dx = \frac{-8a^2 - b^2 - 3b^2c^2x^2 + 4ab\sqrt{\frac{1-cx}{1+cx}} + 4abcx\sqrt{\frac{1-cx}{1+cx}} + 6abc^2x^2\sqrt{\frac{1-cx}{1+cx}} + 6abc^3x^3\sqrt{\frac{1-cx}{1+cx}} + 2b\left(-8a + b\sqrt{\frac{1-cx}{1+cx}}\right.}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])^2/x^5,x]
```

```
[Out] (-8*a^2 - b^2 - 3*b^2*c^2*x^2 + 4*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*a*b*c*x
*Sqrt[(1 - c*x)/(1 + c*x)] + 6*a*b*c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)] + 6*a*
b*c^3*x^3*Sqrt[(1 - c*x)/(1 + c*x)] + 2*b*(-8*a + b*Sqrt[(1 - c*x)/(1 + c*x
)]*(2 + 2*c*x + 3*c^2*x^2 + 3*c^3*x^3))*ArcSech[c*x] + b^2*(-8 + 3*c^4*x^4)
*ArcSech[c*x]^2 - 6*a*b*c^4*x^4*Log[x] + 6*a*b*c^4*x^4*Log[1 + Sqrt[(1 - c*
x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(32*x^4)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.74

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\operatorname{arcsech}(cx)^2}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{8c^3x^3}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{16cx}+\frac{3\operatorname{arcsech}(cx}{32}\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{8c^3x^3}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{16cx}+\frac{3\operatorname{arcsec}}{3}\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arcsech}(cx)^2}{4c^4x^4}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{8c^3x^3}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)}{16cx}+\frac{3\operatorname{arcsec}}{3}\right.\right.$

```
[In] int((a+b*arcsech(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^2/x^4+b^2*c^4*(-1/4/c^4/x^4*arcsech(c*x)^2+1/8/c^3/x^3*(-(c*x-1)/c/x
)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)+3/16*(-(c*x-1)/c/x)^(1/2)*((c*x+1)
/c/x)^(1/2)/c/x*arcsech(c*x)+3/32*arcsech(c*x)^2-1/32/c^4/x^4-3/32/c^2/x^2)
+2*a*b*c^4*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/c^3/x^3*((c
*x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*(-c^2*x^2+1)^(1
/2)*c^2*x^2+2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x^5}\,dx =$$

$$-\frac{3\,b^2c^2x^2-(3\,b^2c^4x^4-8\,b^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2+8\,a^2+b^2-2\left(3\,abc^4x^4-8\,ab+(3\,b^2c^3x^3+2\,b^2cx)\sqrt{\right.}{32\,x^4}$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/32*(3*b^2*c^2*x^2 - (3*b^2*c^4*x^4 - 8*b^2)*log((c*x*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) + 1)/(c*x))^2 + 8*a^2 + b^2 - 2*(3*a*b*c^4*x^4 - 8*a*b + (3*b^2
*c^3*x^3 + 2*b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(3*a*b*c^3*x^3 + 2*a*b*c*x)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)))/x^4
```

Sympy [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5}\,dx=\int \frac{(a+b\,\text{asech}\,(cx))^2}{x^5}\,dx$$

```
[In] integrate((a+b*asech(c*x))**2/x**5,x)
[Out] Integral((a + b*asech(c*x))**2/x**5, x)
```

Maxima [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)^2}{x^5}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="maxima")
[Out] 1/32*a*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(1/
(c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt(1
/(c^2*x^2) - 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1)
 + 1))/c - 16*arcsech(c*x)/x^4) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt(
1/(c*x) - 1) + 1/(c*x))^2/x^5, x) - 1/4*a^2/x^4
```

Giac [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)^2}{x^5}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{x^5}\,dx=\int \frac{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^2/x^5,x)
[Out] int((a + b*acosh(1/(c*x)))^2/x^5, x)
```

3.42 $\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 223

$$\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx = \frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4} - \frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^2} + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^3 + \frac{b^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{c^4} + \frac{b^3\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)}{2c^4}$$

```
[Out] -1/4*b^2*x^2*(a+b*arcsech(c*x))/c^2-1/2*b*(a+b*arcsech(c*x))^2/c^4+1/4*x^4*
(a+b*arcsech(c*x))^3+b^2*(a+b*arcsech(c*x))*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))^2)/c^4+1/2*b^3*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))^2)/c^4+1/4*b^3*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/c^4-1/2*b*(c*x+1)*(a
+b*arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/c^4-1/4*b*x^2*(c*x+1)*(a+b*arcs
ech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6420, 5559, 4271, 3852, 8, 4269, 3799, 2221, 2317, 2438}

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx=\frac{b^2\log\left(e^{2\operatorname{sech}^{-1}(cx)}+1\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{c^4}-\frac{b^2x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4c^2}-\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2c^4}-\frac{b\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2c^4}-\frac{bx^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{4c^2}+\frac{1}{4}x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)^3+\frac{b^3\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(cx)}\right)}{2c^4}+\frac{b^3\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{4c^4}$$

```
[In] Int[x^3*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(4*c^4) - (b^2*x^2*(a + b*ArcSech
[c*x]))/(4*c^2) - (b*(a + b*ArcSech[c*x])^2)/(2*c^4) - (b*Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*c^4) - (b*x^2*Sqrt[(1 - c*x)/
(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(4*c^2) + (x^4*(a + b*ArcSech[
c*x])^3)/4 + (b^2*(a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/c^4 + (
b^3*PolyLog[2, -E^(2*ArcSech[c*x])])/(2*c^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
```

```
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int (a+bx)^3\text{sech}^4(x)\tanh(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{c^4}$$

$$= \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2\text{sech}^4(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{4c^4}$$

$$= -\frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^2} + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{b\text{Subst}\left(\int (a+bx)^2\text{sech}^2(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{2c^4} + \frac{b^3\text{Subst}\left(\int \text{sech}^2(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{4c^4}$$

$$= -\frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^2} + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^3 + \frac{b^2\text{Subst}\left(\int (a+bx)\tanh(x)\,dx,x,\text{sech}^{-1}(cx)\right)}{c^4} + \frac{(ib^3)\,\text{Subst}\left(\int 1\,dx,x,-i\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{4c^4}$$

$$= \frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4} - \frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^2} + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^3 + \frac{(2b^2)\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\text{sech}^{-1}(cx)\right)}{c^4}$$

$$= \frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4} - \frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^2} + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^3 + \frac{b^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{c^4} - \frac{b^3\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx,x,\text{sech}^{-1}(cx)\right)}{c^4}$$

$$
\begin{aligned}
&= \frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4} - \frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} \\
&\quad - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^2} \\
&\quad + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^3 + \frac{b^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{c^4} \\
&\quad - \frac{b^3\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{sech}^{-1}(cx)}\right)}{2c^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4c^4} - \frac{b^2x^2\left(a+b\text{sech}^{-1}(cx)\right)}{4c^2} \\
&\quad - \frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^4} \\
&\quad - \frac{bx^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4c^2} + \frac{1}{4}x^4\left(a+b\text{sech}^{-1}(cx)\right)^3 \\
&\quad + \frac{b^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{c^4} + \frac{b^3\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)}{2c^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.51

$$
\begin{aligned}
&\int x^3\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx \\
&= \frac{1}{4}\left(a^3x^4 + b^3x^4\text{sech}^{-1}(cx)^3 + a^2b\left(-\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)(2+c^2x^2)}{c^4} + 3x^4\text{sech}^{-1}(cx)\right)\right. \\
&\quad + \frac{ab^2\left(-c^2x^2 - 2\sqrt{\frac{1-cx}{1+cx}}(2+2cx+c^2x^2+c^3x^3)\,\text{sech}^{-1}(cx) + 3c^4x^4\text{sech}^{-1}(cx)^2 + 4\log\left(\frac{1}{cx}\right)\right)}{c^4} \\
&\quad - \frac{b^3\left(-\sqrt{\frac{1-cx}{1+cx}}(1+cx) + \left(-2+2\sqrt{\frac{1-cx}{1+cx}} + 2cx\sqrt{\frac{1-cx}{1+cx}} + c^2x^2\sqrt{\frac{1-cx}{1+cx}} + c^3x^3\sqrt{\frac{1-cx}{1+cx}}\right)\text{sech}^{-1}(cx)^2 + \text{sech}^{-1}(\cdots\right.}{c^4}
\end{aligned}
$$

```
[In] Integrate[x^3*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (a^3*x^4 + b^3*x^4*ArcSech[c*x]^3 + a^2*b*(-((Sqrt[(1 - c*x)/(1 + c*x)]*(1
+ c*x)*(2 + c^2*x^2))/c^4) + 3*x^4*ArcSech[c*x]) + (a*b^2*(-(c^2*x^2) - 2*S
qrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^3)*ArcSech[c*x] + 3*c
^4*x^4*ArcSech[c*x]^2 + 4*Log[1/(c*x)]))/c^4 - (b^3*(-(Sqrt[(1 - c*x)/(1 +
c*x)]*(1 + c*x)) + (-2 + 2*Sqrt[(1 - c*x)/(1 + c*x)] + 2*c*x*Sqrt[(1 - c*x)
```

```
/(1 + c*x)] + c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)] + c^3*x^3*Sqrt[(1 - c*x)/(1
 + c*x)])*ArcSech[c*x]^2 + ArcSech[c*x]*(c^2*x^2 - 4*Log[1 + E^(-2*ArcSech[
c*x])]) + 2*PolyLog[2, -E^(-2*ArcSech[c*x])]))/c^4)/4
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.08

method	result
derivativedivides	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsech}(cx)^3c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,c^3x^3}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx}{2}-\frac{c^2x^2\operatorname{arcsech}(cx)}{4}+\sqrt{\cdots}\right.}{\cdots}$
default	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsech}(cx)^3c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,c^3x^3}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx}{2}-\frac{c^2x^2\operatorname{arcsech}(cx)}{4}+\sqrt{\cdots}\right.}{\cdots}$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{\operatorname{arcsech}(cx)^3c^4x^4}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,c^3x^3}{4}-\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx}{2}-\frac{c^2x^2\operatorname{arcsech}(cx)}{4}+\sqrt{\cdots}\right.}{\cdots}$

```
[In] int(x^3*(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*a^3*c^4*x^4+b^3*(1/4*arcsech(c*x)^3*c^4*x^4-1/4*arcsech(c*x)^2*(
-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^3*x^3-1/2*arcsech(c*x)^2*(-(c*x-1
)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1/4*c^2*x^2*arcsech(c*x)+1/4*(-(c*x-1)
/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-1/2*arcsech(c*x)^2-1/4+arcsech(c*x)*ln(
1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+1/2*polylog(2,-(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))^2))+3*a*b^2*(-1/3*arcsech(c*x)+1/4*arcsech(c*x)^
2*c^4*x^4-1/6*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^3*x^3
-1/3*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1/12*c^2*x^2
+1/3*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+3*b*a^2*(1/4*c^4*x^4
*arcsech(c*x)-1/12*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(c^2*x^2+2)
))
```

Fricas [F]

$$\int x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx=\int\left(b\operatorname{arsech}\left(cx\right)+a\right)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arcsech(c*x)^3 + 3*a*b^2*x^3*arcsech(c*x)^2 + 3*a^2*b*x^3*
arcsech(c*x) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int x^3 (a + b \operatorname{asech}{(cx)})^3 \, dx$$

```
[In] integrate(x**3*(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))**3, x)
```

Maxima [F]

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arsech}{(cx)} + a)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*x^4 + 1/4*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*a^2*b + integrate(b^3*x^3*log(sqrt(1/(c*x)
+ 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3 + 3*a*b^2*x^3*log(sqrt(1/(c*x) + 1)*sqr
t(1/(c*x) - 1) + 1/(c*x))^2, x)
```

Giac [F]

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arsech}{(cx)} + a)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int x^3 \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int(x^3*(a + b*acosh(1/(c*x)))^3,x)
```

```
[Out] int(x^3*(a + b*acosh(1/(c*x)))^3, x)
```

3.43 $\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3 dx$

Optimal result

Integrand size = 14, antiderivative size = 242

$$\int x^2\left(a+b\text{sech}^{-1}(cx)\right)^3 dx=-\frac{b^2x\left(a+b\text{sech}^{-1}(cx)\right)}{c^2}-\frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2}+\frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{c^3}+\frac{b^3\arctan\left(\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{cx}\right)}{c^3}+\frac{ib^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{c^3}-\frac{ib^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{c^3}-\frac{ib^3\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(cx)}\right)}{c^3}+\frac{ib^3\text{PolyLog}\left(3,ie^{\text{sech}^{-1}(cx)}\right)}{c^3}$$

```
[Out] -b^2*x*(a+b*arcsech(c*x))/c^2+1/3*x^3*(a+b*arcsech(c*x))^3-b*(a+b*arcsech(c
*x))^2*arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c^3+b^3*arctan((c*x+1
)*((-c*x+1)/(c*x+1))^(1/2)/c/x)/c^3+I*b^2*(a+b*arcsech(c*x))*polylog(2,-I*(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-I*b^2*(a+b*arcsech(c*x))*polyl
og(2,I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-I*b^3*polylog(3,-I*(1/
c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3+I*b^3*polylog(3,I*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2)))/c^3-1/2*b*x*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x
+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6420, 5559, 4271, 3855, 4265, 2611, 2320, 6724}

$$\begin{aligned}\int x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx=&-\frac{b\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{c^3}\\&+\frac{ib^2\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(cx)}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{c^3}\\&-\frac{ib^2\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(cx)}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{c^3}\\&-\frac{b^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{c^2}\\&-\frac{bx\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2c^2}\\&+\frac{1}{3}x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^3+\frac{b^3\arctan\left(\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{cx}\right)}{c^3}\\&-\frac{ib^3\operatorname{PolyLog}\left(3,-ie^{\operatorname{sech}^{-1}(cx)}\right)}{c^3}+\frac{ib^3\operatorname{PolyLog}\left(3,ie^{\operatorname{sech}^{-1}(cx)}\right)}{c^3}\end{aligned}$$

```
[In] Int[x^2*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] -((b^2*x*(a + b*ArcSech[c*x]))/c^2) - (b*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c
*x)*(a + b*ArcSech[c*x])^2)/(2*c^2) + (x^3*(a + b*ArcSech[c*x])^3)/3 - (b*(
a + b*ArcSech[c*x])^2*ArcTan[E^ArcSech[c*x]])/c^3 + (b^3*ArcTan[(Sqrt[(1 -
c*x)/(1 + c*x)]*(1 + c*x))/(c*x)])/c^3 + (I*b^2*(a + b*ArcSech[c*x])*PolyLo
g[2, (-I)*E^ArcSech[c*x]])/c^3 - (I*b^2*(a + b*ArcSech[c*x])*PolyLog[2, I*E
^ArcSech[c*x]])/c^3 - (I*b^3*PolyLog[3, (-I)*E^ArcSech[c*x]])/c^3 + (I*b^3*
PolyLog[3, I*E^ArcSech[c*x]])/c^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int (a+bx)^3\text{sech}^3(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3}$$

$$= \frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{b\text{Subst}\left(\int (a+bx)^2\text{sech}^3(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3}$$

$$= -\frac{b^2x\left(a+b\text{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{b\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{2c^3} + \frac{b^3\text{Subst}\left(\int \text{sech}(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3}$$

$$= -\frac{b^2x\left(a+b\text{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{c^3} + \frac{b^3\arctan\left(\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{cx}\right)}{c^3} + \frac{(ib^2)\,\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3} - \frac{(ib^2)\,\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3}$$

$$= -\frac{b^2x\left(a+b\text{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{b\left(a+b\text{sech}^{-1}(cx)\right)^2\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{c^3} + \frac{b^3\arctan\left(\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{cx}\right)}{c^3} + \frac{ib^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{c^3} - \frac{ib^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{c^3} - \frac{(ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3} + \frac{(ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^3}$$

$$= -\frac{b^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^3 - \frac{b\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c^3} + \frac{b^3\arctan\left(\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{cx}\right)}{c^3} + \frac{ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(cx)}\right)}{c^3} - \frac{ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(cx)}\right)}{c^3} - \frac{(ib^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{c^3} + \frac{(ib^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{c^3}$$

$$= -\frac{b^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{3}x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)^3 - \frac{b\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c^3} + \frac{b^3\arctan\left(\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{cx}\right)}{c^3} + \frac{ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(cx)}\right)}{c^3} - \frac{ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(cx)}\right)}{c^3} - \frac{ib^3\operatorname{PolyLog}\left(3,-ie^{\operatorname{sech}^{-1}(cx)}\right)}{c^3} + \frac{ib^3\operatorname{PolyLog}\left(3,ie^{\operatorname{sech}^{-1}(cx)}\right)}{c^3}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.82

$$\int x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx = \frac{2a^3c^3x^3 - 3a^2bcx\sqrt{\frac{1-cx}{1+cx}}(1+cx) + 6a^2bc^3x^3\operatorname{sech}^{-1}(cx) + 3ia^2b\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right) - 6ab^2\Big(c$$

```
[In] Integrate[x^2*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (2*a^3*c^3*x^3 - 3*a^2*b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 6*a^2*b*
c^3*x^3*ArcSech[c*x] + (3*I)*a^2*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c
*x)]*(1 + c*x)] - 6*a*b^2*(c*x + c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Ar
```

```
cSech[c*x] - c^3*x^3*ArcSech[c*x]^2 - I*ArcSech[c*x]*Log[1 - I/E^ArcSech[c*
x]] + I*ArcSech[c*x]*Log[1 + I/E^ArcSech[c*x]] - I*PolyLog[2, (-I)/E^ArcSec
h[c*x]] + I*PolyLog[2, I/E^ArcSech[c*x]]) - b^3*(6*c*x*ArcSech[c*x] + 3*c*x
*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*ArcSech[c*x]^2 - 2*c^3*x^3*ArcSech[c*x
]^3 - (3*I)*((-4*I)*ArcTan[Tanh[ArcSech[c*x]/2]] + ArcSech[c*x]^2*Log[1 - I
/E^ArcSech[c*x]] - ArcSech[c*x]^2*Log[1 + I/E^ArcSech[c*x]] + 2*ArcSech[c*x
]*PolyLog[2, (-I)/E^ArcSech[c*x]] - 2*ArcSech[c*x]*PolyLog[2, I/E^ArcSech[c
*x]] + 2*PolyLog[3, (-I)/E^ArcSech[c*x]] - 2*PolyLog[3, I/E^ArcSech[c*x]]))
)/(6*c^3)
```

Maple [F]

$$\int x^2(a+b\ \mathrm{arcsech}\left(cx\right))^3\,dx$$

```
[In] int(x^2*(a+b*arcsech(c*x))^3,x)
[Out] int(x^2*(a+b*arcsech(c*x))^3,x)
```

Fricas [F]

$$\int x^2\left(a+b\mathrm{sech}^{-1}(cx)\right)^3\,dx=\int\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
[Out] integral(b^3*x^2*arcsech(c*x)^3 + 3*a*b^2*x^2*arcsech(c*x)^2 + 3*a^2*b*x^2*
arcsech(c*x) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\mathrm{sech}^{-1}(cx)\right)^3\,dx=\int x^2(a+b\,\mathrm{asech}\left(cx\right))^3\,dx$$

```
[In] integrate(x**2*(a+b*asech(c*x))**3,x)
[Out] Integral(x**2*(a + b*asech(c*x))**3, x)
```

Maxima [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arsech}\,(cx)+a)^3 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2
*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*a^2*b + integrate
(b^3*x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3 + 3*a*b^2*x^2
*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)
```

Giac [F]

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arsech}\,(cx)+a)^3 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int x^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int(x^2*(a + b*acosh(1/(c*x)))^3,x)
```

```
[Out] int(x^2*(a + b*acosh(1/(c*x)))^3, x)
```

3.44 $\int x(a+b\text{sech}^{-1}(cx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 126

$$\int x(a+b\text{sech}^{-1}(cx))^3\,dx=-\frac{3b(a+b\text{sech}^{-1}(cx))^2}{2c^2}-\frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\,(a+b\text{sech}^{-1}(cx))^2}{2c^2}+\frac{1}{2}x^2(a+b\text{sech}^{-1}(cx))^3+\frac{3b^2(a+b\text{sech}^{-1}(cx))\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{c^2}+\frac{3b^3\,\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)}{2c^2}$$

```
[Out] -3/2*b*(a+b*arcsech(c*x))^2/c^2+1/2*x^2*(a+b*arcsech(c*x))^3+3*b^2*(a+b*arc
sech(c*x))*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/c^2+3/2*b^3*pol
ylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/c^2-3/2*b*(c*x+1)*(a+b*
arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used

= {6420, 5559, 4269, 3799, 2221, 2317, 2438}

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx=\frac{3b^2\log\left(e^{2\operatorname{sech}^{-1}(cx)}+1\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{c^2}-\frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2c^2}-\frac{3b\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)^3+\frac{3b^3\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(cx)}\right)}{2c^2}$$

```
[In] Int[x*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (-3*b*(a + b*ArcSech[c*x])^2)/(2*c^2) - (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)*(a + b*ArcSech[c*x])^2)/(2*c^2) + (x^2*(a + b*ArcSech[c*x])^3)/2 + (3
*b^2*(a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/c^2 + (3*b^3*PolyLog
[2, -E^(2*ArcSech[c*x])])/(2*c^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^3\text{sech}^2(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^2}\\
&= \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2\text{sech}^2(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{2c^2}\\
&= -\frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^3\\
&\quad + \frac{(3b^2)\text{ Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^2}\\
&= -\frac{3b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2}\\
&\quad + \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^3 + \frac{(6b^2)\text{ Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(cx)\right)}{c^2}\\
&= -\frac{3b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2}\\
&\quad + \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^3 + \frac{3b^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{c^2}\\
&\quad - \frac{(3b^3)\text{ Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{c^2}\\
&= -\frac{3b\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a+b\text{sech}^{-1}(cx)\right)^3\\
&\quad + \frac{3b^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)}{c^2} - \frac{(3b^3)\text{ Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\text{sech}^{-1}(cx)}\right)}{2c^2}
\end{aligned}$$

$$= -\frac{3b\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a + b\operatorname{sech}^{-1}(cx)\right)^3 + \frac{3b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right)}{c^2} + \frac{3b^3\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(cx)}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.74

$$\int x\left(a+b\operatorname{sech}^{-1}(cx)\right)^3\,dx$$

$$= \frac{-3b^2\left(-ac^2x^2+b\left(-1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)\right)\operatorname{sech}^{-1}(cx)^2+b^3c^2x^2\operatorname{sech}^{-1}(cx)^3+3b\operatorname{sech}^{-1}(cx)\left(a\left(ac^2x^2\right.\right.}{}$$

```
[In] Integrate[x*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] (-3*b^2*(-(a*c^2*x^2) + b*(-1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c
*x)/(1 + c*x)]))*ArcSech[c*x]^2 + b^3*c^2*x^2*ArcSech[c*x]^3 + 3*b*ArcSech[
c*x]*(a*(a*c^2*x^2 - 2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + 2*b^2*Log[1
 + E^(-2*ArcSech[c*x])]) + a*(a*(a*c^2*x^2 - 3*b*Sqrt[(1 - c*x)/(1 + c*x)]*
(1 + c*x)) + 6*b^2*Log[1/(c*x)]) - 3*b^3*PolyLog[2, -E^(-2*ArcSech[c*x])])/
(2*c^2)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.52

method	result
derivativedivides	$\frac{\frac{a^3c^2x^2}{2}+b^3\left(\frac{\operatorname{arcsech}(cx)^2\left(c^2x^2\operatorname{arcsech}(cx)-3\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+3\right)}{2}-3\operatorname{arcsech}(cx)^2+3\operatorname{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\right.\right.\right.}{}$
default	$\frac{\frac{a^3c^2x^2}{2}+b^3\left(\frac{\operatorname{arcsech}(cx)^2\left(c^2x^2\operatorname{arcsech}(cx)-3\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+3\right)}{2}-3\operatorname{arcsech}(cx)^2+3\operatorname{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\right.\right.\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{\operatorname{arcsech}(cx)^2\left(c^2x^2\operatorname{arcsech}(cx)-3\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+3\right)}{2}-3\operatorname{arcsech}(cx)^2+3\operatorname{arcsech}(cx)\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\right.\right.\right.}{c^2}$

```
[In] int(x*(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)
```

[Out] 1/c^2*(1/2*a^3*c^2*x^2+b^3*(1/2*arcsech(c*x)^2*(c^2*x^2*arcsech(c*x)-3*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+3)-3*arcsech(c*x)^2+3*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+3*a*b^2*(-2*arcsech(c*x)+1/2*arcsech(c*x)*(c^2*x^2*arcsech(c*x)-2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+2)+ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+3*b*a^2*(1/2*c^2*x^2*arcsech(c*x)-1/2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)))

Fricas [F]

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int\left(b\,\text{arsech}\left(cx\right)+a\right)^3x\,dx$$

[In] integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral(b^3*x*arcsech(c*x)^3 + 3*a*b^2*x*arcsech(c*x)^2 + 3*a^2*b*x*arcsech(c*x) + a^3*x, x)

Sympy [F]

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int x(a+b\,\text{asech}\left(cx\right))^3\,dx$$

[In] integrate(x*(a+b*asech(c*x))**3,x)

[Out] Integral(x*(a + b*asech(c*x))**3, x)

Maxima [F]

$$\int x\left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int\left(b\,\text{arsech}\left(cx\right)+a\right)^3x\,dx$$

[In] integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="maxima")

[Out] 3/2*a*b^2*x^2*arcsech(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*a^2*b - 3*(x*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x)/c + log(x)/c^2)*a*b^2 + b^3*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3, x)

Giac [F]

$$\int x(a+b\text{sech}^{-1}(cx))^3\,dx=\int (b\operatorname{arsech}(cx)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{sech}^{-1}(cx))^3\,dx=\int x\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

```
[In] int(x*(a + b*acosh(1/(c*x)))^3,x)
[Out] int(x*(a + b*acosh(1/(c*x)))^3, x)
```

3.45 $\int \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 140

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = x\left(a+b\text{sech}^{-1}(cx)\right)^3-\frac{6b\left(a+b\text{sech}^{-1}(cx)\right)^2\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{c}+\frac{6ib^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)}{c}-\frac{6ib^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)}{c}-\frac{6ib^3\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(cx)}\right)}{c}+\frac{6ib^3\text{PolyLog}\left(3,ie^{\text{sech}^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arcsech(c*x))^3-6*b*(a+b*arcsech(c*x))^2*arctan(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))/c+6*I*b^2*(a+b*arcsech(c*x))*polylog(2,-I*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2)))/c-6*I*b^2*(a+b*arcsech(c*x))*polylog(2,I*(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/c-6*I*b^3*polylog(3,-I*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2)))/c+6*I*b^3*polylog(3,I*(1/c/x+(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2)))/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6414, 5559, 4265, 2611, 2320, 6724}

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = -\frac{6b\arctan\left(e^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)^2}{c} + \frac{6ib^2\,\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)}{c} - \frac{6ib^2\,\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(cx)}\right)\left(a+b\text{sech}^{-1}(cx)\right)}{c} + x\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{6ib^3\,\text{PolyLog}\left(3,-ie^{\text{sech}^{-1}(cx)}\right)}{c} + \frac{6ib^3\,\text{PolyLog}\left(3,ie^{\text{sech}^{-1}(cx)}\right)}{c}$$

```
[In] Int[(a + b*ArcSech[c*x])^3,x]
```

```
[Out] x*(a + b*ArcSech[c*x])^3 - (6*b*(a + b*ArcSech[c*x])^2*ArcTan[E^ArcSech[c*x
]])/c + ((6*I)*b^2*(a + b*ArcSech[c*x])*PolyLog[2, (-I)*E^ArcSech[c*x]])/c
- ((6*I)*b^2*(a + b*ArcSech[c*x])*PolyLog[2, I*E^ArcSech[c*x]])/c - ((6*I)*
b^3*PolyLog[3, (-I)*E^ArcSech[c*x]])/c + ((6*I)*b^3*PolyLog[3, I*E^ArcSech[
c*x]])/c
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6414

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^3\text{sech}(x)\tanh(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}\\
&= x\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}\\
&= x\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{6b\left(a+b\text{sech}^{-1}(cx)\right)^2\arctan\left(e^{\text{sech}^{-1}(cx)}\right)}{c}\\
&\quad + \frac{(6ib^2)\,\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}\\
&\quad - \frac{(6ib^2)\,\text{Subst}\left(\int (a+bx)\log\left(1+ie^x\right)\,dx, x, \text{sech}^{-1}(cx)\right)}{c}
\end{aligned}
$$

$$
\begin{aligned}
&= x\left(a+b\operatorname{sech}^{-1}(cx)\right)^3-\frac{6b\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad+\frac{6ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad-\frac{6ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad-\frac{(6ib^3)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-ie^x\right)\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{c}\\
&\quad+\frac{(6ib^3)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,ie^x\right)\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{c}
\end{aligned}
$$

$$
\begin{aligned}
&= x\left(a+b\operatorname{sech}^{-1}(cx)\right)^3-\frac{6b\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad+\frac{6ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad-\frac{6ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad-\frac{(6ib^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad+\frac{(6ib^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{c}
\end{aligned}
$$

$$
\begin{aligned}
&= x\left(a+b\operatorname{sech}^{-1}(cx)\right)^3-\frac{6b\left(a+b\operatorname{sech}^{-1}(cx)\right)^2\arctan\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad+\frac{6ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad-\frac{6ib^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,ie^{\operatorname{sech}^{-1}(cx)}\right)}{c}\\
&\quad-\frac{6ib^3\operatorname{PolyLog}\left(3,-ie^{\operatorname{sech}^{-1}(cx)}\right)}{c}+\frac{6ib^3\operatorname{PolyLog}\left(3,ie^{\operatorname{sech}^{-1}(cx)}\right)}{c}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 282 vs. 2(140) = 280.

Time = 0.57 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.01

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = a^3x+3a^2bx\text{sech}^{-1}(cx)-\frac{3a^2b\arctan\left(\frac{cx\sqrt{\frac{1-cx}{1+cx}}}{-1+cx}\right)}{c}$$
$$+\frac{3iab^2\left(\text{sech}^{-1}(cx)\left(-icx\text{sech}^{-1}(cx)+2\log\left(1-ie^{-\text{sech}^{-1}(cx)}\right)-2\log\left(1+ie^{-\text{sech}^{-1}(cx)}\right)\right)+2\,\text{PolyLog}\left(\right.\right.}{c}$$
$$+\frac{b^3\left(cx\text{sech}^{-1}(cx)^3-3i\left(-\text{sech}^{-1}(cx)^2\left(\log\left(1-ie^{-\text{sech}^{-1}(cx)}\right)-\log\left(1+ie^{-\text{sech}^{-1}(cx)}\right)\right)-2\text{sech}^{-1}(cx)\left(\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcSech[c*x] - (3*a^2*b*ArcTan[(c*x*Sqrt[(1 - c*x)/(1 + c
*x)])/(-1 + c*x)])/c + ((3*I)*a*b^2*(ArcSech[c*x]*((-I)*c*x*ArcSech[c*x] +
2*Log[1 - I/E^ArcSech[c*x]] - 2*Log[1 + I/E^ArcSech[c*x]]) + 2*PolyLog[2, (
-I)/E^ArcSech[c*x]] - 2*PolyLog[2, I/E^ArcSech[c*x]]))/c + (b^3*(c*x*ArcSec
h[c*x]^3 - (3*I)*(-(ArcSech[c*x]^2*(Log[1 - I/E^ArcSech[c*x]] - Log[1 + I/E
^ArcSech[c*x]])) - 2*ArcSech[c*x]*(PolyLog[2, (-I)/E^ArcSech[c*x]] - PolyLo
g[2, I/E^ArcSech[c*x]]) - 2*(PolyLog[3, (-I)/E^ArcSech[c*x]] - PolyLog[3, I
/E^ArcSech[c*x]]))))/c
```

Maple [F]

$$\int \left(a+b\,\text{arcsech}\left(cx\right)\right)^3 dx$$

```
[In] int((a+b*arcsech(c*x))^3,x)
```

```
[Out] int((a+b*arcsech(c*x))^3,x)
```

Fricas [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int \left(b\,\text{arsech}\left(cx\right)+a\right)^3 dx$$

```
[In] integrate((a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x)
 + a^3, x)
```

Sympy [F]

$$\int \left(a + b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (a + b\,\text{asech}\,(cx))^3 \, dx$$

[In] integrate((a+b*asech(c*x))**3,x)

[Out] Integral((a + b*asech(c*x))**3, x)

Maxima [F]

$$\int \left(a + b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arsech}\,(cx) + a)^3 \, dx$$

[In] integrate((a+b*arcsech(c*x))^3,x, algorithm="maxima")

[Out] b^3*x*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^3 + a^3*x + 3*(c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*a^2*b/c - integrate(-(b^3*log(c)^3 - 3*a*b^2*log(c)^2 - (b^3*c^2*x^2 - b^3)*log(x)^3 - (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*log(c) - a*b^2 - (b^3*c^2*(log(c) + 1) - a*b^2*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^3*c^2*x^2 - b^3)*log(x))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 + 3*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 - (b^3*c^2*x^2 - b^3)*log(x)^3 - (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 - (b^3*c^2*x^2 - b^3)*log(x)^2 + (b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 - (b^3*c^2*x^2 - b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) + 2*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) - (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))/(c^2*x^2 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 1), x)

Giac [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arsech}\,(cx)+a)^3\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^3,x)
[Out] int((a + b*acosh(1/(c*x)))^3, x)
```

3.46 $\int \frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^3}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 114

$$\begin{aligned}\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx &= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^4}{4b}-\left(a+b\text{sech}^{-1}(cx)\right)^3\log\left(1+e^{2\text{sech}^{-1}(cx)}\right)\\&\quad-\frac{3}{2}b\left(a+b\text{sech}^{-1}(cx)\right)^2\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right)\\&\quad+\frac{3}{2}b^2\left(a+b\text{sech}^{-1}(cx)\right)\text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right)\\&\quad-\frac{3}{4}b^3\,\text{PolyLog}\left(4,-e^{2\text{sech}^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/4*(a+b*arcsech(c*x))^4/b-(a+b*arcsech(c*x))^3*ln(1+(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2)-3/2*b*(a+b*arcsech(c*x))^2*polylog(2,-(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*b^2*(a+b*arcsech(c*x))*polylog(3,-(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-3/4*b^3*polylog(4,-(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

= {6420, 3799, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x}\,dx = \frac{3}{2}b^2\operatorname{PolyLog}\left(3,-e^{2\operatorname{sech}^{-1}(cx)}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right) - \frac{3}{2}b\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(cx)}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - \log\left(e^{2\operatorname{sech}^{-1}(cx)}+1\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)^3 - \frac{3}{4}b^3\operatorname{PolyLog}\left(4,-e^{2\operatorname{sech}^{-1}(cx)}\right)$$

```
[In] Int[(a + b*ArcSech[c*x])^3/x,x]
```

```
[Out] (a + b*ArcSech[c*x])^4/(4*b) - (a + b*ArcSech[c*x])^3*Log[1 + E^(2*ArcSech[
c*x])] - (3*b*(a + b*ArcSech[c*x])^2*PolyLog[2, -E^(2*ArcSech[c*x])])/2 + (
3*b^2*(a + b*ArcSech[c*x])*PolyLog[3, -E^(2*ArcSech[c*x])])/2 - (3*b^3*Poly
Log[4, -E^(2*ArcSech[c*x])])/4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
```

```
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int (a+bx)^3 \tanh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - 2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^3 \log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right)\\
&\quad + (3b)\text{Subst}\left(\int (a+bx)^2 \log\left(1+e^{2x}\right)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^4}{4b} - \left(a+b\operatorname{sech}^{-1}(cx)\right)^3 \log\left(1+e^{2\operatorname{sech}^{-1}(cx)}\right)\\
&\quad - \frac{3}{2}b\left(a+b\operatorname{sech}^{-1}(cx)\right)^2 \text{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(cx)}\right)\\
&\quad + \left(3b^2\right)\text{Subst}\left(\int (a+bx)\,\text{PolyLog}\left(2,-e^{2x}\right)\,dx, x, \operatorname{sech}^{-1}(cx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(a+b\text{sech}^{-1}(cx))^4}{4b} - (a+b\text{sech}^{-1}(cx))^3 \log\left(1+e^{2\text{sech}^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b(a+b\text{sech}^{-1}(cx))^2 \text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}b^2(a+b\text{sech}^{-1}(cx)) \text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right) \\
&\quad - \frac{1}{2}(3b^3) \text{Subst}\left(\int \text{PolyLog}\left(3,-e^{2x}\right) dx, x, \text{sech}^{-1}(cx)\right) \\
&= \frac{(a+b\text{sech}^{-1}(cx))^4}{4b} - (a+b\text{sech}^{-1}(cx))^3 \log\left(1+e^{2\text{sech}^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b(a+b\text{sech}^{-1}(cx))^2 \text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}b^2(a+b\text{sech}^{-1}(cx)) \text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right) \\
&\quad - \frac{1}{4}(3b^3) \text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x} dx, x, e^{2\text{sech}^{-1}(cx)}\right) \\
&= \frac{(a+b\text{sech}^{-1}(cx))^4}{4b} - (a+b\text{sech}^{-1}(cx))^3 \log\left(1+e^{2\text{sech}^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b(a+b\text{sech}^{-1}(cx))^2 \text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}b^2(a+b\text{sech}^{-1}(cx)) \text{PolyLog}\left(3,-e^{2\text{sech}^{-1}(cx)}\right) - \frac{3}{4}b^3 \text{PolyLog}\left(4,-e^{2\text{sech}^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.60

$$
\begin{aligned}
\int \frac{(a+b\text{sech}^{-1}(cx))^3}{x} dx = \frac{1}{4}\Big(&-6a^2b\text{sech}^{-1}(cx)^2 - 4ab^2\text{sech}^{-1}(cx)^3 - b^3\text{sech}^{-1}(cx)^4 \\
&- 12a^2b\text{sech}^{-1}(cx) \log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) \\
&- 12ab^2\text{sech}^{-1}(cx)^2 \log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) \\
&- 4b^3\text{sech}^{-1}(cx)^3 \log\left(1+e^{-2\text{sech}^{-1}(cx)}\right) + 4a^3\log(cx) \\
&+ 6b(a+b\text{sech}^{-1}(cx))^2 \text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right) \\
&+ 6b^2(a+b\text{sech}^{-1}(cx)) \text{PolyLog}\left(3,-e^{-2\text{sech}^{-1}(cx)}\right) \\
&+ 3b^3 \text{PolyLog}\left(4,-e^{-2\text{sech}^{-1}(cx)}\right)\Big)
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x,x]
```

```
[Out] (-6*a^2*b*ArcSech[c*x]^2 - 4*a*b^2*ArcSech[c*x]^3 - b^3*ArcSech[c*x]^4 - 12
*a^2*b*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - 12*a*b^2*ArcSech[c*x]^2*
```

```
Log[1 + E^(-2*ArcSech[c*x])] - 4*b^3*ArcSech[c*x]^3*Log[1 + E^(-2*ArcSech[c
*x])] + 4*a^3*Log[c*x] + 6*b*(a + b*ArcSech[c*x])^2*PolyLog[2, -E^(-2*ArcSe
ch[c*x])] + 6*b^2*(a + b*ArcSech[c*x])*PolyLog[3, -E^(-2*ArcSech[c*x])] + 3
*b^3*PolyLog[4, -E^(-2*ArcSech[c*x])])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(206) = 412$.

Time = 0.58 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.76

method	result
parts	$a^3 \ln(x) + b^3 \left(\frac{\operatorname{arcsech}(cx)^4}{4} - \operatorname{arcsech}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right) - \frac{3\,\mathrm{arcs}}{}\right.$
derivativedivides	$a^3 \ln(cx) + b^3 \left(\frac{\operatorname{arcsech}(cx)^4}{4} - \operatorname{arcsech}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right) - \frac{3\,\mathrm{ar}}{}\right.$
default	$a^3 \ln(cx) + b^3 \left(\frac{\operatorname{arcsech}(cx)^4}{4} - \operatorname{arcsech}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2\right) - \frac{3\,\mathrm{ar}}{}\right.$

```
[In] int((a+b*arcsech(c*x))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*ln(x)+b^3*(1/4*arcsech(c*x)^4-arcsech(c*x)^3*ln(1+(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2)-3/2*arcsech(c*x)^2*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2)+3/2*arcsech(c*x)*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))^2)-3/4*polylog(4,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^
2))+3*a*b^2*(1/3*arcsech(c*x)^3-arcsech(c*x)^2*ln(1+(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))^2)-arcsech(c*x)*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))^2)+1/2*polylog(3,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))+3
*b*a^2*(1/2*arcsech(c*x)^2-arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))^2)-1/2*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))
```

Fricas [F]

$$\int \frac{\left(a + b\operatorname{sech}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{\left(b\operatorname{arsech}(cx) + a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x
) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx=\int \frac{\left(a+b\operatorname{asech}\left(cx\right)\right)^3}{x}\,dx$$

```
[In] integrate((a+b*asech(c*x))**3/x,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx=\int \frac{\left(b\operatorname{arsech}\left(cx\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(b^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x
))^3/x + 3*a*b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x + 3
*a^2*b*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

Giac [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx=\int \frac{\left(b\operatorname{arsech}\left(cx\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x}\,dx=\int \frac{\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^3/x,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^3/x, x)
```

3.47 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^2}\,dx=\frac{6b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x}-\frac{6b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}+\frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x}$$

```
[Out] -6*b^2*(a+b*arcsech(c*x))/x-(a+b*arcsech(c*x))^3/x+6*b^3*(c*x+1)*((-c*x+1)/
(c*x+1))^(1/2)/x+3*b*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1)/(c*x+1))^(1/2)/
x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6420, 3377, 2717}

$$\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^2}\,dx=-\frac{6b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}+\frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{x}-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x}+\frac{6b^3\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{x}$$

```
[In] Int[(a + b*ArcSech[c*x])^3/x^2,x]
```

```
[Out] (6*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/x - (6*b^2*(a + b*ArcSech[c*x])
)/x + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/x -
(a + b*ArcSech[c*x])^3/x
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int (a+bx)^3\sinh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x} + (3bc)\text{Subst}\left(\int (a+bx)^2\cosh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x}\\
&\quad - \left(6b^2c\right)\text{Subst}\left(\int (a+bx)\sinh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= -\frac{6b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x}\\
&\quad - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x} + \left(6b^3c\right)\text{Subst}\left(\int \cosh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{6b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x} - \frac{6b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\\
&\quad + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{x} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^2}\,dx =$$

$$-\frac{a^3+6ab^2-3a^2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)-6b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)+3b\left(a^2+2b^2-2ab\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\text{sech}^{-1}(cx)}{x}$$

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x^2,x]
```

```
[Out] -((a^3 + 6*a*b^2 - 3*a^2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 6*b^3*Sqrt
[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 3*b*(a^2 + 2*b^2 - 2*a*b*Sqrt[(1 - c*x)/(
1 + c*x)]*(1 + c*x))*ArcSech[c*x] - 3*b^2*(-a + b*Sqrt[(1 - c*x)/(1 + c*x)]
*(1 + c*x))*ArcSech[c*x]^2 + b^3*ArcSech[c*x]^3)/x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(98) = 196.

Time = 0.54 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.21

method	result
parts	$-\frac{a^3}{x}+b^3c\left(-\frac{\text{arcsech}(cx)^3}{cx}+3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\ \text{arcsech}\left(cx\right)^2-\frac{6\ \text{arcsech}(cx)}{cx}+6\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{c}{}}\right.$
derivativedivides	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\text{arcsech}(cx)^3}{cx}+3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\ \text{arcsech}\left(cx\right)^2-\frac{6\ \text{arcsech}(cx)}{cx}+6\sqrt{-\frac{cx-1}{cx}}\right.\right.$
default	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\text{arcsech}(cx)^3}{cx}+3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\ \text{arcsech}\left(cx\right)^2-\frac{6\ \text{arcsech}(cx)}{cx}+6\sqrt{-\frac{cx-1}{cx}}\right.\right.$

```
[In] int((a+b*arcsech(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3/x+b^3*c*(-1/c/x*arcsech(c*x)^3+3*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1
/2)*arcsech(c*x)^2-6/c/x*arcsech(c*x)+6*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^
(1/2))+3*a*b^2*c*(-1/c/x*arcsech(c*x)^2+2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)
*((c*x+1)/c/x)^(1/2)-2/c/x)+3*b*a^2*c*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(
1/2)*((c*x+1)/c/x)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(98) = 196$.

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.24

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^3}{x^2}\, dx =$$

$$-\frac{b^3 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3 - 3\,(a^2b + 2\,b^3)cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + a^3 + 6\,ab^2 - 3\left(b^3cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - ab^2\right)\log\left(\frac{cx\sqrt{-\ldots}}{\ldots}\right.\ldots}{x}$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^3 - 3*(a^2*b + 2*
b^3)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + a^3 + 6*a*b^2 - 3*(b^3*c*x*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) - a*b^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
 1)/(c*x))^2 - 3*(2*a*b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a^2*b - 2*b^
3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

Sympy [F]

$$\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^3}{x^2}\, dx = \int \frac{\left(a + b\,\text{asech}\,(cx)\right)^3}{x^2}\, dx$$

```
[In] integrate((a+b*asech(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{\left(a + b\text{sech}^{-1}(cx)\right)^3}{x^2}\, dx &= -\frac{b^3\,\text{arsech}\,(cx)^3}{x} + 3\left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\text{arsech}\,(cx)}{x}\right)a^2b \\ &\quad + 6\left(c\sqrt{\frac{1}{c^2x^2}-1}\,\text{arsech}\,(cx) - \frac{1}{x}\right)ab^2 \\ &\quad + 3\left(c\sqrt{\frac{1}{c^2x^2}-1}\,\text{arsech}\,(cx)^2 + 2\,c\sqrt{\frac{1}{c^2x^2}-1} - \frac{2\,\text{arsech}\,(cx)}{x}\right)b^3 \\ &\quad - \frac{3\,ab^2\,\text{arsech}\,(cx)^2}{x} - \frac{a^3}{x}\end{aligned}$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*arcsech(c*x)^3/x + 3*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*a^2*b
+ 6*(c*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x) - 1/x)*a*b^2 + 3*(c*sqrt(1/(c^2*x
^2) - 1)*arcsech(c*x)^2 + 2*c*sqrt(1/(c^2*x^2) - 1) - 2*arcsech(c*x)/x)*b^3
 - 3*a*b^2*arcsech(c*x)^2/x - a^3/x
```

Giac [F]

$$\int \frac{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}{x^2}\,dx=\int \frac{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^2}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^3/x^2,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^3/x^2, x)
```

3.48 $\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 163

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^3}\,dx = \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8x^2} - \frac{3}{8}b^3c^2\text{sech}^{-1}(cx) - \frac{3b^2(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{4x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^2} - \frac{1}{4}c^2\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2x^2}$$

```
[Out] -3/8*b^3*c^2*arcsech(c*x)-3/4*b^2*(-c*x+1)*(c*x+1)*(a+b*arcsech(c*x))/x^2-1
/4*c^2*(a+b*arcsech(c*x))^3-1/2*(-c*x+1)*(c*x+1)*(a+b*arcsech(c*x))^3/x^2+3
/8*b^3*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/x^2+3/4*b*(c*x+1)*(a+b*arcsech(c*x)
)^2*((-c*x+1)/(c*x+1))^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

= {6420, 5554, 3392, 32, 2715, 8}

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^3}\,dx = -\frac{3b^2(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)}{4x^2} - \frac{1}{4}c^2\left(a+b\text{sech}^{-1}(cx)\right)^3 + \frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^2} - \frac{(1-cx)(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2x^2} - \frac{3}{8}b^3c^2\text{sech}^{-1}(cx) + \frac{3b^3\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{8x^2}$$

```
[In] Int[(a + b*ArcSech[c*x])^3/x^3,x]
```

```
[Out] (3*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(8*x^2) - (3*b^3*c^2*ArcSech[c*
x])/8 - (3*b^2*(1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x]))/(4*x^2) + (3*b*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(4*x^2) - (c^2*(a
+ b*ArcSech[c*x])^3)/4 - ((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^3)/(2*x^
2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\left(c^2\text{Subst}\left(\int (a+bx)^3\cosh(x)\sinh(x)\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2x^2} + \frac{1}{2}\left(3bc^2\right)\text{Subst}\left(\int (a+bx)^2\sinh^2(x)\,dx, x, \text{sech}^{-1}(cx)\right)$$

$$= -\frac{3b^2(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{4x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^2} - \frac{(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2x^2} - \frac{1}{4}\left(3bc^2\right)\text{Subst}\left(\int (a+bx)^2\,dx, x, \text{sech}^{-1}(cx)\right) + \frac{1}{4}\left(3b^3c^2\right)\text{Subst}\left(\int \sinh^2(x)\,dx, x, \text{sech}^{-1}(cx)\right)$$

$$= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8x^2} - \frac{3b^2(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{4x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^2} - \frac{1}{4}c^2\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2x^2} - \frac{1}{8}\left(3b^3c^2\right)\text{Subst}\left(\int 1\,dx, x, \text{sech}^{-1}(cx)\right)$$

$$= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8x^2} - \frac{3}{8}b^3c^2\text{sech}^{-1}(cx) - \frac{3b^2(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)}{4x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{4x^2} - \frac{1}{4}c^2\left(a+b\text{sech}^{-1}(cx)\right)^3 - \frac{(1-cx)(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^3}{2x^2}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^3}\,dx = \frac{-4a^3-6ab^2+3b(2a^2+b^2)\sqrt{\frac{1-cx}{1+cx}}(1+cx)-6b\left(2a^2+b^2-2ab\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)\text{sech}^{-1}(cx)+6b^2\left(b\sqrt{\frac{1-}{1+}}\right.}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x^3,x]
```

```
[Out] (-4*a^3 - 6*a*b^2 + 3*b*(2*a^2 + b^2)*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) -
 6*b*(2*a^2 + b^2 - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x]
 + 6*b^2*(b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + a*(-2 + c^2*x^2))*ArcSech
[c*x]^2 + 2*b^3*(-2 + c^2*x^2)*ArcSech[c*x]^3 - 3*b*(2*a^2 + b^2)*c^2*x^2*L
og[x] + 3*b*(2*a^2 + b^2)*c^2*x^2*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*S
qrt[(1 - c*x)/(1 + c*x)]])/(8*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(147) = 294.

Time = 0.50 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.97

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\text{arcsech}(cx)^3}{2c^2x^2}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,\text{arcsech}(cx)^2}{4cx}+\frac{\text{arcsech}(cx)^3}{4}-\frac{3\,\text{arcsech}(cx)}{4c^2x^2}+\frac{3\sqrt{}}{}\right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\text{arcsech}(cx)^3}{2c^2x^2}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,\text{arcsech}(cx)^2}{4cx}+\frac{\text{arcsech}(cx)^3}{4}-\frac{3\,\text{arcsech}(cx)}{4c^2x^2}+\frac{3\sqrt{}}{}\right.\right.$
parts	$-\frac{a^3}{2x^2}+b^3c^2\left(-\frac{\text{arcsech}(cx)^3}{2c^2x^2}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,\text{arcsech}(cx)^2}{4cx}+\frac{\text{arcsech}(cx)^3}{4}-\frac{3\,\text{arcsech}(cx)}{4c^2x^2}+\frac{3\sqrt{-\frac{c}{c}}}{}\right.$

```
[In] int((a+b*arcsech(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/2/c^2/x^2*arcsech(c*x)^3+3/4*(-(c*x-1)/c/x)^(
1/2)*((c*x+1)/c/x)^(1/2)/c/x*arcsech(c*x)^2+1/4*arcsech(c*x)^3-3/4/c^2/x^2*
arcsech(c*x)+3/8*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)/c/x+3/8*arcsech(c
*x))+3*a*b^2*(-1/2/c^2/x^2*arcsech(c*x)^2+1/2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)
/c/x)^(1/2)/c/x*arcsech(c*x)+1/4*arcsech(c*x)^2-1/4/c^2/x^2)+3*b*a^2*(-1/2/
c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c/x*((c*x+1)/c/x)^(1/2)*(arct
anh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.66

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^3}\,dx$$
$$= \frac{2\left(b^3c^2x^2-2\,b^3\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3+3\left(2\,a^2b+b^3\right)cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-4\,a^3-6\,ab^2+6\left(ab^2c^2x^2+b^3cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right.}{}$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(b^3*c^2*x^2 - 2*b^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(
c*x))^3 + 3*(2*a^2*b + b^3)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*a^3 - 6*
a*b^2 + 6*(a*b^2*c^2*x^2 + b^3*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*a*b^2
)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + 3*(4*a*b^2*c*x*sq
rt(-(c^2*x^2 - 1)/(c^2*x^2)) + (2*a^2*b + b^3)*c^2*x^2 - 4*a^2*b - 2*b^3)*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

Sympy [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\operatorname{asech}\left(cx\right)\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*asech(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(b\,\operatorname{arsech}\left(cx\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1)
- c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1
) - 1))/c + 4*arcsech(c*x)/x^2) - 1/2*a^3/x^2 + integrate(b^3*log(sqrt(1/(c
*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3/x^3 + 3*a*b^2*log(sqrt(1/(c*x) + 1)
*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(b\,\operatorname{arsech}\left(cx\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^3/x^3,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^3/x^3, x)
```

3.49 $\int \frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 213

$$\int \frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx=\frac{14b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{9x}+\frac{2b^3\left(\frac{1-cx}{1+cx}\right)^{3/2}(1+cx)^3}{27x^3}-\frac{2b^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{9x^3}-\frac{4b^2c^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{3x}+\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{3x^3}+\frac{2bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{3x}-\frac{\left(a+b\,\text{sech}^{-1}(cx)\right)^3}{3x^3}$$

```
[Out] 2/27*b^3*((-c*x+1)/(c*x+1))^(3/2)*(c*x+1)^3/x^3-2/9*b^2*(a+b*arcsech(c*x))/
x^3-4/3*b^2*c^2*(a+b*arcsech(c*x))/x-1/3*(a+b*arcsech(c*x))^3/x^3+14/9*b^3*
c^2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/x+1/3*b*(c*x+1)*(a+b*arcsech(c*x))^2*(
(-c*x+1)/(c*x+1))^(1/2)/x^3+2/3*b*c^2*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1
)/(c*x+1))^(1/2)/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

= {6420, 5555, 3392, 3377, 2717, 2713}

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^4}\,dx = -\frac{4b^2c^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x} - \frac{2b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{9x^3} + \frac{2bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x} + \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x^3} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{3x^3} + \frac{14b^3c^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{9x} + \frac{2b^3\left(\frac{1-cx}{cx+1}\right)^{3/2}(cx+1)^3}{27x^3}$$

```
[In] Int[(a + b*ArcSech[c*x])^3/x^4,x]
```

```
[Out] (14*b^3*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(9*x) + (2*b^3*((1 - c*x)/
(1 + c*x))^(3/2)*(1 + c*x)^3)/(27*x^3) - (2*b^2*(a + b*ArcSech[c*x]))/(9*x^
3) - (4*b^2*c^2*(a + b*ArcSech[c*x]))/(3*x) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*
(1 + c*x)*(a + b*ArcSech[c*x])^2)/(3*x^3) + (2*b*c^2*Sqrt[(1 - c*x)/(1 + c*
x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(3*x) - (a + b*ArcSech[c*x])^3/(3*x^3
)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^3 \text{Subst}\left(\int (a+bx)^3 \cosh^2(x) \sinh(x)\, dx, x, \text{sech}^{-1}(cx)\right)\right) \\
&= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3x^3} + \left(bc^3\right) \text{Subst}\left(\int (a+bx)^2 \cosh^3(x)\, dx, x, \text{sech}^{-1}(cx)\right) \\
&= -\frac{2b^2\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3} \\
&\quad - \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3x^3} + \frac{1}{3}\left(2bc^3\right) \text{Subst}\left(\int (a+bx)^2 \cosh(x)\, dx, x, \text{sech}^{-1}(cx)\right) \\
&\quad + \frac{1}{9}\left(2b^3c^3\right) \text{Subst}\left(\int \cosh^3(x)\, dx, x, \text{sech}^{-1}(cx)\right) \\
&= -\frac{2b^2\left(a+b\text{sech}^{-1}(cx)\right)}{9x^3} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x^3} \\
&\quad + \frac{2bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\text{sech}^{-1}(cx)\right)^2}{3x} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{3x^3} \\
&\quad - \frac{1}{3}\left(4b^2c^3\right) \text{Subst}\left(\int (a+bx) \sinh(x)\, dx, x, \text{sech}^{-1}(cx)\right) \\
&\quad + \frac{1}{9}\left(2ib^3c^3\right) \text{Subst}\left(\int \left(1-x^2\right)\, dx, x, -\frac{i\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{cx}\right)
\end{aligned}$$

$$= \frac{2b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{9x} + \frac{2b^3\left(\frac{1-cx}{1+cx}\right)^{3/2}(1+cx)^3}{27x^3} - \frac{2b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{9x^3} - \frac{4b^2c^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x^3} + \frac{2bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{3x^3} + \frac{1}{3}\left(4b^3c^3\right)\operatorname{Subst}\left(\int\cosh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)$$

$$= \frac{14b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{9x} + \frac{2b^3\left(\frac{1-cx}{1+cx}\right)^{3/2}(1+cx)^3}{27x^3} - \frac{2b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{9x^3} - \frac{4b^2c^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x^3} + \frac{2bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{3x} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{3x^3}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^4}\,dx$$

$$= \frac{-9a^3 - 6ab^2(1+6c^2x^2) + 9a^2b\sqrt{\frac{1-cx}{1+cx}}(1+cx+2c^2x^2+2c^3x^3) + 2b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx+20c^2x^2+20c^3x^3) -}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x^4,x]
```

```
[Out] (-9*a^3 - 6*a*b^2*(1 + 6*c^2*x^2) + 9*a^2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
c*x + 2*c^2*x^2 + 2*c^3*x^3) + 2*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2
0*c^2*x^2 + 20*c^3*x^3) - 3*b*(9*a^2 + 2*b^2*(1 + 6*c^2*x^2) - 6*a*b*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3))*ArcSech[c*x] + 9*b^2
*(-3*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3))*Arc
Sech[c*x]^2 - 9*b^3*ArcSech[c*x]^3)/(27*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(191) = 382.

Time = 0.83 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.82

method	result
derivativedivides	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\text{arcsech}(cx)^3}{3c^3x^3}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\ \text{arcsech}(cx)^2}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\ \text{arcsech}(cx)^2}{3c^2x^2}-\frac{4\ \text{arc}}{}\right.\right.$
default	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\text{arcsech}(cx)^3}{3c^3x^3}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\ \text{arcsech}(cx)^2}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\ \text{arcsech}(cx)^2}{3c^2x^2}-\frac{4\ \text{arc}}{}\right.\right.$
parts	$-\frac{a^3}{3x^3}+b^3c^3\left(-\frac{\text{arcsech}(cx)^3}{3c^3x^3}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\ \text{arcsech}(cx)^2}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\ \text{arcsech}(cx)^2}{3c^2x^2}-\frac{4\ \text{arcsech}}{3cx}\right.$

```
[In] int((a+b*arcsech(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arcsech(c*x)^3+2/3*(-(c*x-1)/c/x)^(
1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)^2+1/3/c^2/x^2*(-(c*x-1)/c/x)^(1/2)*((
c*x+1)/c/x)^(1/2)*arcsech(c*x)^2-4/3/c/x*arcsech(c*x)+40/27*(-(c*x-1)/c/x)^
(1/2)*((c*x+1)/c/x)^(1/2)-2/9/c^3/x^3*arcsech(c*x)+2/27*(-(c*x-1)/c/x)^(1/2
)*((c*x+1)/c/x)^(1/2)/c^2/x^2)+3*a*b^2*(-1/3/c^3/x^3*arcsech(c*x)^2+4/9*arc
sech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+2/9/c^2/x^2*(-(c*x-1)/c/
x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)-4/9/c/x-2/27/c^3/x^3)+3*b*a^2*(-1
/3/c^3/x^3*arcsech(c*x)+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^2*((c*x+1)/c/x)^(1/2
)*(2*c^2*x^2+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^4}\,dx =$$

$$-\frac{36\,ab^2c^2x^2+9\,b^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3+9\,a^3+6\,ab^2+9\left(3\,ab^2-\left(2\,b^3c^3x^3+b^3cx\right)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{}}{}\right.}{}$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(36*a*b^2*c^2*x^2 + 9*b^3*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1
)/(c*x))^3 + 9*a^3 + 6*a*b^2 + 9*(3*a*b^2 - (2*b^3*c^3*x^3 + b^3*c*x)*sqrt(
-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*
x))^2 + 3*(12*b^3*c^2*x^2 + 9*a^2*b + 2*b^3 - 6*(2*a*b^2*c^3*x^3 + a*b^2*c*
x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
```

```
+ 1)/(c*x)) - (2*(9*a^2*b + 20*b^3)*c^3*x^3 + (9*a^2*b + 2*b^3)*c*x)*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)))/x^3
```

Sympy [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^4}\,dx=\int \frac{\left(a+b\operatorname{asech}\left(cx\right)\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*asech(c*x))**3/x**4,x)
[Out] Integral((a + b*asech(c*x))**3/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^4}\,dx=\int \frac{\left(b\operatorname{arsech}\left(cx\right)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="maxima")
[Out] 1/3*a^2*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c -
3*arcsech(c*x)/x^3) - 1/3*a^3/x^3 + integrate(b^3*log(sqrt(1/(c*x) + 1)*sqr
t(1/(c*x) - 1) + 1/(c*x))^3/x^4 + 3*a*b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x
) - 1) + 1/(c*x))^2/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^4}\,dx=\int \frac{\left(b\operatorname{arsech}\left(cx\right)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^4}\,dx=\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))^3/x^4,x)
[Out] int((a + b*acosh(1/(c*x)))^3/x^4, x)
```

3.50 $\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 242

$$\int \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{x^5}\,dx = \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{128x^4}+\frac{45b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{256x^2}+\frac{45}{256}b^3c^4\mathrm{sech}^{-1}(cx) - \frac{3b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{32x^4}-\frac{9b^2c^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{32x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{16x^4} + \frac{9bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{32x^2} + \frac{3}{32}c^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3-\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}{4x^4}$$

```
[Out] 45/256*b^3*c^4*arcsech(c*x)-3/32*b^2*(a+b*arcsech(c*x))/x^4-9/32*b^2*c^2*(a
+b*arcsech(c*x))/x^2+3/32*c^4*(a+b*arcsech(c*x))^3-1/4*(a+b*arcsech(c*x))^3
/x^4+3/128*b^3*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/x^4+45/256*b^3*c^2*(c*x+1)*
((-c*x+1)/(c*x+1))^(1/2)/x^2+3/16*b*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1)/
(c*x+1))^(1/2)/x^4+9/32*b*c^2*(c*x+1)*(a+b*arcsech(c*x))^2*((-c*x+1)/(c*x+1
))^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6420, 5555, 3392, 32, 2715, 8}

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^5}\,dx = -\frac{9b^2c^2\left(a+b\text{sech}^{-1}(cx)\right)}{32x^2}-\frac{3b^2\left(a+b\text{sech}^{-1}(cx)\right)}{32x^4}+\frac{3}{32}c^4\left(a+b\text{sech}^{-1}(cx)\right)^3+\frac{9bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{32x^2}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{4x^4}+\frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\left(a+b\text{sech}^{-1}(cx)\right)^2}{16x^4}+\frac{45}{256}b^3c^4\text{sech}^{-1}(cx)+\frac{45b^3c^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{256x^2}+\frac{3b^3\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{128x^4}$$

```
[In] Int[(a + b*ArcSech[c*x])^3/x^5,x]
```

```
[Out] (3*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(128*x^4) + (45*b^3*c^2*Sqrt[(1
 - c*x)/(1 + c*x)]*(1 + c*x))/(256*x^2) + (45*b^3*c^4*ArcSech[c*x])/256 - (
3*b^2*(a + b*ArcSech[c*x]))/(32*x^4) - (9*b^2*c^2*(a + b*ArcSech[c*x]))/(32
*x^2) + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(1
6*x^4) + (9*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^
2)/(32*x^2) + (3*c^4*(a + b*ArcSech[c*x])^3)/32 - (a + b*ArcSech[c*x])^3/(4
*x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c^4\text{Subst}\left(\int (a+bx)^3\cosh^3(x)\sinh(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)^2\cosh^4(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= -\frac{3b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^4}+\frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{16x^4}\\
&\quad -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{4x^4}+\frac{1}{16}\left(9bc^4\right)\text{Subst}\left(\int (a+bx)^2\cosh^2(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&\quad +\frac{1}{32}\left(3b^3c^4\right)\text{Subst}\left(\int \cosh^4(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{128x^4} - \frac{3b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^4} - \frac{9b^2c^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^2} \\
&\quad + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{16x^4} + \frac{9bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{32x^2} \\
&\quad - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{4x^4} + \frac{1}{32}\left(9bc^4\right)\text{Subst}\left(\int(a+bx)^2\,dx, x, \operatorname{sech}^{-1}(cx)\right) \\
&\quad + \frac{1}{128}\left(9b^3c^4\right)\text{Subst}\left(\int\cosh^2(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right) \\
&\quad + \frac{1}{32}\left(9b^3c^4\right)\text{Subst}\left(\int\cosh^2(x)\,dx, x, \operatorname{sech}^{-1}(cx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{128x^4} + \frac{45b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{256x^2} - \frac{3b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^4} \\
&\quad - \frac{9b^2c^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^2} + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{16x^4} \\
&\quad + \frac{9bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{32x^2} + \frac{3}{32}c^4\left(a+b\operatorname{sech}^{-1}(cx)\right)^3 \\
&\quad - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{4x^4} + \frac{1}{256}\left(9b^3c^4\right)\text{Subst}\left(\int 1\,dx, x, \operatorname{sech}^{-1}(cx)\right) \\
&\quad + \frac{1}{64}\left(9b^3c^4\right)\text{Subst}\left(\int 1\,dx, x, \operatorname{sech}^{-1}(cx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3b^3\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{128x^4} + \frac{45b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{256x^2} + \frac{45}{256}b^3c^4\operatorname{sech}^{-1}(cx) \\
&\quad - \frac{3b^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^4} - \frac{9b^2c^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{32x^2} \\
&\quad + \frac{3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{16x^4} + \frac{9bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{32x^2} \\
&\quad + \frac{3}{32}c^4\left(a+b\operatorname{sech}^{-1}(cx)\right)^3 - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{4x^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}{x^5}\,dx$$
$$=\frac{-8a(8a^2+3b^2)-72ab^2c^2x^2+3b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(8a^2(2+3c^2x^2)+b^2(2+15c^2x^2)\right)-24b\Big(8a^2+b^2(1+3}{}$$

[In] Integrate[(a + b*ArcSech[c*x])^3/x^5,x]

[Out] (-8*a*(8*a^2 + 3*b^2) - 72*a*b^2*c^2*x^2 + 3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(8*a^2*(2 + 3*c^2*x^2) + b^2*(2 + 15*c^2*x^2)) - 24*b*(8*a^2 + b^2*(1 + 3*c^2*x^2) - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + 3*c^2*x^2 + 3*c^3*x^3))*ArcSech[c*x] + 24*b^2*(b*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + 3*c^2*x^2 + 3*c^3*x^3) + a*(-8 + 3*c^4*x^4))*ArcSech[c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcSech[c*x]^3 - 9*b*(8*a^2 + 5*b^2)*c^4*x^4*Log[x] + 9*b*(8*a^2 + 5*b^2)*c^4*x^4*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(256*x^4)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. 2(216) = 432.

Time = 0.99 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.00

method	result
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{4c^4x^4}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{16c^3x^3}+\frac{9\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{32cx}+3\right.\right.$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arcsech}(cx)^3}{4c^4x^4}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{16c^3x^3}+\frac{9\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{32cx}+3\right.\right.$
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\operatorname{arcsech}(cx)^3}{4c^4x^4}+\frac{3\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{16c^3x^3}+\frac{9\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{arcsech}(cx)^2}{32cx}+\frac{3\operatorname{arcse}}{}\right.$

[In] int((a+b*arcsech(c*x))^3/x^5,x,method=_RETURNVERBOSE)

[Out] c^4*(-1/4*a^3/c^4/x^4+b^3*(-1/4/c^4/x^4*arcsech(c*x)^3+3/16/c^3/x^3*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)^2+9/32*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)/c/x*arcsech(c*x)^2+3/32*arcsech(c*x)^3-3/32/c^4/x^4*arcsech(c*x)+3/128*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)/c^3/x^3+45/256*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)/c/x+45/256*arcsech(c*x)-9/32/c^2/x^2*arcsech(c*x))+3*a*b^2*(-1/4/c^4/x^4*arcsech(c*x)^2+1/8/c^3/x^3*(-(c*x-1)/c/

```
x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)+3/16*(-(c*x-1)/c/x)^(1/2)*((c*x+1
)/c/x)^(1/2)/c/x*arcsech(c*x)+3/32*arcsech(c*x)^2-1/32/c^4/x^4-3/32/c^2/x^2
)+3*b*a^2*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/c^3/x^3*((c*
x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*(-c^2*x^2+1)^(1/
2)*c^2*x^2+2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.45

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^5}\,dx =$$

$$-\frac{72\,ab^2c^2x^2 - 8\,(3\,b^3c^4x^4 - 8\,b^3)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3 + 64\,a^3 + 24\,ab^2 - 24\left(3\,ab^2c^4x^4 - 8\,ab^2 + (3\,b^3c^3x\right.}{}$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/256*(72*a*b^2*c^2*x^2 - 8*(3*b^3*c^4*x^4 - 8*b^3)*log((c*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 1)/(c*x))^3 + 64*a^3 + 24*a*b^2 - 24*(3*a*b^2*c^4*x^4 -
 8*a*b^2 + (3*b^3*c^3*x^3 + 2*b^3*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 - 3*(3*(8*a^2*b + 5*b^3)*
c^4*x^4 - 24*b^3*c^2*x^2 - 64*a^2*b - 8*b^3 + 16*(3*a*b^2*c^3*x^3 + 2*a*b^2
*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) + 1)/(c*x)) - 3*(3*(8*a^2*b + 5*b^3)*c^3*x^3 + 2*(8*a^2*b + b^3)*c*x)*sq
rt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^4
```

Sympy [F]

$$\int \frac{\left(a+b\text{sech}^{-1}(cx)\right)^3}{x^5}\,dx = \int \frac{\left(a+b\operatorname{asech}\left(cx\right)\right)^3}{x^5}\,dx$$

```
[In] integrate((a+b*asech(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x**5, x)
```

Maxima [F]

$$\int \frac{\left(a + b\mathrm{sech}^{-1}(cx)\right)^3}{x^5}\, dx = \int \frac{(b\,\mathrm{arsech}\,(cx) + a)^3}{x^5}\, dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(
1/(c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt
(1/(c^2*x^2) - 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) -
1) + 1))/c - 16*arcsech(c*x)/x^4) - 1/4*a^3/x^4 + integrate(b^3*log(sqrt(1/
(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3/x^5 + 3*a*b^2*log(sqrt(1/(c*x) +
1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x^5, x)
```

Giac [F]

$$\int \frac{\left(a + b\mathrm{sech}^{-1}(cx)\right)^3}{x^5}\, dx = \int \frac{(b\,\mathrm{arsech}\,(cx) + a)^3}{x^5}\, dx$$

```
[In] integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\mathrm{sech}^{-1}(cx)\right)^3}{x^5}\, dx = \int \frac{\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\, dx$$

```
[In] int((a + b*acosh(1/(c*x)))^3/x^5,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))^3/x^5, x)
```

3.51 $\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx=\mathrm{Int}\left(\frac{x}{a+b\,\mathrm{sech}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x/(a+b*arcsech(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx=\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$$

```
[In] Int[x/(a + b*ArcSech[c*x]),x]
```

```
[Out] Defer[Int][x/(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\text{integral}=\int \frac{x}{a+b\,\mathrm{sech}^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{x}{a+b\text{sech}^{-1}(cx)}\,dx$$

```
[In] Integrate[x/(a + b*ArcSech[c*x]),x]
[Out] Integrate[x/(a + b*ArcSech[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\ \mathrm{arcsech}\left(cx\right)}dx$$

```
[In] int(x/(a+b*arcsech(c*x)),x)
[Out] int(x/(a+b*arcsech(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{x}{b\,\mathrm{arsech}\left(cx\right)+a}\,dx$$

```
[In] integrate(x/(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral(x/(b*arcsech(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{x}{a+b\,\mathrm{asech}\left(cx\right)}\,dx$$

```
[In] integrate(x/(a+b*asech(c*x)),x)
[Out] Integral(x/(a + b*asech(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\text{sech}^{-1}(cx)}\,dx = \int \frac{x}{b\,\text{arsech}\,(cx) + a}\,dx$$

```
[In] integrate(x/(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate(x/(b*arcsech(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\text{sech}^{-1}(cx)}\,dx = \int \frac{x}{b\,\text{arsech}\,(cx) + a}\,dx$$

```
[In] integrate(x/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(x/(b*arcsech(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 4.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{a + b\text{sech}^{-1}(cx)}\,dx = \int \frac{x}{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int(x/(a + b*acosh(1/(c*x))),x)
[Out] int(x/(a + b*acosh(1/(c*x))), x)
```

3.52 $\int \frac{1}{a+b\operatorname{sech}^{-1}(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{a+b\operatorname{sech}^{-1}(cx)}\,dx=\operatorname{Int}\left(\frac{1}{a+b\operatorname{sech}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsech(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\operatorname{sech}^{-1}(cx)}\,dx=\int \frac{1}{a+b\operatorname{sech}^{-1}(cx)}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])^(-1),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])^(-1), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{a+b\operatorname{sech}^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{1}{a+b\text{sech}^{-1}(cx)}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])^(-1),x]
[Out] Integrate[(a + b*ArcSech[c*x])^(-1), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\ \text{arcsech}\,(cx)}dx$$

```
[In] int(1/(a+b*arcsech(c*x)),x)
[Out] int(1/(a+b*arcsech(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{1}{b\,\text{arsech}\,(cx)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arcsech(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\text{asech}\,(cx)}\,dx$$

```
[In] integrate(1/(a+b*asech(c*x)),x)
[Out] Integral(1/(a + b*asech(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a + b\text{sech}^{-1}(cx)}\, dx = \int \frac{1}{b\,\text{arsech}\,(cx) + a}\, dx$$

```
[In] integrate(1/(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsech(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a + b\text{sech}^{-1}(cx)}\, dx = \int \frac{1}{b\,\text{arsech}\,(cx) + a}\, dx$$

```
[In] integrate(1/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arcsech(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{a + b\text{sech}^{-1}(cx)}\, dx = \int \frac{1}{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}\, dx$$

```
[In] int(1/(a + b*acosh(1/(c*x))),x)
[Out] int(1/(a + b*acosh(1/(c*x))), x)
```

3.53 $\int \frac{1}{x\left(a+b\mathbf{sech}^{-1}(cx)\right)} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)} dx = \text{Int}\left(\frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsech(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)} dx = \int \frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)} dx$$

```
[In] Int[1/(x*(a + b*ArcSech[c*x])),x]
[Out] Defer[Int][1/(x*(a + b*ArcSech[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\operatorname{sech}^{-1}(cx)\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcSech[c*x])),x]
[Out] Integrate[1/(x*(a + b*ArcSech[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \text{arcsech}\left(cx\right)\right)}dx$$

```
[In] int(1/x/(a+b*arcsech(c*x)),x)
[Out] int(1/x/(a+b*arcsech(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arsech}\left(cx\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x*arcsech(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\,\text{asech}\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*asech(c*x)),x)
[Out] Integral(1/(x*(a + b*asech(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arcsech(c*x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arsech}\,(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

```
[In] int(1/(x*(a + b*acosh(1/(c*x)))),x)
[Out] int(1/(x*(a + b*acosh(1/(c*x)))), x)
```

3.54 $\int \frac{1}{x^2\left(a+b\mathbf{sech}^{-1}(cx)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \frac{c\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}$$

```
[Out] -c*cosh(a/b)*Shi(a/b+arcsech(c*x))/b+c*Chi(a/b+arcsech(c*x))*sinh(a/b)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6420, 3384, 3379, 3382}

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \frac{c\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}$$

```
[In] Int[1/(x^2*(a + b*ArcSech[c*x])),x]
```

```
[Out] (c*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/b - (c*Cosh[a/b]*SinhIntegra
l[a/b + ArcSech[c*x]])/b
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\left(\left(c\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&\quad+\left(c\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= \frac{c\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b} - \frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \frac{c\left(\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)}{b}$$

```
[In] Integrate[1/(x^2*(a + b*ArcSech[c*x])),x]
[Out] (c*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
 + ArcSech[c*x]]))/b
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$c\left(-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\frac{a}{b}+\mathrm{arcsech}(cx)\right)}{2b}+\frac{e^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\,\mathrm{arcsech}(cx)-\frac{a}{b}\right)}{2b}\right)$	54
default	$c\left(-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\frac{a}{b}+\mathrm{arcsech}(cx)\right)}{2b}+\frac{e^{-\frac{a}{b}}\,\mathrm{Ei}_1\left(-\,\mathrm{arcsech}(cx)-\frac{a}{b}\right)}{2b}\right)$	54

[In] int(1/x^2/(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)

[Out] c*(-1/2/b*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/2/b*exp(-a/b)*Ei(1,-arcsech(c*x
)-a/b))

Fricas [F]

$$\int \frac{1}{x^2\left(a+b\mathrm{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arsech}\left(cx\right)+a)x^2}\,dx$$

[In] integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral(1/(b*x^2*arcsech(c*x) + a*x^2), x)

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\mathrm{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(a+b\,\mathrm{asech}\left(cx\right)\right)}\,dx$$

[In] integrate(1/x**2/(a+b*asech(c*x)),x)

[Out] Integral(1/(x**2*(a + b*asech(c*x))), x)

Maxima [F]

$$\int \frac{1}{x^2\left(a+b\mathrm{sech}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arsech}\left(cx\right)+a)x^2}\,dx$$

[In] integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] integrate(1/((b*arcsech(c*x) + a)*x^2), x)

Giac [F]

$$\int \frac{1}{x^2 \left(a + b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \, \text{arsech}\,(cx) + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a + b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{x^2 \left(a + b \, \text{acosh}\left(\frac{1}{c\,x}\right)\right)} \, dx$$

```
[In] int(1/(x^2*(a + b*acosh(1/(c*x)))),x)
[Out] int(1/(x^2*(a + b*acosh(1/(c*x)))), x)
```

3.55 $\int \frac{1}{x^3\left(a+b\,\text{sech}^{-1}(cx)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{1}{x^3\left(a+b\,\text{sech}^{-1}(cx)\right)}\,dx=\frac{c^2\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{2b}$$

```
[Out] -1/2*c^2*cosh(2*a/b)*Shi(2*a/b+2*arcsech(c*x))/b+1/2*c^2*Chi(2*a/b+2*arcsec
h(c*x))*sinh(2*a/b)/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6420, 5556, 12, 3384, 3379, 3382}

$$\int \frac{1}{x^3\left(a+b\,\text{sech}^{-1}(cx)\right)}\,dx=\frac{c^2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{2b}$$

```
[In] Int[1/(x^3*(a + b*ArcSech[c*x])),x]
```

```
[Out] (c^2*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/(2*b) - (c^2*Cos
h[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSech[c*x]])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\left(c^2\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{2}c^2\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\right)
\end{aligned}$$

$$= -\left(\frac{1}{2}\left(c^2\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\text{sech}^{-1}(cx)\right)\right)$$
$$+\frac{1}{2}\left(c^2\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\text{sech}^{-1}(cx)\right)$$
$$=\frac{c^2\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{2b}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int\frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)}\,dx$$
$$=\frac{c^2\left(\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\right)}{2b}$$

```
[In] Integrate[1/(x^3*(a + b*ArcSech[c*x])),x]
[Out] (c^2*(CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*
SinhIntegral[(2*a)/b + 2*ArcSech[c*x]]))/(2*b)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$c^2\left(-\frac{e^{\frac{2a}{b}}\text{Ei}_1\left(\frac{2a}{b}+2\,\text{arcsech}(cx)\right)}{4b}+\frac{e^{-\frac{2a}{b}}\text{Ei}_1\left(-2\,\text{arcsech}(cx)-\frac{2a}{b}\right)}{4b}\right)$	60
default	$c^2\left(-\frac{e^{\frac{2a}{b}}\text{Ei}_1\left(\frac{2a}{b}+2\,\text{arcsech}(cx)\right)}{4b}+\frac{e^{-\frac{2a}{b}}\text{Ei}_1\left(-2\,\text{arcsech}(cx)-\frac{2a}{b}\right)}{4b}\right)$	60

```
[In] int(1/x^3/(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
[Out] c^2*(-1/4/b*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))+1/4/b*exp(-2*a/b)*Ei(1,-2
*arcsech(c*x)-2*a/b))
```

Fricas [F]

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \operatorname{arsech}(cx) + a)x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*arcsech(c*x) + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{asech}(cx)\right)} \, dx$$

```
[In] integrate(1/x**3/(a+b*asech(c*x)),x)
[Out] Integral(1/(x**3*(a + b*asech(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \operatorname{arsech}(cx) + a)x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arcsech(c*x) + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \operatorname{arsech}(cx) + a)x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)} \, dx = \int \frac{1}{x^3 \, \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)} \, dx$$

```
[In] int(1/(x^3*(a + b*acosh(1/(c*x)))),x)
[Out] int(1/(x^3*(a + b*acosh(1/(c*x)))), x)
```

3.56 $\int \frac{1}{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}\,dx = \frac{c^3\,\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b} + \frac{c^3\,\mathrm{Chi}\left(\frac{3a}{b}+3\,\mathrm{sech}^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4b} - \frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{4b} - \frac{c^3\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3a}{b}+3\,\mathrm{sech}^{-1}(cx)\right)}{4b}$$

```
[Out] -1/4*c^3*cosh(a/b)*Shi(a/b+arcsech(c*x))/b-1/4*c^3*cosh(3*a/b)*Shi(3*a/b+3*
arcsech(c*x))/b+1/4*c^3*Chi(a/b+arcsech(c*x))*sinh(a/b)/b+1/4*c^3*Chi(3*a/b
+3*arcsech(c*x))*sinh(3*a/b)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used

= {6420, 5556, 3384, 3379, 3382}

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = \frac{c^3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b} + \frac{c^3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b} - \frac{c^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b} - \frac{c^3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b}$$

```
[In] Int[1/(x^4*(a + b*ArcSech[c*x])),x]
```

```
[Out] (c^3*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/(4*b) + (c^3*CoshIntegral[
(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(4*b) - (c^3*Cosh[a/b]*SinhIntegra
l[a/b + ArcSech[c*x]])/(4*b) - (c^3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcSech[c*x]])/(4*b)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^3\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\left(c^3\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{4}c^3\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\right)-\frac{1}{4}c^3\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= -\left(\frac{1}{4}\left(c^3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&\quad -\frac{1}{4}\left(c^3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\\
&\quad +\frac{1}{4}\left(c^3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\\
&\quad +\frac{1}{4}\left(c^3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= \frac{c^3\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b}+\frac{c^3\text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4b}\\
&\quad -\frac{c^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b}-\frac{c^3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)}\,dx = -\frac{c^3\left(-\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right.}{4b}$$

```
[In] Integrate[1/(x^4*(a + b*ArcSech[c*x])),x]
```

```
[Out] -1/4*(c^3*(-(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b]) - CoshIntegral[3*(
a/b + ArcSech[c*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSech[c
*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])]))/b
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

method	result
derivativedivides	$c^{3}\left(-\frac{e^{\frac{3a}{b}} \operatorname{Ei}_{1}\left(\frac{3a}{b}+3 \operatorname{arcsech}(cx)\right)}{8b}-\frac{e^{\frac{a}{b}} \operatorname{Ei}_{1}\left(\frac{a}{b}+\operatorname{arcsech}(cx)\right)}{8b}+\frac{e^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsech}(cx)-\frac{a}{b}\right)}{8b}+\frac{e^{-\frac{3a}{b}} \operatorname{Ei}_{1}\left(-3 \operatorname{arc}\right.}{8b}\right.$
default	$c^{3}\left(-\frac{e^{\frac{3a}{b}} \operatorname{Ei}_{1}\left(\frac{3a}{b}+3 \operatorname{arcsech}(cx)\right)}{8b}-\frac{e^{\frac{a}{b}} \operatorname{Ei}_{1}\left(\frac{a}{b}+\operatorname{arcsech}(cx)\right)}{8b}+\frac{e^{-\frac{a}{b}} \operatorname{Ei}_{1}\left(-\operatorname{arcsech}(cx)-\frac{a}{b}\right)}{8b}+\frac{e^{-\frac{3a}{b}} \operatorname{Ei}_{1}\left(-3 \operatorname{arc}\right.}{8b}\right.$

```
[In] int(1/x^4/(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/8/b*exp(3*a/b)*Ei(1,3*a/b+3*arcsech(c*x))-1/8/b*exp(a/b)*Ei(1,a/b+a
rcsech(c*x))+1/8/b*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b)+1/8/b*exp(-3*a/b)*Ei(1
,-3*arcsech(c*x)-3*a/b))
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a+b\operatorname{sech}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\operatorname{arsech}(cx)+a)x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^4*arcsech(c*x) + a*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a+b\operatorname{sech}^{-1}(cx)\right)}\,dx = \int \frac{1}{x^4 \left(a+b\operatorname{asech}(cx)\right)}\,dx$$

```
[In] integrate(1/x**4/(a+b*asech(c*x)),x)
```

```
[Out] Integral(1/(x**4*(a + b*asech(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a + b\text{sech}^{-1}(cx)\right)}\, dx = \int \frac{1}{(b \operatorname{arsech}(cx) + a)x^4}\, dx$$

```
[In] integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arcsech(c*x) + a)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(a + b\text{sech}^{-1}(cx)\right)}\, dx = \int \frac{1}{(b \operatorname{arsech}(cx) + a)x^4}\, dx$$

```
[In] integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a + b\text{sech}^{-1}(cx)\right)}\, dx = \int \frac{1}{x^4 \left(a + b \operatorname{acosh}\left(\frac{1}{cx}\right)\right)}\, dx$$

```
[In] int(1/(x^4*(a + b*acosh(1/(c*x)))),x)
[Out] int(1/(x^4*(a + b*acosh(1/(c*x)))), x)
```

3.57 $\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}\,dx=\operatorname{Int}\left(\frac{x}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(a+b*arcsech(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{x}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}\,dx$$

```
[In] Int[x/(a + b*ArcSech[c*x])^2,x]
```

```
[Out] Defer[Int][x/(a + b*ArcSech[c*x])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{x}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{(a+b\text{sech}^{-1}(cx))^2}\,dx=\int \frac{x}{(a+b\text{sech}^{-1}(cx))^2}\,dx$$

```
[In] Integrate[x/(a + b*ArcSech[c*x])^2,x]
[Out] Integrate[x/(a + b*ArcSech[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\ \text{arcsech}\,(cx))^2}dx$$

```
[In] int(x/(a+b*arcsech(c*x))^2,x)
[Out] int(x/(a+b*arcsech(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{x}{(a+b\text{sech}^{-1}(cx))^2}\,dx=\int \frac{x}{(b\,\text{arsech}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral(x/(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\text{sech}^{-1}(cx))^2}\,dx=\int \frac{x}{(a+b\,\text{asech}\,(cx))^2}\,dx$$

```
[In] integrate(x/(a+b*asech(c*x))**2,x)
[Out] Integral(x/(a + b*asech(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 546, normalized size of antiderivative = 45.50

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{x}{\left(b\,\text{arsech}\,(cx)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x + (c^2*x^3 - x)*x)/((b^2*c^2
*log(c) - a*b*c^2)*x^2 - b^2*log(c) - (b^2*log(c) + b^2*log(x) - a*b)*sqrt(
c*x + 1)*sqrt(-c*x + 1) + a*b - (b^2*c^2*x^2 - sqrt(c*x + 1)*sqrt(-c*x + 1)
*b^2 - b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^2*x^2 - b^2)*log
(x)) + integrate((2*(2*c^2*x^2 - 1)*(c*x + 1)*(c*x - 1)*x + (3*c^4*x^4 - 8*
c^2*x^2 + 4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x + 2*(c^4*x^4 - 2*c^2*x^2 + 1)*x
)/((b^2*c^4*log(c) - a*b*c^4)*x^4 - (b^2*log(c) + b^2*log(x) - a*b)*(c*x +
1)*(c*x - 1) - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - 2*((b^2*c^2*
log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b + (b^2*c^2*x^2 - b^2)*log(x))*sqrt
(c*x + 1)*sqrt(-c*x + 1) - a*b - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 - (c*x + 1)*(
c*x - 1)*b^2 - 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + b^2)*lo
g(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*l
og(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{x}{\left(b\,\text{arsech}\,(cx)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsech(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{x}{\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int(x/(a + b*acosh(1/(c*x)))^2,x)
[Out] int(x/(a + b*acosh(1/(c*x)))^2, x)
```

3.58 $\int \frac{1}{\left(a+b\,\text{sech}^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\text{Int}\left(\frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsech(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])^(-2),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])^(-2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 76.82 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])^(-2),x]
[Out] Integrate[(a + b*ArcSech[c*x])^(-2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\,\operatorname{arcsech}\left(cx\right)\right)^{2}}dx$$

```
[In] int(1/(a+b*arcsech(c*x))^2,x)
[Out] int(1/(a+b*arcsech(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\operatorname{arsech}\left(cx\right)+a\right)^{2}}\,dx$$

```
[In] integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\operatorname{asech}\left(cx\right)\right)^{2}}\,dx$$

```
[In] integrate(1/(a+b*asech(c*x))**2,x)
[Out] Integral((a + b*asech(c*x))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 535, normalized size of antiderivative = 53.50

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*log(c
) - a*b*c^2)*x^2 - b^2*log(c) - (b^2*log(c) + b^2*log(x) - a*b)*sqrt(c*x +
1)*sqrt(-c*x + 1) + a*b - (b^2*c^2*x^2 - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2 -
 b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^2*c^2*x^2 - b^2)*log(x)) +
 integrate((c^4*x^4 - 2*c^2*x^2 + (3*c^2*x^2 - 1)*(c*x + 1)*(c*x - 1) + (2*
c^4*x^4 - 5*c^2*x^2 + 2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/((b^2*c^4*log(c)
 - a*b*c^4)*x^4 - (b^2*log(c) + b^2*log(x) - a*b)*(c*x + 1)*(c*x - 1) - 2*(
b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - 2*((b^2*c^2*log(c) - a*b*c^2)*
x^2 - b^2*log(c) + a*b + (b^2*c^2*x^2 - b^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*
x + 1) - a*b - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 - (c*x + 1)*(c*x - 1)*b^2 - 2*(
b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + b^2)*log(sqrt(c*x + 1)*sq
rt(-c*x + 1) + 1) + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*acosh(1/(c*x)))^2,x)
[Out] int(1/(a + b*acosh(1/(c*x)))^2, x)
```

3.59 $\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx=\mathrm{Int}\left(\frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsech(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcSech[c*x])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcSech[c*x])^2),x]
[Out] Integrate[1/(x*(a + b*ArcSech[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \text{arcsech}\left(cx\right)\right)^{2}}dx$$

```
[In] int(1/x/(a+b*arcsech(c*x))^2,x)
[Out] int(1/x/(a+b*arcsech(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(b\,\text{arsech}\left(cx\right)+a)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*arcsech(c*x)^2 + 2*a*b*x*arcsech(c*x) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\text{asech}\left(cx\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*asech(c*x))**2,x)
[Out] Integral(1/(x*(a + b*asech(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 544, normalized size of antiderivative = 38.86

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2 -
 b^2)*x*log(x) - (b^2*x*log(x) + (b^2*log(c) - a*b)*x)*sqrt(c*x + 1)*sqrt(-
c*x + 1) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x + (sqrt(c*
x + 1)*sqrt(-c*x + 1)*b^2*x - (b^2*c^2*x^2 - b^2)*x)*log(sqrt(c*x + 1)*sqrt
(-c*x + 1) + 1)) + integrate(-(2*(c*x + 1)*(c*x - 1)*c^2*x^2 + (c^4*x^4 - 2
*c^2*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1))/((b^2*x*log(x) + (b^2*log(c) - a*b)
*x)*(c*x + 1)*(c*x - 1) - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x*log(x) + 2*
((b^2*c^2*x^2 - b^2)*x*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c
) + a*b)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b^2*c^4*log(c) - a*b*c^4)*x^4
- 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x - ((c*x + 1)*(c*x
- 1)*b^2*x + 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x - (b^2*c^
4*x^4 - 2*b^2*c^2*x^2 + b^2)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 3.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int(1/(x*(a + b*acosh(1/(c*x)))^2),x)
[Out] int(1/(x*(a + b*acosh(1/(c*x)))^2), x)
```

3.60 $\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 86

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{bx\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2} + \frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2}$$

```
[Out] -c*Chi(a/b+arcsech(c*x))*cosh(a/b)/b^2+c*Shi(a/b+arcsech(c*x))*sinh(a/b)/b^
2+(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/b/x/(a+b*arcsech(c*x))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6420, 3378, 3384, 3379, 3382}

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = -\frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2} + \frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2} + \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{bx\left(a+b\text{sech}^{-1}(cx)\right)}$$

```
[In] Int[1/(x^2*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(b*x*(a + b*ArcSech[c*x])) - (c*Cosh[
a/b]*CoshIntegral[a/b + ArcSech[c*x]])/b^2 + (c*Sinh[a/b]*SinhIntegral[a/b
+ ArcSech[c*x]])/b^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\sinh(x)}{(a+bx)^2}\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$

$$= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{bx\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{c\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b}$$

$$= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{bx\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{\left(c\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b} + \frac{\left(c\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b}$$

$$= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{bx\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2} + \frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$
$$= \frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x\left(a+b\text{sech}^{-1}(cx)\right)} - c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right) + c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{b^2}$$

```
[In] Integrate[1/(x^2*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x*(a + b*ArcSech[c*x])) - c*Cosh[
a/b]*CoshIntegral[a/b + ArcSech[c*x]] + c*Sinh[a/b]*SinhIntegral[a/b + ArcS
ech[c*x]])/b^2
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.91

method	result
derivativedivides	$c\left(\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-1}{2cxb(a+b\,\text{arcsech}(cx))} + \frac{e^{\frac{a}{b}}\,\text{Ei}_1\left(\frac{a}{b}+\text{arcsech}(cx)\right)}{2b^2} + \frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+1}{2bcx(a+b\,\text{arcsech}(cx))} + \frac{e^{-\frac{a}{b}}\,\text{Ei}_1\left(-\text{arcsech}(cx)-\frac{a}{b}\right)}{2b^2}\right)$
default	$c\left(\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-1}{2cxb(a+b\,\text{arcsech}(cx))} + \frac{e^{\frac{a}{b}}\,\text{Ei}_1\left(\frac{a}{b}+\text{arcsech}(cx)\right)}{2b^2} + \frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+1}{2bcx(a+b\,\text{arcsech}(cx))} + \frac{e^{-\frac{a}{b}}\,\text{Ei}_1\left(-\text{arcsech}(cx)-\frac{a}{b}\right)}{2b^2}\right)$

```
[In] int(1/x^2/(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/2*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-1)/c/x/b/(a+b*arcsech(
c*x))+1/2/b^2*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/2/b*((-(c*x-1)/c/x)^(1/2)*c
*x*((c*x+1)/c/x)^(1/2)+1)/c/x/(a+b*arcsech(c*x))+1/2/b^2*exp(-a/b)*Ei(1,-ar
csech(c*x)-a/b))
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*arcsech(c*x)^2 + 2*a*b*x^2*arcsech(c*x) + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\,\mathrm{asech}\left(cx\right)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*asech(c*x))**2,x)
[Out] Integral(1/(x**2*(a + b*asech(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2 -
 b^2)*x^2*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^2
- (b^2*x^2*log(x) + (b^2*log(c) - a*b)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) +
(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^2 - (b^2*c^2*x^2 - b^2)*x^2)*log(sqrt(c
*x + 1)*sqrt(-c*x + 1) + 1)) + integrate(-(c^4*x^4 - 2*c^2*x^2 - (c^2*x^2 +
 1)*(c*x + 1)*(c*x - 1) - (c^2*x^2 - 2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(
(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^2*log(x) - (b^2*x^2*log(x) + (b^2*log
(c) - a*b)*x^2)*(c*x + 1)*(c*x - 1) + ((b^2*c^4*log(c) - a*b*c^4)*x^4 - 2*(
b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x^2 - 2*((b^2*c^2*x^2 - b
^2)*x^2*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^2)*s
qrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1)*b^2*x^2 + 2*(b^2*c^2*x^2
 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2 - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b
^2)*x^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int(1/(x^2*(a + b*acosh(1/(c*x)))^2),x)
[Out] int(1/(x^2*(a + b*acosh(1/(c*x)))^2), x)
```

3.61 $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^2}+\frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{c^2\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^2}$$

```
[Out] -c^2*Chi(2*a/b+2*arcsech(c*x))*cosh(2*a/b)/b^2+c^2*Shi(2*a/b+2*arcsech(c*x)
)*sinh(2*a/b)/b^2+1/2*c^2*sinh(2*arcsech(c*x))/b/(a+b*arcsech(c*x))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6420, 5556, 12, 3378, 3384, 3379, 3382}

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^2}+\frac{c^2\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^2}+\frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)}$$

```
[In] Int[1/(x^3*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] -((c^2*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]])/b^2) + (c^2*Si
nh[2*ArcSech[c*x]])/(2*b*(a + b*ArcSech[c*x])) + (c^2*Sinh[(2*a)/b]*SinhInt
egral[(2*a)/b + 2*ArcSech[c*x]])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c^2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{(a+bx)^2}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\left(c^2\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)^2}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{2}c^2\text{Subst}\left(\int \frac{\sinh(2x)}{(a+bx)^2}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{c^2\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b}\\
&= \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{\left(c^2\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b}\\
&\quad + \frac{\left(c^2\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b}\\
&= -\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^2} + \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{2b\left(a+b\text{sech}^{-1}(cx)\right)}\\
&\quad + \frac{c^2\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$
\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx
= \frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x^2\left(a+b\text{sech}^{-1}(cx)\right)} - c^2\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right) + c^2\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)}{b^2}
$$

```
[In] Integrate[1/(x^3*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^2*(a + b*ArcSech[c*x])) - c^2*C
osh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSech[c*x])] + c^2*Sinh[(2*a)/b]*SinhI
ntegral[2*(a/b + ArcSech[c*x])])/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(83) = 166.

Time = 0.76 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.19

method	result
derivativedivides	$c^2\left(\frac{2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2-2}{4c^2x^2b(a+b\,\mathrm{arcsech}(cx))}+\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(\frac{2a}{b}+2\,\mathrm{arcsech}(cx)\right)}{2b^2}-\frac{c^2x^2-2-2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{4b\,c^2x^2(a+b\,\mathrm{arcsech}(cx))}+\frac{\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_1(\right.$
default	$c^2\left(\frac{2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2-2}{4c^2x^2b(a+b\,\mathrm{arcsech}(cx))}+\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(\frac{2a}{b}+2\,\mathrm{arcsech}(cx)\right)}{2b^2}-\frac{c^2x^2-2-2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{4b\,c^2x^2(a+b\,\mathrm{arcsech}(cx))}+\frac{\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}_1(\right.$

```
[In] int(1/x^3/(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
[Out] c^2*(1/4*(2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+c^2*x^2-2)/c^2/x^2
/b/(a+b*arcsech(c*x))+1/2/b^2*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))-1/4/b*(
c^2*x^2-2-2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2))/c^2/x^2/(a+b*arcs
ech(c*x))+1/2/b^2*exp(-2*a/b)*Ei(1,-2*arcsech(c*x)-2*a/b))
```

Fricas [F]

$$\int \frac{1}{x^3\left(a+b\mathrm{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\,(cx)+a\right)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^3*arcsech(c*x)^2 + 2*a*b*x^3*arcsech(c*x) + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+b\mathrm{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\,\mathrm{asech}\,(cx)\right)^2}\,dx$$

```
[In] integrate(1/x**3/(a+b*asech(c*x))**2,x)
[Out] Integral(1/(x**3*(a + b*asech(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)^2}\, dx = \int \frac{1}{\left(b\,\text{arsech}\,(cx) + a\right)^2 x^3}\, dx$$

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2 -
 b^2)*x^3*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^3
- (b^2*x^3*log(x) + (b^2*log(c) - a*b)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1) +
(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^3 - (b^2*c^2*x^2 - b^2)*x^3)*log(sqrt(c
*x + 1)*sqrt(-c*x + 1) + 1)) + integrate(-(2*c^4*x^4 - 4*c^2*x^2 - 2*(c*x +
 1)*(c*x - 1) + (c^4*x^4 - 4*c^2*x^2 + 4)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 2)
/((b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^3*log(x) + ((b^2*c^4*log(c) - a*b*c
^4)*x^4 - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x^3 - (b^2*x
^3*log(x) + (b^2*log(c) - a*b)*x^3)*(c*x + 1)*(c*x - 1) - 2*((b^2*c^2*x^2 -
 b^2)*x^3*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^3)
*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1)*b^2*x^3 + 2*(b^2*c^2*x
^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3 - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 +
 b^2)*x^3)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)^2}\, dx = \int \frac{1}{\left(b\,\text{arsech}\,(cx) + a\right)^2 x^3}\, dx$$

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)^2}\, dx = \int \frac{1}{x^3 \left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\, dx$$

```
[In] int(1/(x^3*(a + b*acosh(1/(c*x)))^2),x)
```

```
[Out] int(1/(x^3*(a + b*acosh(1/(c*x)))^2), x)
```

3.62 $\int \frac{1}{x^4\left(a+b\,\mathbf{sech}^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 190

$$\begin{aligned}\int \frac{1}{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}\,dx =& \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4bx\left(a+b\operatorname{sech}^{-1}(cx)\right)}-\frac{c^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{4b^2}\\&-\frac{3c^3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{sech}^{-1}(cx)\right)}{4b^2}\\&+\frac{c^3\sinh\left(3\operatorname{sech}^{-1}(cx)\right)}{4b\left(a+b\operatorname{sech}^{-1}(cx)\right)}+\frac{c^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{4b^2}\\&+\frac{3c^3\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3a}{b}+3\operatorname{sech}^{-1}(cx)\right)}{4b^2}\end{aligned}$$

```
[Out] -1/4*c^3*Chi(a/b+arcsech(c*x))*cosh(a/b)/b^2-3/4*c^3*Chi(3*a/b+3*arcsech(c*
x))*cosh(3*a/b)/b^2+1/4*c^3*Shi(a/b+arcsech(c*x))*sinh(a/b)/b^2+3/4*c^3*Shi
(3*a/b+3*arcsech(c*x))*sinh(3*a/b)/b^2+1/4*c^3*sinh(3*arcsech(c*x))/b/(a+b*
arcsech(c*x))+1/4*c^2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/b/x/(a+b*arcsech(c*x
))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{6420, 5556, 3378, 3384, 3379, 3382\}$

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx = -\frac{c^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b^2} - \frac{3c^3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b^2} + \frac{c^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{4b^2} + \frac{3c^3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{4b^2} + \frac{c^3\sinh\left(3\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{4bx\left(a+b\text{sech}^{-1}(cx)\right)}$$

```
[In] Int[1/(x^4*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] (c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(4*b*x*(a + b*ArcSech[c*x])) - (c
^3*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]])/(4*b^2) - (3*c^3*Cosh[(3*a)/
b]*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b^2) + (c^3*Sinh[3*ArcSech[c*
x]])/(4*b*(a + b*ArcSech[c*x])) + (c^3*Sinh[a/b]*SinhIntegral[a/b + ArcSech
[c*x]])/(4*b^2) + (3*c^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSech[c*x
]])/(4*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^3\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{(a+bx)^2}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)\\
&= -\left(c^3\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)^2}+\frac{\sinh(3x)}{4(a+bx)^2}\right)\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{4}c^3\text{Subst}\left(\int \frac{\sinh(x)}{(a+bx)^2}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)-\frac{1}{4}c^3\text{Subst}\left(\int \frac{\sinh(3x)}{(a+bx)^2}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\\
&= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4bx\left(a+b\operatorname{sech}^{-1}(cx)\right)}+\frac{c^3\sinh\left(3\operatorname{sech}^{-1}(cx)\right)}{4b\left(a+b\operatorname{sech}^{-1}(cx)\right)}\\
&\quad-\frac{c^3\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4b}-\frac{(3c^3)\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4b}\\
&= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4bx\left(a+b\operatorname{sech}^{-1}(cx)\right)}+\frac{c^3\sinh\left(3\operatorname{sech}^{-1}(cx)\right)}{4b\left(a+b\operatorname{sech}^{-1}(cx)\right)}\\
&\quad-\frac{\left(c^3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4b}\\
&\quad-\frac{\left(3c^3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4b}\\
&\quad+\frac{\left(c^3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4b}\\
&\quad+\frac{\left(3c^3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4b}
\end{aligned}$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{4bx\left(a+b\operatorname{sech}^{-1}(cx)\right)} - \frac{c^3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{4b^2} - \frac{3c^3\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\operatorname{sech}^{-1}(cx)\right)}{4b^2} + \frac{c^3\sinh\left(3\operatorname{sech}^{-1}(cx)\right)}{4b\left(a+b\operatorname{sech}^{-1}(cx)\right)} + \frac{c^3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right)}{4b^2} + \frac{3c^3\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3a}{b}+3\operatorname{sech}^{-1}(cx)\right)}{4b^2}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}\,dx = \frac{4b\sqrt{\frac{1-cx}{1+cx}} + 4bcx\sqrt{\frac{1-cx}{1+cx}} - c^3x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\operatorname{sech}^{-1}(cx)\right) - 3c^3x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\operatorname{c}}{}$$

```
[In] Integrate[1/(x^4*(a + b*ArcSech[c*x])^2),x]
```

```
[Out] (4*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*b*c*x*Sqrt[(1 - c*x)/(1 + c*x)] - c^3*x^
3*(a + b*ArcSech[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]] - 3*c^3*x
^3*(a + b*ArcSech[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSech[c*x])]
+ a*c^3*x^3*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] + b*c^3*x^3*ArcSech[
c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] + 3*a*c^3*x^3*Sinh[(3*a)/b]
*SinhIntegral[3*(a/b + ArcSech[c*x])] + 3*b*c^3*x^3*ArcSech[c*x]*Sinh[(3*a)
/b]*SinhIntegral[3*(a/b + ArcSech[c*x])])/(4*b^2*x^3*(a + b*ArcSech[c*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(176) = 352.

Time = 0.94 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.21

method	result
derivativedivides	$c^3\left(-\frac{\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}c^3x^3-4\sqrt{-\frac{cx-1}{cx}}cx\sqrt{\frac{cx+1}{cx}}-3c^2x^2+4}{8c^3x^3b(a+b\operatorname{arcsech}(cx))} + \frac{3e^{\frac{3a}{b}}\operatorname{Ei}_1\left(\frac{3a}{b}+3\operatorname{arcsech}(cx)\right)}{8b^2} + \frac{\sqrt{-\frac{cx-1}{cx}}cx\sqrt{\frac{cx-}{c}}}{8cxb(a+b\operatorname{arcsech}}\right.$
default	$c^3\left(-\frac{\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}c^3x^3-4\sqrt{-\frac{cx-1}{cx}}cx\sqrt{\frac{cx+1}{cx}}-3c^2x^2+4}{8c^3x^3b(a+b\operatorname{arcsech}(cx))} + \frac{3e^{\frac{3a}{b}}\operatorname{Ei}_1\left(\frac{3a}{b}+3\operatorname{arcsech}(cx)\right)}{8b^2} + \frac{\sqrt{-\frac{cx-1}{cx}}cx\sqrt{\frac{cx-}{c}}}{8cxb(a+b\operatorname{arcsech}}\right.$

```
[In] int(1/x^4/(a+b*arcsech(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/8*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x
)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-3*c^2*x^2+4)/c^3/x^3/b/(a+b*arcsech(c*x))+3
```

```
/8/b^2*exp(3*a/b)*Ei(1,3*a/b+3*arcsech(c*x))+1/8*((-(c*x-1)/c/x)^(1/2)*c*x*
((c*x+1)/c/x)^(1/2)-1)/c/x/b/(a+b*arcsech(c*x))+1/8/b^2*exp(a/b)*Ei(1,a/b+a
rcsech(c*x))+1/8/b*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+1)/c/x/(a+
b*arcsech(c*x))+1/8/b^2*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b)-1/8/b*(((c*x+1)/c
/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/
c/x)^(1/2)+3*c^2*x^2-4)/c^3/x^3/(a+b*arcsech(c*x))+3/8/b^2*exp(-3*a/b)*Ei(1
,-3*arcsech(c*x)-3*a/b))
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^4*arcsech(c*x)^2 + 2*a*b*x^4*arcsech(c*x) + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^4\left(a+b\,\text{asech}\,(cx)\right)^2}\,dx$$

```
[In] integrate(1/x**4/(a+b*asech(c*x))**2,x)
```

```
[Out] Integral(1/(x**4*(a + b*asech(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^2x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2 -
 b^2)*x^4*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^4
- (b^2*x^4*log(x) + (b^2*log(c) - a*b)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) +
(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^4 - (b^2*c^2*x^2 - b^2)*x^4)*log(sqrt(c
*x + 1)*sqrt(-c*x + 1) + 1)) - integrate((3*c^4*x^4 - 6*c^2*x^2 + (c^2*x^2
- 3)*(c*x + 1)*(c*x - 1) + (2*c^4*x^4 - 7*c^2*x^2 + 6)*sqrt(c*x + 1)*sqrt(-
c*x + 1) + 3)/((b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^4*log(x) + ((b^2*c^4*l
og(c) - a*b*c^4)*x^4 - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)
```

```
*x^4 - (b^2*x^4*log(x) + (b^2*log(c) - a*b)*x^4)*(c*x + 1)*(c*x - 1) - 2*((
b^2*c^2*x^2 - b^2)*x^4*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c
) + a*b)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1)*b^2*x^4 +
 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4 - (b^2*c^4*x^4 - 2*
b^2*c^2*x^2 + b^2)*x^4)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(a + b\text{sech}^{-1}(cx)\right)^2} \, dx = \int \frac{1}{\left(b \operatorname{arsech}\left(cx\right) + a\right)^2 x^4} \, dx$$

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a + b\text{sech}^{-1}(cx)\right)^2} \, dx = \int \frac{1}{x^4 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)^2} \, dx$$

```
[In] int(1/(x^4*(a + b*acosh(1/(c*x)))^2),x)
[Out] int(1/(x^4*(a + b*acosh(1/(c*x)))^2), x)
```

3.63 $\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\mathrm{Int}\left(\frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3},x\right)$$

```
[Out] Unintegrable(x/(a+b*arcsech(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

```
[In] Int[x/(a + b*ArcSech[c*x])^3,x]
```

```
[Out] Defer[Int][x/(a + b*ArcSech[c*x])^3, x]
```

Rubi steps

$$\text{integral}=\int \frac{x}{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$$

```
[In] Integrate[x/(a + b*ArcSech[c*x])^3,x]
[Out] Integrate[x/(a + b*ArcSech[c*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b\,\operatorname{arcsech}\left(cx\right)\right)^{3}}dx$$

```
[In] int(x/(a+b*arcsech(c*x))^3,x)
[Out] int(x/(a+b*arcsech(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(b\,\operatorname{arsech}\left(cx\right)+a\right)^{3}}\,dx$$

```
[In] integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
[Out] integral(x/(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c
*x) + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(a+b\,\text{asech}\,(cx)\right)^3}\,dx$$

```
[In] integrate(x/(a+b*asech(c*x))**3,x)
[Out] Integral(x/(a + b*asech(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 2818, normalized size of antiderivative = 234.83

$$\int \frac{x}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{x}{\left(b\,\text{arsech}\,(cx)+a\right)^3}\,dx$$

```
[In] integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
[Out] -1/2*((2*(2*b*c^4*x^5 - 3*b*c^2*x^3 + b*x)*x*log(x) + (4*(b*c^4*log(c) - a*
c^4)*x^5 - (b*c^2*(6*log(c) + 1) - 6*a*c^2)*x^3 + (b*(2*log(c) + 1) - 2*a)*
x)*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + (3*(b*c^6*x^7 - 5*b*c^4*x^5 + 6*b*
c^2*x^3 - 2*b*x)*x*log(x) + (3*(b*c^6*log(c) - a*c^6)*x^7 - (b*c^4*(15*log(
c) + 2) - 15*a*c^4)*x^5 + (b*c^2*(18*log(c) + 5) - 18*a*c^2)*x^3 - 3*(b*(2*
log(c) + 1) - 2*a)*x)*x)*(c*x + 1)*(c*x - 1) - 2*(b*c^6*x^7 - 3*b*c^4*x^5 +
 3*b*c^2*x^3 - b*x)*x*log(x) - ((5*b*c^6*x^7 - 17*b*c^4*x^5 + 18*b*c^2*x^3
- 6*b*x)*x*log(x) + ((b*c^6*(5*log(c) + 1) - 5*a*c^6)*x^7 - (b*c^4*(17*log(
c) + 5) - 17*a*c^4)*x^5 + (b*c^2*(18*log(c) + 7) - 18*a*c^2)*x^3 - 3*(b*(2*
log(c) + 1) - 2*a)*x)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b*c^6*(2*log(c) +
 1) - 2*a*c^6)*x^7 - 3*(b*c^4*(2*log(c) + 1) - 2*a*c^4)*x^5 + 3*(b*c^2*(2*l
og(c) + 1) - 2*a*c^2)*x^3 - (b*(2*log(c) + 1) - 2*a)*x)*x - (2*(2*b*c^4*x^5
 - 3*b*c^2*x^3 + b*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*x + 3*(b*c^6*x^7 - 5
*b*c^4*x^5 + 6*b*c^2*x^3 - 2*b*x)*(c*x + 1)*(c*x - 1)*x - (5*b*c^6*x^7 - 17
*b*c^4*x^5 + 18*b*c^2*x^3 - 6*b*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x - 2*(b*c^
6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1
) + 1))/((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) + a^2*b^2*c^6)*x^6 - b^4*lo
g(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4 + 2*a*
b^3*log(c) - (b^4*log(c)^2 + b^4*log(x)^2 - 2*a*b^3*log(c) + a^2*b^2 + 2*(b
^4*log(c) - a*b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - a^2*b^2 + 3*(
b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*l
og(c) + a^2*b^2*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*log(x)^2 + 2*(b^4*log(c) - a
```

```
*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*log(x))*(c*x + 1)*(c*x - 1) + 3*(b
^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2 + (b^4*c^6*x^6 - 3*
b^4*c^4*x^4 + 3*b^4*c^2*x^2 - (c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*b^4 - b^4 -
3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1) - 3*(b^4*c^4*x^4 - 2*b^4*c^2*x^2
+ b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^
2 + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*log(x)^2 - 3*(b^4*l
og(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4 - 2*a*b
^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^
2)*x^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*log(x)^2 + 2*((b^4*c^4*log(c)
- a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)
*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6
 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) - (b^4*log(c) + b^4*log(
x) - a*b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + a*b^3 + 3*(b^4*log(c) - a*b^
3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*log(x))*(c*x + 1
)*(c*x - 1) + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2 - 3*((b^4*c^4*log(c) - a*b
^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x^2 + (b^
4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) + (b^
4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*log(x))*log(sqrt(c*x + 1)*
sqrt(-c*x + 1) + 1) + 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(
c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x
^2)*log(x)) + integrate(-1/2*(4*(6*c^4*x^4 - 6*c^2*x^2 + 1)*(c*x + 1)^2*(c*
x - 1)^2*x - (33*c^6*x^6 - 108*c^4*x^4 + 88*c^2*x^2 - 16)*(c*x + 1)^(3/2)*(
-c*x + 1)^(3/2)*x - 12*(c^8*x^8 - 7*c^6*x^6 + 14*c^4*x^4 - 10*c^2*x^2 + 2)*
(c*x + 1)*(c*x - 1)*x + (15*c^8*x^8 - 67*c^6*x^6 + 108*c^4*x^4 - 72*c^2*x^2
 + 16)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x + 4*(c^8*x^8 - 4*c^6*x^6 + 6*c^4*x^4
- 4*c^2*x^2 + 1)*x)/((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) -
 a*b^2*c^6)*x^6 + (b^3*log(c) + b^3*log(x) - a*b^2)*(c*x + 1)^2*(c*x - 1)^2
 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + 4*(b^3*log(c) - a*b^2 - (b^3*c^2*lo
g(c) - a*b^2*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x +
 1)^(3/2) + b^3*log(c) - 6*((b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) -
 a*b^2 - 2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^4*x^4 - 2*b^3*c^2*x^2
+ b^3)*log(x))*(c*x + 1)*(c*x - 1) - a*b^2 - 4*(b^3*c^2*log(c) - a*b^2*c^2)
*x^2 - 4*((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^3*c^4*log(c) - a*b^2*c^4)
*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^6*x
^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x +
 1) - (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 + (c*x + 1)^2*(c*x - 1)^
2*b^3 - 4*b^3*c^2*x^2 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3
/2) - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1) + b^3 - 4*(
b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x
+ 1))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^3*c^8*x^8 - 4*b^3*c^6*x^6
+ 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{(a+b\text{sech}^{-1}(cx))^3} \, dx = \int \frac{x}{(b\,\text{arsech}\,(cx)+a)^3} \, dx$$

```
[In] integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate(x/(b*arcsech(c*x) + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{(a+b\text{sech}^{-1}(cx))^3} \, dx = \int \frac{x}{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3} \, dx$$

```
[In] int(x/(a + b*acosh(1/(c*x)))^3,x)
[Out] int(x/(a + b*acosh(1/(c*x)))^3, x)
```

3.64 $\int \frac{1}{\left(a+b\,\mathbf{sech}^{-1}(cx)\right)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx=\operatorname{Int}\left(\frac{1}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsech(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])^(-3),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])^(-3), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+b\operatorname{sech}^{-1}(cx)\right)^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 93.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])^(-3),x]
[Out] Integrate[(a + b*ArcSech[c*x])^(-3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsech}\left(cx\right)\right)^{3}}dx$$

```
[In] int(1/(a+b*arcsech(c*x))^3,x)
[Out] int(1/(a+b*arcsech(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 4.00

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^{3}}\,dx$$

```
[In] integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c
*x) + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(a+b\,\mathrm{asech}\,(cx)\right)^3}\,dx$$

```
[In] integrate(1/(a+b*asech(c*x))**3,x)
[Out] Integral((a + b*asech(c*x))**(-3), x)
```

Maxima [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 2771, normalized size of antiderivative = 277.10

$$\int \frac{1}{\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\,(cx)+a\right)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
[Out] 1/2*((b*c^6*(log(c) + 1) - a*c^6)*x^7 - 3*(b*c^4*(log(c) + 1) - a*c^4)*x^5
- (3*(b*c^4*log(c) - a*c^4)*x^5 - (b*c^2*(4*log(c) + 1) - 4*a*c^2)*x^3 + (b
*(log(c) + 1) - a)*x + (3*b*c^4*x^5 - 4*b*c^2*x^3 + b*x)*log(x))*(c*x + 1)^
(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(log(c) + 1) - a*c^2)*x^3 - (2*(b*c^6*log
(c) - a*c^6)*x^7 - 2*(b*c^4*(5*log(c) + 1) - 5*a*c^4)*x^5 + (b*c^2*(11*log(
c) + 5) - 11*a*c^2)*x^3 - 3*(b*(log(c) + 1) - a)*x + (2*b*c^6*x^7 - 10*b*c^
4*x^5 + 11*b*c^2*x^3 - 3*b*x)*log(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*(3*log(
c) + 1) - 3*a*c^6)*x^7 - 5*(b*c^4*(2*log(c) + 1) - 2*a*c^4)*x^5 + (b*c^2*(1
0*log(c) + 7) - 10*a*c^2)*x^3 - 3*(b*(log(c) + 1) - a)*x + (3*b*c^6*x^7 - 1
0*b*c^4*x^5 + 10*b*c^2*x^3 - 3*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) -
(b*(log(c) + 1) - a)*x - (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - (3*b*c^4*
x^5 - 4*b*c^2*x^3 + b*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - (2*b*c^6*x^7 -
10*b*c^4*x^5 + 11*b*c^2*x^3 - 3*b*x)*(c*x + 1)*(c*x - 1) + (3*b*c^6*x^7 - 1
0*b*c^4*x^5 + 10*b*c^2*x^3 - 3*b*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - b*x)*log
(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3
 - b*x)*log(x))/((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) + a^2*b^2*c^6)*x^6
- b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^
4 + 2*a*b^3*log(c) - (b^4*log(c)^2 + b^4*log(x)^2 - 2*a*b^3*log(c) + a^2*b^
2 + 2*(b^4*log(c) - a*b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - a^2*b
^2 + 3*(b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b
^3*c^2*log(c) + a^2*b^2*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*log(x)^2 + 2*(b^4*lo
g(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*log(x))*(c*x + 1)*(c*x - 1
```

```
) + 3*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2 + (b^4*c^6*
x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - (c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*b^4
- b^4 - 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1) - 3*(b^4*c^4*x^4 - 2*b^4*
c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x +
1) + 1)^2 + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*log(x)^2 -
3*(b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4
 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^
2*b^2*c^2)*x^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*log(x)^2 + 2*((b^4*c^4
*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c
^2)*x^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2*((b^4*c^6*log(c) - a*b^3*
c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) - (b^4*log(c) +
b^4*log(x) - a*b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + a*b^3 + 3*(b^4*log(c
) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*log(x))*
(c*x + 1)*(c*x - 1) + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2 - 3*((b^4*c^4*log(
c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x
^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*log(x))*sqrt(c*x + 1)*sqrt(-c*x +
1) + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*log(x))*log(sqrt(c
*x + 1)*sqrt(-c*x + 1) + 1) + 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*
c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^
3*c^2)*x^2)*log(x)) + integrate(-1/2*(c^8*x^8 - 4*c^6*x^6 + 6*c^4*x^4 + (15
*c^4*x^4 - 12*c^2*x^2 + 1)*(c*x + 1)^2*(c*x - 1)^2 - (18*c^6*x^6 - 57*c^4*x
^4 + 40*c^2*x^2 - 4)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 4*c^2*x^2 - 3*(2*c^
8*x^8 - 13*c^6*x^6 + 25*c^4*x^4 - 16*c^2*x^2 + 2)*(c*x + 1)*(c*x - 1) + (6*
c^8*x^8 - 25*c^6*x^6 + 39*c^4*x^4 - 24*c^2*x^2 + 4)*sqrt(c*x + 1)*sqrt(-c*x
 + 1) + 1)/((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2*c^
6)*x^6 + (b^3*log(c) + b^3*log(x) - a*b^2)*(c*x + 1)^2*(c*x - 1)^2 + 6*(b^3
*c^4*log(c) - a*b^2*c^4)*x^4 + 4*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*
b^2*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2)
 + b^3*log(c) - 6*((b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 -
2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*lo
g(x))*(c*x + 1)*(c*x - 1) - a*b^2 - 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 - 4*
((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 - b^
3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^6*x^6 - 3*b^
3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^
3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 + (c*x + 1)^2*(c*x - 1)^2*b^3 - 4
*b^3*c^2*x^2 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 6*(
b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1) + b^3 - 4*(b^3*c^6*x
^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log
(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c
^4*x^4 - 4*b^3*c^2*x^2 + b^3)*log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(b\,\text{arsech}\left(cx\right)+a\right)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^(-3), x)
```

Mupad [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \int \frac{1}{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

```
[In] int(1/(a + b*acosh(1/(c*x)))^3,x)
[Out] int(1/(a + b*acosh(1/(c*x)))^3, x)
```

3.65 $\int \frac{1}{x\left(a+b\mathbf{sech}^{-1}(cx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\mathrm{Int}\left(\frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsech(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

```
[In] Int[1/(x*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcSech[c*x])^3), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcSech[c*x])^3),x]
[Out] Integrate[1/(x*(a + b*ArcSech[c*x])^3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \operatorname{arcsech}\left(cx\right)\right)^{3}}dx$$

```
[In] int(1/x/(a+b*arcsech(c*x))^3,x)
[Out] int(1/x/(a+b*arcsech(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.21

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\operatorname{arsech}\left(cx\right)+a\right)^{3}x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*x*arcsech(c*x)^3 + 3*a*b^2*x*arcsech(c*x)^2 + 3*a^2*b*x*arc
sech(c*x) + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\operatorname{asech}\left(cx\right)\right)^3}\,dx$$

```
[In] integrate(1/x/(a+b*asech(c*x))**3,x)
[Out] Integral(1/(x*(a + b*asech(c*x))**3), x)
```

Maxima [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 2638, normalized size of antiderivative = 188.43

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\operatorname{arsech}\left(cx\right)+a\right)^3x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
[Out] -1/2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - (2*(b*c^4*log(c) - a*c^4)*x^5
 - (b*c^2*(2*log(c) + 1) - 2*a*c^2)*x^3 + b*x + 2*(b*c^4*x^5 - b*c^2*x^3)*l
og(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - ((b*c^6*log(c) - a*c^6)*x^7 - (b*
c^4*(5*log(c) + 2) - 5*a*c^4)*x^5 + (b*c^2*(4*log(c) + 5) - 4*a*c^2)*x^3 -
3*b*x + (b*c^6*x^7 - 5*b*c^4*x^5 + 4*b*c^2*x^3)*log(x))*(c*x + 1)*(c*x - 1)
 + ((b*c^6*(log(c) + 1) - a*c^6)*x^7 - (b*c^4*(3*log(c) + 5) - 3*a*c^4)*x^5
 + (b*c^2*(2*log(c) + 7) - 2*a*c^2)*x^3 - 3*b*x + (b*c^6*x^7 - 3*b*c^4*x^5
+ 2*b*c^2*x^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - b*x + (2*(b*c^4*x^5 -
 b*c^2*x^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + (b*c^6*x^7 - 5*b*c^4*x^5 + 4
*b*c^2*x^3)*(c*x + 1)*(c*x - 1) - (b*c^6*x^7 - 3*b*c^4*x^5 + 2*b*c^2*x^3)*s
qrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1))/((b^4*x
*log(x)^2 + 2*(b^4*log(c) - a*b^3)*x*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c
) + a^2*b^2)*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - (b^4*c^6*x^6 - 3*b^4*c^4
*x^4 + 3*b^4*c^2*x^2 - b^4)*x*log(x)^2 + 3*((b^4*c^2*x^2 - b^4)*x*log(x)^2
- 2*(b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x*log(x) - (b^4
*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(
c) + a^2*b^2*c^2)*x^2)*x)*(c*x + 1)*(c*x - 1) + ((c*x + 1)^(3/2)*(-c*x + 1)
^(3/2)*b^4*x + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x + 3*(b^4*c^4*x^4
 - 2*b^4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x - (b^4*c^6*x^6 - 3*b
^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^
2 - 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^
4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x*log(x) + 3*(
```

```
(b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x*log(x)^2 + 2*((b^4*c^4*log(c) - a*b^3
*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x*log(
x) + (b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*
x^4 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) +
 a^2*b^2*c^2)*x^2)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b^4*c^6*log(c)^2 - 2
*a*b^3*c^6*log(c) + a^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 -
 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - a^2*b^2 + 3*(b^4*
c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x - 2*((b^4*x*log(x)
+ (b^4*log(c) - a*b^3)*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*((b^4*c^2*x^
2 - b^4)*x*log(x) - (b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)
*x)*(c*x + 1)*(c*x - 1) - (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^
4)*x*log(x) + 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x*log(x) + ((b^4*c^4*l
og(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2
)*x^2)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((b^4*c^6*log(c) - a*b^3*c^6)*x^6
- 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(
c) - a*b^3*c^2)*x^2)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)) + integrate(
-1/2*(4*(2*c^4*x^4 - c^2*x^2)*(c*x + 1)^2*(c*x - 1)^2 - (7*c^6*x^6 - 22*c^4
*x^4 + 12*c^2*x^2)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 2*(c^8*x^8 - 5*c^6*x^
6 + 10*c^4*x^4 - 6*c^2*x^2)*(c*x + 1)*(c*x - 1) + (c^8*x^8 - 3*c^6*x^6 + 6*
c^4*x^4 - 4*c^2*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1))/((b^3*x*log(x) + (b^3*lo
g(c) - a*b^2)*x)*(c*x + 1)^2*(c*x - 1)^2 - 4*((b^3*c^2*x^2 - b^3)*x*log(x)
- (b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x)*(c*x + 1)^(3/2
)*(-c*x + 1)^(3/2) - 6*((b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x*log(x) + ((b^
3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3*c^2*log(c) - a*
b^2*c^2)*x^2)*x)*(c*x + 1)*(c*x - 1) + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3
*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x*log(x) - 4*((b^3*c^6*x^6 - 3*b^3*c^4*x^4
+ 3*b^3*c^2*x^2 - b^3)*x*log(x) + ((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^
3*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*
b^2*c^2)*x^2)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((b^3*c^8*log(c) - a*b^2*c^
8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2*c^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4
)*x^4 + b^3*log(c) - a*b^2 - 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x - ((c*x
+ 1)^2*(c*x - 1)^2*b^3*x - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)
^(3/2)*x - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x - 4*
(b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x
 + 1)*x + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^
3)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \int \frac{1}{(b\,\text{arsech}\,(cx)+a)^3x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \int \frac{1}{x\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

```
[In] int(1/(x*(a + b*acosh(1/(c*x)))^3),x)
[Out] int(1/(x*(a + b*acosh(1/(c*x)))^3), x)
```

3.66 $\int \frac{1}{x^2\left(a+b\,\text{sech}^{-1}(cx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 114

$$\int \frac{1}{x^2\left(a+b\,\text{sech}^{-1}(cx)\right)^3}\,dx=\frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx\left(a+b\,\text{sech}^{-1}(cx)\right)^2}+\frac{1}{2b^2x\left(a+b\,\text{sech}^{-1}(cx)\right)}+\frac{c\,\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{2b^3}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{2b^3}$$

```
[Out] 1/2/b^2/x/(a+b*arcsech(c*x))-1/2*c*cosh(a/b)*Shi(a/b+arcsech(c*x))/b^3+1/2*
c*Chi(a/b+arcsech(c*x))*sinh(a/b)/b^3+1/2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/
b/x/(a+b*arcsech(c*x))^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6420, 3378, 3384, 3379, 3382}

$$\int \frac{1}{x^2\left(a+b\,\text{sech}^{-1}(cx)\right)^3}\,dx=\frac{c\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{2b^3}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{2b^3}+\frac{1}{2b^2x\left(a+b\,\text{sech}^{-1}(cx)\right)}+\frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bx\left(a+b\,\text{sech}^{-1}(cx)\right)^2}$$

```
[In] Int[1/(x^2*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(2*b*x*(a + b*ArcSech[c*x])^2) + 1/(2
*b^2*x*(a + b*ArcSech[c*x])) + (c*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b
])/(2*b^3) - (c*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(2*b^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\left(c\text{Subst}\left(\int \frac{\sinh(x)}{(a+bx)^3}\,dx, x, \operatorname{sech}^{-1}(cx)\right)\right)$$

$$= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx\left(a+b\operatorname{sech}^{-1}(cx)\right)^2} - \frac{c\text{Subst}\left(\int \frac{\cosh(x)}{(a+bx)^2}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{2b}$$

$$= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx\left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{1}{2b^2x\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{c\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{2b^2}$$

$$= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx\left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{1}{2b^2x\left(a+b\text{sech}^{-1}(cx)\right)} - \frac{\left(c\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{2b^2} + \frac{\left(c\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{2b^2}$$

$$= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx\left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{1}{2b^2x\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{2b^3} - \frac{c\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{2b^3}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \frac{\frac{b^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x\left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{b}{ax+bx\text{sech}^{-1}(cx)} + c\left(\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right) - \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)}{2b^3}$$

```
[In] Integrate[1/(x^2*(a + b*ArcSech[c*x])^3),x]
[Out] ((b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x*(a + b*ArcSech[c*x])^2) + b/(
a*x + b*x*ArcSech[c*x]) + c*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] - C
osh[a/b]*SinhIntegral[a/b + ArcSech[c*x]]))/(2*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(104) = 208.

Time = 0.61 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.14

method	result
derivativedivides	$c\left(-\frac{\left(\sqrt{-\frac{cx-1}{cx}}\, cx\sqrt{\frac{cx+1}{cx}}-1\right)(b\,\mathrm{arcsech}(cx)+a-b)}{4cx\,b^2\left(b^2\,\mathrm{arcsech}(cx)^2+2ab\,\mathrm{arcsech}(cx)+a^2\right)}-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\frac{a}{b}+\mathrm{arcsech}(cx)\right)}{4b^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\, cx\sqrt{\frac{cx+1}{cx}}+1}{4bcx(a+b\,\mathrm{arcsech}(cx))^2}+\frac{\sqrt{}}{4b^{}}\right.$
default	$c\left(-\frac{\left(\sqrt{-\frac{cx-1}{cx}}\, cx\sqrt{\frac{cx+1}{cx}}-1\right)(b\,\mathrm{arcsech}(cx)+a-b)}{4cx\,b^2\left(b^2\,\mathrm{arcsech}(cx)^2+2ab\,\mathrm{arcsech}(cx)+a^2\right)}-\frac{e^{\frac{a}{b}}\,\mathrm{Ei}_1\left(\frac{a}{b}+\mathrm{arcsech}(cx)\right)}{4b^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\, cx\sqrt{\frac{cx+1}{cx}}+1}{4bcx(a+b\,\mathrm{arcsech}(cx))^2}+\frac{\sqrt{}}{4b^{}}\right.$

[In] int(1/x^2/(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)

[Out] c*(-1/4*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-1)*(b*arcsech(c*x)+a-b)/c/x/b^2/(b^2*arcsech(c*x)^2+2*a*b*arcsech(c*x)+a^2)-1/4/b^3*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/4/b*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+1)/c/x/(a+b*arcsech(c*x))^2+1/4/b^2*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+1)/c/x/(a+b*arcsech(c*x))+1/4/b^3*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b))

Fricas [F]

$$\int \frac{1}{x^2\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^3x^2}\,dx$$

[In] integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3*x^2*arcsech(c*x)^3 + 3*a*b^2*x^2*arcsech(c*x)^2 + 3*a^2*b*x^2*arcsech(c*x) + a^3*x^2), x)

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^2\left(a+b\,\mathrm{asech}\left(cx\right)\right)^3}\,dx$$

[In] integrate(1/x**2/(a+b*asech(c*x))**3,x)

[Out] Integral(1/(x**2*(a + b*asech(c*x))**3), x)

Maxima [F]

$$\int \frac{1}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^3x^2}\,dx$$

[In] integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="maxima")

[Out] -1/2*((b*c^6*(log(c) - 1) - a*c^6)*x^7 - 3*(b*c^4*(log(c) - 1) - a*c^4)*x^5 - (b*c^2*x^3 - (b*c^4*log(c) - a*c^4)*x^5 + (b*(log(c) - 1) - a)*x - (b*c^4*x^5 - b*x)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(log(c) - 1) - a*c^2)*x^3 - (2*b*c^4*x^5 + (b*c^2*(3*log(c) - 5) - 3*a*c^2)*x^3 - 3*(b*(log(c) - 1) - a)*x + 3*(b*c^2*x^3 - b*x)*log(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*(log(c) - 1) - a*c^6)*x^7 - (b*c^4*(4*log(c) - 5) - 4*a*c^4)*x^5 + (b*c^2*(6*log(c) - 7) - 6*a*c^2)*x^3 - 3*(b*(log(c) - 1) - a)*x + (b*c^6*x^7 - 4*b*c^4*x^5 + 6*b*c^2*x^3 - 3*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b*(log(c) - 1) - a)*x - (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 + (b*c^4*x^5 - b*x)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 3*(b*c^2*x^3 - b*x)*(c*x + 1)*(c*x - 1) + (b*c^6*x^7 - 4*b*c^4*x^5 + 6*b*c^2*x^3 - 3*b*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^2*log(x)^2 - (b^4*x^2*log(x)^2 + 2*(b^4*log(c) - a*b^3)*x^2*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2)*x^2)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 2*((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^2*log(x) - 3*((b^4*c^2*x^2 - b^4)*x^2*log(x)^2 - 2*(b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^2*log(x) - (b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^2)*(c*x + 1)*(c*x - 1) + ((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) + a^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - a^2*b^2 + 3*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^2 - ((c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*b^4*x^2 + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x^2 + 3*(b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2 - (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^2*log(x)^2 + 2*((b^4*c^4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^2*log(x) + (b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c) + a^2*b^2*c^4)*x^4 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 2*((b^4*x^2*log(x) + (b^4*log(c) - a*b^3)*x^2)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^2*log(x) + 3*((b^4*c^2*x^2 - b^4)*x^2*log(x) - (b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^2)*(c*x + 1)*(c*x - 1) - ((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c

```
^2)*x^2)*x^2 + 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^2*log(x) + ((b^4*c^
4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*
c^2)*x^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x +
1) + 1)) + integrate(-1/2*(c^8*x^8 - 4*c^6*x^6 + 6*c^4*x^4 + (3*c^4*x^4 + 1
)*(c*x + 1)^2*(c*x - 1)^2 + (3*c^4*x^4 - 4*c^2*x^2 + 4)*(c*x + 1)^(3/2)*(-c
*x + 1)^(3/2) - 4*c^2*x^2 - 3*(c^6*x^6 + c^4*x^4 - 4*c^2*x^2 + 2)*(c*x + 1)
*(c*x - 1) - (c^6*x^6 - 9*c^4*x^4 + 12*c^2*x^2 - 4)*sqrt(c*x + 1)*sqrt(-c*x
 + 1) + 1)/((b^3*x^2*log(x) + (b^3*log(c) - a*b^2)*x^2)*(c*x + 1)^2*(c*x -
1)^2 - 4*((b^3*c^2*x^2 - b^3)*x^2*log(x) - (b^3*log(c) - a*b^2 - (b^3*c^2*l
og(c) - a*b^2*c^2)*x^2)*x^2)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + (b^3*c^8*x^
8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x^2*log(x) - 6*((b
^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x^2*log(x) + ((b^3*c^4*log(c) - a*b^2*c^4
)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x^2)*(c*x
+ 1)*(c*x - 1) + ((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*
b^2*c^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 4*
(b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x^2 - 4*((b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3
*b^3*c^2*x^2 - b^3)*x^2*log(x) + ((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^3
*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*b
^2*c^2)*x^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((c*x + 1)^2*(c*x - 1)^2*b
^3*x^2 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*x^2 - 6*(b^
3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x^2 - 4*(b^3*c^6*x^6 -
 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2 + (b
^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x^2)*log(
sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{(b\,\text{arsech}\,(cx)+a)^3 x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^3} \, dx$$

```
[In] int(1/(x^2*(a + b*acosh(1/(c*x)))^3),x)
[Out] int(1/(x^2*(a + b*acosh(1/(c*x)))^3), x)
```

3.67 $\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\frac{c^2\cosh\left(2\text{sech}^{-1}(cx)\right)}{2b^2\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{c^2\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^3}+\frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^3}$$

```
[Out] 1/2*c^2*cosh(2*arcsech(c*x))/b^2/(a+b*arcsech(c*x))-c^2*cosh(2*a/b)*Shi(2*a
/b+2*arcsech(c*x))/b^3+c^2*Chi(2*a/b+2*arcsech(c*x))*sinh(2*a/b)/b^3+1/4*c^
2*sinh(2*arcsech(c*x))/b/(a+b*arcsech(c*x))^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6420, 5556, 12, 3378, 3384, 3379, 3382}

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\frac{c^2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^3}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^3}+\frac{c^2\cosh\left(2\text{sech}^{-1}(cx)\right)}{2b^2\left(a+b\text{sech}^{-1}(cx)\right)}+\frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2}$$

```
[In] Int[1/(x^3*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] (c^2*Cosh[2*ArcSech[c*x]])/(2*b^2*(a + b*ArcSech[c*x])) + (c^2*CoshIntegral
[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/b^3 + (c^2*Sinh[2*ArcSech[c*x]])/
(4*b*(a + b*ArcSech[c*x])^2) - (c^2*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*
ArcSech[c*x]])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
```

```
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\left(c^2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{(a+bx)^3}\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$
$$= -\left(c^2\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)^3}\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$
$$= -\left(\frac{1}{2}c^2\text{Subst}\left(\int \frac{\sinh(2x)}{(a+bx)^3}\,dx, x, \text{sech}^{-1}(cx)\right)\right)$$
$$= \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{c^2\text{Subst}\left(\int \frac{\cosh(2x)}{(a+bx)^2}\,dx, x, \text{sech}^{-1}(cx)\right)}{2b}$$
$$= \frac{c^2\cosh\left(2\text{sech}^{-1}(cx)\right)}{2b^2\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{c^2\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b^2}$$
$$= \frac{c^2\cosh\left(2\text{sech}^{-1}(cx)\right)}{2b^2\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{\left(c^2\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b^2} + \frac{\left(c^2\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{b^2}$$
$$= \frac{c^2\cosh\left(2\text{sech}^{-1}(cx)\right)}{2b^2\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^2\text{Chi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^3} + \frac{c^2\sinh\left(2\text{sech}^{-1}(cx)\right)}{4b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{sech}^{-1}(cx)\right)}{b^3}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx = \frac{\frac{b^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x^2\left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{b(2-c^2x^2)}{x^2\left(a+b\text{sech}^{-1}(cx)\right)} + 2c^2\left(\text{Chi}\left(2\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - \cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{sech}^{-1}\right.\right.\right.}{2b^3}$$

[In] Integrate[1/(x^3*(a + b*ArcSech[c*x])^3),x]

[Out] ((b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^2*(a + b*ArcSech[c*x])^2) + (b*(2 - c^2*x^2))/(x^2*(a + b*ArcSech[c*x])) + 2*c^2*(CoshIntegral[2*(a/b + ArcSech[c*x])]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSech[c*x])]))/(2*b^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(108) = 216$.

Time = 0.79 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.47

method	result
derivativedivides	$c^2\left(-\frac{\left(2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2-2\right)(2b\,\mathrm{arcsech}(cx)+2a-b)}{8c^2x^2b^2\left(b^2\,\mathrm{arcsech}(cx)^2+2ab\,\mathrm{arcsech}(cx)+a^2\right)}-\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(\frac{2a}{b}+2\,\mathrm{arcsech}(cx)\right)}{2b^3}-\frac{c^2x^2-2-2\sqrt{-\text{?}}}{8b\,c^2x^2(a+b\,\text{a}}\right.$
default	$c^2\left(-\frac{\left(2\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2-2\right)(2b\,\mathrm{arcsech}(cx)+2a-b)}{8c^2x^2b^2\left(b^2\,\mathrm{arcsech}(cx)^2+2ab\,\mathrm{arcsech}(cx)+a^2\right)}-\frac{\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}_1\left(\frac{2a}{b}+2\,\mathrm{arcsech}(cx)\right)}{2b^3}-\frac{c^2x^2-2-2\sqrt{-\text{?}}}{8b\,c^2x^2(a+b\,\text{a}}\right.$

[In] int(1/x^3/(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)

[Out] c^2*(-1/8*(2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+c^2*x^2-2)*(2*b*arcsech(c*x)+2*a-b)/c^2/x^2/b^2/(b^2*arcsech(c*x)^2+2*a*b*arcsech(c*x)+a^2)-1/2/b^3*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))-1/8/b*(c^2*x^2-2-2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2))/c^2/x^2/(a+b*arcsech(c*x))^2-1/4/b^2*(c^2*x^2-2-2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2))/c^2/x^2/(a+b*arcsech(c*x))+1/2/b^3*exp(-2*a/b)*Ei(1,-2*arcsech(c*x)-2*a/b))

Fricas [F]

$$\int \frac{1}{x^3\left(a+b\mathrm{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^3x^3}\,dx$$

[In] integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3*x^3*arcsech(c*x)^3 + 3*a*b^2*x^3*arcsech(c*x)^2 + 3*a^2*b*x^3*arcsech(c*x) + a^3*x^3), x)

Sympy [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^3\left(a+b\,\text{asech}\,(cx)\right)^3}\,dx$$

```
[In] integrate(1/x**3/(a+b*asech(c*x))**3,x)
[Out] Integral(1/(x**3*(a + b*asech(c*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\text{arsech}\,(cx)+a\right)^3x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
[Out] -1/2*((b*c^6*(2*log(c) - 1) - 2*a*c^6)*x^7 - 3*(b*c^4*(2*log(c) - 1) - 2*a*
c^4)*x^5 + ((b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 - (b*(2*log(c) - 1) - 2*a)
*x + 2*(b*c^2*x^3 - b*x)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^
2*(2*log(c) - 1) - 2*a*c^2)*x^3 - ((b*c^6*log(c) - a*c^6)*x^7 - (b*c^4*(5*l
og(c) - 2) - 5*a*c^4)*x^5 + 5*(b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 - 3*(b*(
2*log(c) - 1) - 2*a)*x + (b*c^6*x^7 - 5*b*c^4*x^5 + 10*b*c^2*x^3 - 6*b*x)*l
og(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*(3*log(c) - 1) - 3*a*c^6)*x^7 - (b*c^4
*(11*log(c) - 5) - 11*a*c^4)*x^5 + 7*(b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 -
 3*(b*(2*log(c) - 1) - 2*a)*x + (3*b*c^6*x^7 - 11*b*c^4*x^5 + 14*b*c^2*x^3
- 6*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b*(2*log(c) - 1) - 2*a)*x
- (2*b*c^6*x^7 - 6*b*c^4*x^5 + 6*b*c^2*x^3 + 2*(b*c^2*x^3 - b*x)*(c*x + 1)^
(3/2)*(-c*x + 1)^(3/2) - (b*c^6*x^7 - 5*b*c^4*x^5 + 10*b*c^2*x^3 - 6*b*x)*(
c*x + 1)*(c*x - 1) + (3*b*c^6*x^7 - 11*b*c^4*x^5 + 14*b*c^2*x^3 - 6*b*x)*sq
rt(c*x + 1)*sqrt(-c*x + 1) - 2*b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) +
 2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*x^6 - 3*
b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3*log(x)^2 + 2*((b^4*c^6*log(c) - a*b^
3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b
^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3*log(x) - (b^4*x^3*log(x)^2 + 2*(b^4*log
(c) - a*b^3)*x^3*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2)*x^3)*(c
*x + 1)^(3/2)*(-c*x + 1)^(3/2) + ((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c) +
a^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c)
+ a^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - a^2*b^2 + 3*(b^4*c^2*log(c)^2 - 2*a*b
^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^3 - 3*((b^4*c^2*x^2 - b^4)*x^3*log(x)^2
 - 2*(b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3*log(x) - (
b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*l
og(c) + a^2*b^2*c^2)*x^2)*x^3)*(c*x + 1)*(c*x - 1) - ((c*x + 1)^(3/2)*(-c*x
 + 1)^(3/2)*b^4*x^3 + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x^3 + 3*(b^
```

```
4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3 - (b^4*c^
6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3)*log(sqrt(c*x + 1)*sqrt(-c
*x + 1) + 1)^2 - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^3*log(x)^2 + 2*((
b^4*c^4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) -
a*b^3*c^2)*x^2)*x^3*log(x) + (b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^
4*log(c) + a^2*b^2*c^4)*x^4 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^
2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1
) - 2*((b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3*log(x) - (b^
4*x^3*log(x) + (b^4*log(c) - a*b^3)*x^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) +
 ((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b
^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3 - 3*((b^4*c^2*x
^2 - b^4)*x^3*log(x) - (b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x
^2)*x^3)*(c*x + 1)*(c*x - 1) - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^3*l
og(x) + ((b^4*c^4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2
*log(c) - a*b^3*c^2)*x^2)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x +
 1)*sqrt(-c*x + 1) + 1)) + integrate(-1/2*(4*c^8*x^8 - 16*c^6*x^6 + 24*c^4*
x^4 + 4*(c*x + 1)^2*(c*x - 1)^2 + (3*c^6*x^6 - 16*c^2*x^2 + 16)*(c*x + 1)^(
3/2)*(-c*x + 1)^(3/2) - 16*c^2*x^2 - 24*(c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)
*(c*x - 1) + (3*c^8*x^8 - 19*c^6*x^6 + 48*c^4*x^4 - 48*c^2*x^2 + 16)*sqrt(c
*x + 1)*sqrt(-c*x + 1) + 4)/((b^3*x^3*log(x) + (b^3*log(c) - a*b^2)*x^3)*(c
*x + 1)^2*(c*x - 1)^2 + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^
3*c^2*x^2 + b^3)*x^3*log(x) - 4*((b^3*c^2*x^2 - b^3)*x^3*log(x) - (b^3*log(
c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x^3)*(c*x + 1)^(3/2)*(-c*x +
 1)^(3/2) + ((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2*c
^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 4*(b^3*
c^2*log(c) - a*b^2*c^2)*x^2)*x^3 - 6*((b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x
^3*log(x) + ((b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3
*c^2*log(c) - a*b^2*c^2)*x^2)*x^3)*(c*x + 1)*(c*x - 1) - 4*((b^3*c^6*x^6 -
3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*x^3*log(x) + ((b^3*c^6*log(c) - a*b^2*
c^6)*x^6 - 3*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3
*c^2*log(c) - a*b^2*c^2)*x^2)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((c*x + 1
)^2*(c*x - 1)^2*b^3*x^3 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^
(3/2)*x^3 - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x^3 -
 4*(b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-
c*x + 1)*x^3 + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2
 + b^3)*x^3)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{\left(b \operatorname{arsech}\left(cx\right) + a\right)^3 x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a + b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{acosh}\left(\frac{1}{cx}\right)\right)^3} \, dx$$

```
[In] int(1/(x^3*(a + b*acosh(1/(c*x)))^3),x)
[Out] int(1/(x^3*(a + b*acosh(1/(c*x)))^3), x)
```

3.68 $\int \frac{1}{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 240

$$
\begin{aligned}
\int \frac{1}{x^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^3}\,dx = {} & \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8bx\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}+\frac{c^2}{8b^2x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)} \\
& +\frac{3c^3\cosh\left(3\,\mathrm{sech}^{-1}(cx)\right)}{8b^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}+\frac{c^3\mathrm{Chi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^3} \\
& +\frac{9c^3\mathrm{Chi}\left(\frac{3a}{b}+3\,\mathrm{sech}^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{8b^3} \\
& +\frac{c^3\sinh\left(3\,\mathrm{sech}^{-1}(cx)\right)}{8b\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}-\frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{sech}^{-1}(cx)\right)}{8b^3} \\
& -\frac{9c^3\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3a}{b}+3\,\mathrm{sech}^{-1}(cx)\right)}{8b^3}
\end{aligned}
$$

```
[Out] 1/8*c^2/b^2/x/(a+b*arcsech(c*x))+3/8*c^3*cosh(3*arcsech(c*x))/b^2/(a+b*arcs
ech(c*x))-1/8*c^3*cosh(a/b)*Shi(a/b+arcsech(c*x))/b^3-9/8*c^3*cosh(3*a/b)*S
hi(3*a/b+3*arcsech(c*x))/b^3+1/8*c^3*Chi(a/b+arcsech(c*x))*sinh(a/b)/b^3+9/
8*c^3*Chi(3*a/b+3*arcsech(c*x))*sinh(3*a/b)/b^3+1/8*c^3*sinh(3*arcsech(c*x)
)/b/(a+b*arcsech(c*x))^2+1/8*c^2*(c*x+1)*((-c*x+1)/(c*x+1))^(1/2)/b/x/(a+b*
arcsech(c*x))^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6420, 5556, 3378, 3384, 3379, 3382}

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \frac{c^3 \sinh \left(\frac{a}{b}\right) \text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{8b^3} + \frac{9c^3 \sinh \left(\frac{3a}{b}\right) \text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{8b^3} - \frac{c^3 \cosh \left(\frac{a}{b}\right) \text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{8b^3} - \frac{9c^3 \cosh \left(\frac{3a}{b}\right) \text{Shi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{8b^3} + \frac{3c^3 \cosh \left(3\text{sech}^{-1}(cx)\right)}{8b^2 \left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^2}{8b^2 x \left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^3 \sinh \left(3\text{sech}^{-1}(cx)\right)}{8b \left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{c^2 \sqrt{\frac{1-cx}{cx+1}}(cx+1)}{8bx \left(a+b\text{sech}^{-1}(cx)\right)^2}$$

```
[In] Int[1/(x^4*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] (c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(8*b*x*(a + b*ArcSech[c*x])^2) +
c^2/(8*b^2*x*(a + b*ArcSech[c*x])) + (3*c^3*Cosh[3*ArcSech[c*x]])/(8*b^2*(a
 + b*ArcSech[c*x])) + (c^3*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/(8*b
^3) + (9*c^3*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(8*b^3)
+ (c^3*Sinh[3*ArcSech[c*x]])/(8*b*(a + b*ArcSech[c*x])^2) - (c^3*Cosh[a/b]*
SinhIntegral[a/b + ArcSech[c*x]])/(8*b^3) - (9*c^3*Cosh[(3*a)/b]*SinhIntegr
al[(3*a)/b + 3*ArcSech[c*x]])/(8*b^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6420

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^3\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{(a+bx)^3}\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\left(c^3\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)^3}+\frac{\sinh(3x)}{4(a+bx)^3}\right)\,dx, x, \text{sech}^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{4}c^3\text{Subst}\left(\int \frac{\sinh(x)}{(a+bx)^3}\,dx, x, \text{sech}^{-1}(cx)\right)\right)-\frac{1}{4}c^3\text{Subst}\left(\int \frac{\sinh(3x)}{(a+bx)^3}\,dx, x, \text{sech}^{-1}(cx)\right)\\
&= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8bx\left(a+b\text{sech}^{-1}(cx)\right)^2}+\frac{c^3\sinh\left(3\text{sech}^{-1}(cx)\right)}{8b\left(a+b\text{sech}^{-1}(cx)\right)^2}\\
&\quad -\frac{c^3\text{Subst}\left(\int \frac{\cosh(x)}{(a+bx)^2}\,dx, x, \text{sech}^{-1}(cx)\right)}{8b}-\frac{(3c^3)\,\text{Subst}\left(\int \frac{\cosh(3x)}{(a+bx)^2}\,dx, x, \text{sech}^{-1}(cx)\right)}{8b}
\end{aligned}$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8bx\left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{c^2}{8b^2x\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{3c^3\cosh\left(3\text{sech}^{-1}(cx)\right)}{8b^2\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^3\sinh\left(3\text{sech}^{-1}(cx)\right)}{8b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{c^3\text{Subst}\left(\int\frac{\sinh(x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{8b^2} - \frac{(9c^3)\,\text{Subst}\left(\int\frac{\sinh(3x)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{8b^2}$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8bx\left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{c^2}{8b^2x\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{3c^3\cosh\left(3\text{sech}^{-1}(cx)\right)}{8b^2\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^3\sinh\left(3\text{sech}^{-1}(cx)\right)}{8b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{\left(c^3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{8b^2} - \frac{\left(9c^3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{8b^2} + \frac{\left(c^3\sinh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{8b^2} + \frac{\left(9c^3\sinh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \text{sech}^{-1}(cx)\right)}{8b^2}$$

$$= \frac{c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{8bx\left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{c^2}{8b^2x\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{3c^3\cosh\left(3\text{sech}^{-1}(cx)\right)}{8b^2\left(a+b\text{sech}^{-1}(cx)\right)} + \frac{c^3\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^3} + \frac{9c^3\text{Chi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{8b^3} + \frac{c^3\sinh\left(3\text{sech}^{-1}(cx)\right)}{8b\left(a+b\text{sech}^{-1}(cx)\right)^2} - \frac{c^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)}{8b^3} - \frac{9c^3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{sech}^{-1}(cx)\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.85

$$\int\frac{1}{x^4\left(a+b\text{sech}^{-1}(cx)\right)^3}\,dx$$
$$= \frac{\frac{4b^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{x^3\left(a+b\text{sech}^{-1}(cx)\right)^2} + \frac{4b(3-2c^2x^2)}{x^3\left(a+b\text{sech}^{-1}(cx)\right)} - 8c^3\left(\text{Chi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right) - \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{sech}^{-1}(cx)\right)\right)}{}$$

```
[In] Integrate[1/(x^4*(a + b*ArcSech[c*x])^3),x]
```

[Out] ((4*b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^3*(a + b*ArcSech[c*x])^2) + (4*b*(3 - 2*c^2*x^2))/(x^3*(a + b*ArcSech[c*x])) - 8*c^3*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]]) + 9*c^3*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSech[c*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] - Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])]))/(8*b^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(222) = 444$.

Time = 1.00 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.62

method	result
derivativedivides	$c^{3}\left(\frac{\left(\sqrt{\frac{cx+1}{cx}}\,\sqrt{-\frac{cx-1}{cx}}\,c^{3}x^{3}-4\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-3c^{2}x^{2}+4\right)\left(3b\,\operatorname{arcsech}(cx)+3a-b\right)}{16c^{3}x^{3}b^{2}\left(b^{2}\operatorname{arcsech}(cx)^{2}+2ab\,\operatorname{arcsech}(cx)+a^{2}\right)}-\frac{9\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_{1}\left(\frac{3a}{b}+3\,\operatorname{arcsech}(\right.}{16b^{3}}\right.$
default	$c^{3}\left(\frac{\left(\sqrt{\frac{cx+1}{cx}}\,\sqrt{-\frac{cx-1}{cx}}\,c^{3}x^{3}-4\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}-3c^{2}x^{2}+4\right)\left(3b\,\operatorname{arcsech}(cx)+3a-b\right)}{16c^{3}x^{3}b^{2}\left(b^{2}\operatorname{arcsech}(cx)^{2}+2ab\,\operatorname{arcsech}(cx)+a^{2}\right)}-\frac{9\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}_{1}\left(\frac{3a}{b}+3\,\operatorname{arcsech}(\right.}{16b^{3}}\right.$

[In] int(1/x^4/(a+b*arcsech(c*x))^3,x,method=_RETURNVERBOSE)

[Out] c^3*(1/16*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-3*c^2*x^2+4)*(3*b*arcsech(c*x)+3*a-b)/c^3/x^3/b^2/(b^2*arcsech(c*x)^2+2*a*b*arcsech(c*x)+a^2)-9/16/b^3*exp(3*a/b)*Ei(1,3*a/b+3*arcsech(c*x))-1/16*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-1)*(b*arcsech(c*x)+a-b)/c/x/b^2/(b^2*arcsech(c*x)^2+2*a*b*arcsech(c*x)+a^2)-1/16/b^3*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/16/b*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+1)/c/x/(a+b*arcsech(c*x))^2+1/16/b^2*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+1)/c/x/(a+b*arcsech(c*x))+1/16/b^3*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b)-1/16/b*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+3*c^2*x^2-4)/c^3/x^3/(a+b*arcsech(c*x))^2-3/16/b^2*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+3*c^2*x^2-4)/c^3/x^3/(a+b*arcsech(c*x))+9/16/b^3*exp(-3*a/b)*Ei(1,-3*arcsech(c*x)-3*a/b))

Fricas [F]

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{(b \operatorname{arsech}(cx) + a)^3 x^4} \, dx$$

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^4*arcsech(c*x)^3 + 3*a*b^2*x^4*arcsech(c*x)^2 + 3*a^2*b*x
^4*arcsech(c*x) + a^3*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^4 (a + b \operatorname{asech}(cx))^3} \, dx$$

```
[In] integrate(1/x**4/(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(1/(x**4*(a + b*asech(c*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a+b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{(b \operatorname{arsech}(cx) + a)^3 x^4} \, dx$$

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*c^6*(3*log(c) - 1) - 3*a*c^6)*x^7 - 3*(b*c^4*(3*log(c) - 1) - 3*a*
c^4)*x^5 - ((b*c^4*log(c) - a*c^4)*x^5 - (b*c^2*(4*log(c) - 1) - 4*a*c^2)*x
^3 + (b*(3*log(c) - 1) - 3*a)*x + (b*c^4*x^5 - 4*b*c^2*x^3 + 3*b*x)*log(x))
*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(3*log(c) - 1) - 3*a*c^2)*x^3
- (2*(b*c^6*log(c) - a*c^6)*x^7 - 2*(b*c^4*(5*log(c) - 1) - 5*a*c^4)*x^5 +
(b*c^2*(17*log(c) - 5) - 17*a*c^2)*x^3 - 3*(b*(3*log(c) - 1) - 3*a)*x + (2*
b*c^6*x^7 - 10*b*c^4*x^5 + 17*b*c^2*x^3 - 9*b*x)*log(x))*(c*x + 1)*(c*x - 1
) + ((b*c^6*(5*log(c) - 1) - 5*a*c^6)*x^7 - (b*c^4*(18*log(c) - 5) - 18*a*c
^4)*x^5 + (b*c^2*(22*log(c) - 7) - 22*a*c^2)*x^3 - 3*(b*(3*log(c) - 1) - 3*
a)*x + (5*b*c^6*x^7 - 18*b*c^4*x^5 + 22*b*c^2*x^3 - 9*b*x)*log(x))*sqrt(c*x
 + 1)*sqrt(-c*x + 1) - (b*(3*log(c) - 1) - 3*a)*x - (3*b*c^6*x^7 - 9*b*c^4*
x^5 + 9*b*c^2*x^3 - (b*c^4*x^5 - 4*b*c^2*x^3 + 3*b*x)*(c*x + 1)^(3/2)*(-c*x
 + 1)^(3/2) - (2*b*c^6*x^7 - 10*b*c^4*x^5 + 17*b*c^2*x^3 - 9*b*x)*(c*x + 1)
*(c*x - 1) + (5*b*c^6*x^7 - 18*b*c^4*x^5 + 22*b*c^2*x^3 - 9*b*x)*sqrt(c*x +
 1)*sqrt(-c*x + 1) - 3*b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + 3*(b*c^
```

```
6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*x^6 - 3*b^4*c^4*
x^4 + 3*b^4*c^2*x^2 - b^4)*x^4*log(x)^2 + 2*((b^4*c^6*log(c) - a*b^3*c^6)*x
^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b^4*c^2*l
og(c) - a*b^3*c^2)*x^2)*x^4*log(x) + ((b^4*c^6*log(c)^2 - 2*a*b^3*c^6*log(c
) + a^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log
(c) + a^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - a^2*b^2 + 3*(b^4*c^2*log(c)^2 - 2
*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^4 - (b^4*x^4*log(x)^2 + 2*(b^4*log(
c) - a*b^3)*x^4*log(x) + (b^4*log(c)^2 - 2*a*b^3*log(c) + a^2*b^2)*x^4)*(c*
x + 1)^(3/2)*(-c*x + 1)^(3/2) - 3*((b^4*c^2*x^2 - b^4)*x^4*log(x)^2 - 2*(b^
4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^4*log(x) - (b^4*log(
c)^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c)^2 - 2*a*b^3*c^2*log(c) +
a^2*b^2*c^2)*x^2)*x^4)*(c*x + 1)*(c*x - 1) - ((c*x + 1)^(3/2)*(-c*x + 1)^(3
/2)*b^4*x^4 + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x^4 + 3*(b^4*c^4*x^
4 - 2*b^4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4 - (b^4*c^6*x^6 -
3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^4)*log(sqrt(c*x + 1)*sqrt(-c*x + 1)
+ 1)^2 - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^4*log(x)^2 + 2*((b^4*c^4*
log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) - a*b^3*c^
2)*x^2)*x^4*log(x) + (b^4*log(c)^2 + (b^4*c^4*log(c)^2 - 2*a*b^3*c^4*log(c)
 + a^2*b^2*c^4)*x^4 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c)^2 - 2*a*
b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2*((
b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^4*log(x) + ((b^4*c^6*l
og(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) +
a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^4 - (b^4*x^4*log(x) + (b^4*lo
g(c) - a*b^3)*x^4)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 3*((b^4*c^2*x^2 - b^4
)*x^4*log(x) - (b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^4)
*(c*x + 1)*(c*x - 1) - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^4*log(x) +
((b^4*c^4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c)
- a*b^3*c^2)*x^2)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt
(-c*x + 1) + 1)) - integrate(1/2*(9*c^8*x^8 - 36*c^6*x^6 + 54*c^4*x^4 - (c^
4*x^4 + 4*c^2*x^2 - 9)*(c*x + 1)^2*(c*x - 1)^2 + (2*c^6*x^6 + 13*c^4*x^4 -
48*c^2*x^2 + 36)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 36*c^2*x^2 - (2*c^8*x^8
 - 19*c^6*x^6 + 83*c^4*x^4 - 120*c^2*x^2 + 54)*(c*x + 1)*(c*x - 1) + (10*c^
8*x^8 - 57*c^6*x^6 + 123*c^4*x^4 - 112*c^2*x^2 + 36)*sqrt(c*x + 1)*sqrt(-c*
x + 1) + 9)/((b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 +
 b^3)*x^4*log(x) + (b^3*x^4*log(x) + (b^3*log(c) - a*b^2)*x^4)*(c*x + 1)^2*
(c*x - 1)^2 + ((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2
*c^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 4*(b^
3*c^2*log(c) - a*b^2*c^2)*x^2)*x^4 - 4*((b^3*c^2*x^2 - b^3)*x^4*log(x) - (b
^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x^4)*(c*x + 1)^(3/2)*
(-c*x + 1)^(3/2) - 6*((b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x^4*log(x) + ((b^
3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3*c^2*log(c) - a*
b^2*c^2)*x^2)*x^4)*(c*x + 1)*(c*x - 1) - 4*((b^3*c^6*x^6 - 3*b^3*c^4*x^4 +
3*b^3*c^2*x^2 - b^3)*x^4*log(x) + ((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^
3*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*
b^2*c^2)*x^2)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((c*x + 1)^2*(c*x - 1)^2*
```

```
b^3*x^4 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2)*x^4 - 6*(b
^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x^4 - 4*(b^3*c^6*x^6
- 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4 + (
b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x^4)*log
(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(a + b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{\left(b \operatorname{arsech}\left(cx\right) + a\right)^3 x^4} \, dx$$

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate(1/((b*arcsech(c*x) + a)^3*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a + b\text{sech}^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^4 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)^3} \, dx$$

```
[In] int(1/(x^4*(a + b*acosh(1/(c*x)))^3),x)
[Out] int(1/(x^4*(a + b*acosh(1/(c*x)))^3), x)
```

3.69 $\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\text{Int}\left((dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arcsech(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx=\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$$

```
[In] Int[(d*x)^m*(a + b*ArcSech[c*x])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcSech[c*x])^3, x]
```

Rubi steps

$$\text{integral}=\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m (a+b\,\text{arcsech}\,(cx))^3 \, dx$$

```
[In] int((d*x)^m*(a+b*arcsech(c*x))^3,x)
[Out] int((d*x)^m*(a+b*arcsech(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arsech}\,(cx)+a)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="fricas")
[Out] integral((b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x
) + a^3)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 5.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (dx)^m (a+b\,\text{asech}\,(cx))^3 \, dx$$

```
[In] integrate((d*x)**m*(a+b*asech(c*x))**3,x)
[Out] Integral((d*x)**m*(a + b*asech(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 10.45 (sec) , antiderivative size = 1450, normalized size of antiderivative = 90.62

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arsech}(cx) + a)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*d^m*x*x^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^3/(m + 1) + (d*x)^(m +
1)*a^3/(d*(m + 1)) - integrate(((b^3*c^2*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))
*x^m*log(x)^3 - 3*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (b^3*c^2*d^
m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*x^m*log(x)^2 + 3*((b^3*c^2*d
^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(x) + ((b^3*c^2*d^m*(m + 1)*x^2 -
b^3*d^m*(m + 1))*x^m*log(x) - (b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) +
 (a*b^2*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^3*c^2)*x^2)*x^m)*sqr
t(c*x + 1)*sqrt(-c*x + 1) - (b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (
b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*x^m)*log(sqrt(c*x
+ 1)*sqrt(-c*x + 1) + 1)^2 - 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m +
 1)*log(c) + a^2*b*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^
2*d^m*(m + 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*x^m*log(x) + ((b^3*c^2*d
^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(x)^3 - 3*(b^3*d^m*(m + 1)*log(c)
- a*b^2*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*
x^2)*x^m*log(x)^2 - 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c
) + a^2*b*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m
+ 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*x^m*log(x) - (b^3*d^m*(m + 1)*log
(c)^3 - 3*a*b^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*d^m*(m + 1)*log(c) - (b^3*c^
2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*c^2*d^m
*(m + 1)*log(c))*x^2)*x^m)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^3*d^m*(m + 1)*
log(c)^3 - 3*a*b^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*d^m*(m + 1)*log(c) - (b^3
*c^2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*c^2*
d^m*(m + 1)*log(c))*x^2)*x^m - 3*((b^3*c^2*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1
))*x^m*log(x)^2 - 2*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (b^3*c^2*
d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*x^m*log(x) + ((b^3*c^2*d^m
*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(x)^2 - 2*(b^3*d^m*(m + 1)*log(c) -
a*b^2*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^
2)*x^m*log(x) - (b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c) + a^
2*b*d^m*(m + 1) - (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m + 1)*l
og(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*x^m)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^
3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c) + a^2*b*d^m*(m + 1) - (
b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m + 1)*log(c) + a^2*b*c^2*d
^m*(m + 1))*x^2)*x^m)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1))/(c^2*(m + 1)*x
^2 + (c^2*(m + 1)*x^2 - m - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1) - m - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arsech}(cx)+a)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^3 \, dx = \int (d\,x)^m \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int((d*x)^m*(a + b*acosh(1/(c*x)))^3,x)
[Out] int((d*x)^m*(a + b*acosh(1/(c*x)))^3, x)
```

3.70 $\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx=\text{Int}\left((dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arcsech(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx=\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx$$

```
[In] Int[(d*x)^m*(a + b*ArcSech[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcSech[c*x])^2, x]
```

Rubi steps

$$\text{integral}=\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx$$

Mathematica [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b \operatorname{arcsech}\left(cx\right)\right)^2 dx$$

```
[In] int((d*x)^m*(a+b*arcsech(c*x))^2,x)
[Out] int((d*x)^m*(a+b*arcsech(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arsech}\left(cx\right) + a)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral((b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a + b \operatorname{asech}\left(cx\right)\right)^2 \, dx$$

```
[In] integrate((d*x)**m*(a+b*asech(c*x))**2,x)
[Out] Integral((d*x)**m*(a + b*asech(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 704, normalized size of antiderivative = 44.00

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (b \,\text{arsech}\,(cx) + a)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*d^m*x*x^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2/(m + 1) + (d*x)^(m +
1)*a^2/(d*(m + 1)) - integrate(-((b^2*c^2*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1)
)*x^m*log(x)^2 - 2*(b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m + 1) - (b^2*c^2*d^m
*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2)*x^m*log(x) + ((b^2*c^2*d^m*(m +
 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(x)^2 - 2*(b^2*d^m*(m + 1)*log(c) - a*b*d
^m*(m + 1) - (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2)*x^m*lo
g(x) - (b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m + 1)*log(c) - (b^2*c^2*d^m*
(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))*x^2)*x^m)*sqrt(c*x + 1)*sq
rt(-c*x + 1) - (b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m + 1)*log(c) - (b^2*
c^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))*x^2)*x^m - 2*((b^2
*c^2*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(x) + ((b^2*c^2*d^m*(m + 1)*
x^2 - b^2*d^m*(m + 1))*x^m*log(x) - (b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m +
1) + (a*b*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^2*c^2)*x^2)*x^m)*s
qrt(c*x + 1)*sqrt(-c*x + 1) - (b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m + 1) - (
b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2)*x^m)*log(sqrt(c*x +
1)*sqrt(-c*x + 1) + 1))/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 - m - 1)*sqrt(c
*x + 1)*sqrt(-c*x + 1) - m - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (b \,\text{arsech}\,(cx) + a)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 4.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right)^2 \, dx = \int (d\,x)^m \left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2 \, dx$$

```
[In] int((d*x)^m*(a + b*acosh(1/(c*x)))^2,x)
[Out] int((d*x)^m*(a + b*acosh(1/(c*x)))^2, x)
```

3.71 $\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 87

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{(dx)^{1+m} \left(a+b\text{sech}^{-1}(cx)\right)}{d(1+m)} + \frac{b(dx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{d(1+m)^2}$$

```
[Out] (d*x)^(1+m)*(a+b*arcsech(c*x))/d/(1+m)+b*(d*x)^(1+m)*hypergeom([1/2, 1/2+1/
2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/(1+m)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6418, 126, 371}

$$\int (dx)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{(dx)^{m+1} \left(a+b\text{sech}^{-1}(cx)\right)}{d(m+1)} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(dx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{d(m+1)^2}$$

```
[In] Int[(d*x)^m*(a + b*ArcSech[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcSech[c*x]))/(d*(1 + m)) + (b*(d*x)^(1 + m)*Sqrt[(1
 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^
2*x^2])/(d*(1 + m)^2)
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6418

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcSech[c*x])/(d*(m + 1))), x] + Dist[b*(Sqrt[1 +
c*x]/(m + 1))*Sqrt[1/(1 + c*x)], Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
 x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(dx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{d(1+m)}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(dx)^m}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{1+m}\\
&= \frac{(dx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{d(1+m)}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(dx)^m}{\sqrt{1-c^2x^2}}\,dx}{1+m}\\
&= \frac{(dx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{d(1+m)}\\
&\quad+\frac{b(dx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{d(1+m)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)\, dx = \frac{x(dx)^m \left((1+m)(-1+cx)\left(a+b\operatorname{sech}^{-1}(cx)\right)-b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x\right.}{(1+m)^2(-1+cx)}$$

```
[In] Integrate[(d*x)^m*(a + b*ArcSech[c*x]),x]
[Out] (x*(d*x)^m*((1 + m)*(-1 + c*x)*(a + b*ArcSech[c*x]) - b*Sqrt[(1 - c*x)/(1 +
 c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x
^2]))/((1 + m)^2*(-1 + c*x))
```

Maple [F]

$$\int (dx)^m \left(a+b\,\operatorname{arcsech}\left(cx\right)\right) dx$$

```
[In] int((d*x)^m*(a+b*arcsech(c*x)),x)
[Out] int((d*x)^m*(a+b*arcsech(c*x)),x)
```

Fricas [F]

$$\int (dx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)\, dx = \int \left(b\operatorname{arsech}\left(cx\right)+a\right)(dx)^m\, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)\, dx = \int (dx)^m \left(a+b\operatorname{asech}\left(cx\right)\right)\, dx$$

```
[In] integrate((d*x)**m*(a+b*asech(c*x)),x)
[Out] Integral((d*x)**m*(a + b*asech(c*x)), x)
```

Maxima [F]

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int (b \operatorname{arsech}(cx) + a)(dx)^m \, dx$$

[In] integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] (c^2*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 - m - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1) - m - 1), x) + (d^m*x*x^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - d^m*x*x^m*log(x))/(m + 1) - integrate((c^2*d^m*(m + 1)*x^2*log(c) - d^m*(m + 1)*log(c) + d^m)*x^m/(c^2*(m + 1)*x^2 - m - 1), x))*b + (d*x)^(m + 1)*a/(d*(m + 1))

Giac [F]

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int (b \operatorname{arsech}(cx) + a)(dx)^m \, dx$$

[In] integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*(d*x)^m, x)

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b\text{sech}^{-1}(cx)\right) \, dx = \int (d\,x)^m \left(a + b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

[In] int((d*x)^m*(a + b*acosh(1/(c*x))),x)

[Out] int((d*x)^m*(a + b*acosh(1/(c*x))), x)

3.72 $\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx=\text{Int}\left(\frac{(dx)^m}{a+b\text{sech}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arcsech(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcSech[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\text{integral}=\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcSech[c*x]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcSech[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\ \text{arcsech}\left(cx\right)}dx$$

```
[In] int((d*x)^m/(a+b*arcsech(c*x)),x)
[Out] int((d*x)^m/(a+b*arcsech(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\text{arsech}\left(cx\right)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arcsech(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\text{sech}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\,\text{asech}\left(cx\right)}\,dx$$

```
[In] integrate((d*x)**m/(a+b*asech(c*x)),x)
[Out] Integral((d*x)**m/(a + b*asech(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\text{sech}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\text{arsech}\,(cx) + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arcsech(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\text{sech}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\text{arsech}\,(cx) + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arcsech(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 3.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{a + b\text{sech}^{-1}(cx)}\,dx = \int \frac{(d\,x)^m}{a + b\,\text{acosh}\,\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int((d*x)^m/(a + b*acosh(1/(c*x))),x)
[Out] int((d*x)^m/(a + b*acosh(1/(c*x))), x)
```

3.73 $\int \frac{(dx)^m}{\left(a+b\mathbf{sech}^{-1}(cx)\right)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2} dx = \text{Int}\left(\frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}, x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arcsech(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2} dx = \int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2} dx$$

```
[In] Int[(d*x)^m/(a + b*ArcSech[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcSech[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcSech[c*x])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcSech[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{\left(a+b\ \mathrm{arcsech}\left(cx\right)\right)^2}dx$$

```
[In] int((d*x)^m/(a+b*arcsech(c*x))^2,x)
[Out] int((d*x)^m/(a+b*arcsech(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\mathrm{arsech}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\operatorname{asech}{(cx)}\right)^2}\,dx$$

```
[In] integrate((d*x)**m/(a+b*asech(c*x))**2,x)
[Out] Integral((d*x)**m/(a + b*asech(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 616, normalized size of antiderivative = 38.50

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{(b\operatorname{arsech}{(cx)}+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="maxima")
[Out] -((c^2*d^m*x^3 - d^m*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m + (c^2*d^m*x^3 - d
^m*x)*x^m)/((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) - (b^2*log(c) + b^2
*log(x) - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1) + a*b - (b^2*c^2*x^2 - sqrt(c*x
 + 1)*sqrt(-c*x + 1)*b^2 - b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^
2*c^2*x^2 - b^2)*log(x)) + integrate(((c^2*d^m*(m + 3)*x^2 - d^m*(m + 1))*(
c*x + 1)*(c*x - 1)*x^m + (c^4*d^m*(m + 2)*x^4 - c^2*d^m*(3*m + 5)*x^2 + 2*d
^m*(m + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m + (c^4*d^m*(m + 1)*x^4 - 2*c^2
*d^m*(m + 1)*x^2 + d^m*(m + 1))*x^m)/((b^2*c^4*log(c) - a*b*c^4)*x^4 - (b^2
*log(c) + b^2*log(x) - a*b)*(c*x + 1)*(c*x - 1) - 2*(b^2*c^2*log(c) - a*b*c
^2)*x^2 + b^2*log(c) - 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b
 + (b^2*c^2*x^2 - b^2)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - a*b - (b^2*c^
4*x^4 - 2*b^2*c^2*x^2 - (c*x + 1)*(c*x - 1)*b^2 - 2*(b^2*c^2*x^2 - b^2)*sqr
t(c*x + 1)*sqrt(-c*x + 1) + b^2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b
^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{(b\,\text{arsech}\,(cx)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arcsech(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{\left(a+b\text{sech}^{-1}(cx)\right)^2}\,dx=\int \frac{(d\,x)^m}{\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int((d*x)^m/(a + b*acosh(1/(c*x)))^2,x)
[Out] int((d*x)^m/(a + b*acosh(1/(c*x)))^2, x)
```

3.74 $\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 264

$$\begin{aligned}\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = &-\frac{be(9c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} \\ &-\frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} \\ &-\frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} \\ &+\frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e} \\ &+\frac{bd(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{2c^3} \\ &-\frac{bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4e}\end{aligned}$$

```
[Out] 1/4*(e*x+d)^4*(a+b*arcsech(c*x))/e+1/2*b*d*(2*c^2*d^2+e^2)*arcsin(c*x)*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3-1/4*b*d^4*arctanh((-c^2*x^2+1)^(1/2))*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/6*b*e*(9*c^2*d^2+e^2)*(1/(c*x+1))^(1/2)*(c*x
+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/2*b*d*e^2*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1
/2)*(-c^2*x^2+1)^(1/2)/c^2-1/12*b*e^3*x^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(
-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6423, 1823, 858, 222, 272, 65, 214}

$$\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{(d+ex)^4 \left(a+b\text{sech}^{-1}(cx)\right)}{4e} + \frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)\left(2c^2d^2+e^2\right)}{2c^3} - \frac{bd^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4e} - \frac{bde^2x\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{12c^2} - \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(9c^2d^2+e^2)}{6c^4}$$

```
[In] Int[(d + e*x)^3*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/6*(b*e*(9*c^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2
*x^2])/c^4 - (b*d*e^2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/(2*c^2) - (b*e^3*x^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/(12*c^2) + ((d + e*x)^4*(a + b*ArcSech[c*x]))/(4*e) + (b*d*(2*c^2*d^2 +
e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(2*c^3) - (b*d^4*Sqrt[
(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(4*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{4e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^4}{x\sqrt{1-c^2x^2}}\,dx}{4e}$$

$$= -\frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} + \frac{(d+ex)^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{4e} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-3c^2d^4-12c^2d^3ex-2e^2(9c^2d^2+e^2)x^2-12c^2de^3x^3}{x\sqrt{1-c^2x^2}}\,dx}{12c^2e}$$

$$= -\frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} + \frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{6c^4d^4+12c^2de(2c^2d^2+e^2)x+4c^2e^2(9c^2d^2+e^2)x^2}{x\sqrt{1-c^2x^2}}\,dx}{24c^4e}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} + \frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-6c^6d^4-12c^4de(2c^2d^2+e^2)x}{x\sqrt{1-c^2x^2}}\,dx}{24c^6e}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} + \frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e} + \frac{\left(bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-c^2x^2}}\,dx}{4e} + \frac{\left(bd(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx}{2c^2}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} + \frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e} + \frac{bd(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{2c^3} + \frac{\left(bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{8e}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} + \frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e} + \frac{bd(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{2c^3} - \frac{\left(bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{4c^2e}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^2} + \frac{(d+ex)^4\left(a+b\text{sech}^{-1}(cx)\right)}{4e} + \frac{bd(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{2c^3} - \frac{bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.72

$$\int (d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{1}{4}\left(4ad^3x+6ad^2ex^2+4ade^2x^3+ae^3x^4 - \frac{be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(2e^2+c^2(18d^2+6dex+e^2x^2)\right)}{3c^4} + bx\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)\text{sech}^{-1}(cx) + \frac{2ibd(2c^2d^2+e^2)\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{c^3}\right)$$

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSech[c*x]),x]
[Out] (4*a*d^3*x + 6*a*d^2*e*x^2 + 4*a*d*e^2*x^3 + a*e^3*x^4 - (b*e*Sqrt[(1 - c*x
)/(1 + c*x)]*(1 + c*x)*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)))/(3*c^4)
+ b*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcSech[c*x] + ((2*I)*b*d
*(2*c^2*d^2 + e^2)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])
/c^3)/4
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00

method	result
parts	$\frac{a(ex+d)^4}{4e}+\frac{b\left(\frac{c\,e^3\,\mathrm{arcsech}(cx)x^4}{4}+c\,e^2\,\mathrm{arcsech}(cx)x^3d+\frac{3ce\,\mathrm{arcsech}(cx)x^2d^2}{2}+\mathrm{arcsech}(cx)xc\,d^3+\frac{c\,\mathrm{arcsech}(cx)d^4}{4e}+\frac{\sqrt{-\frac{cx}{c}}}{}\right.}{}$
derivativedivides	$\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\mathrm{arcsech}(cx)c^4d^4}{4e}+\mathrm{arcsech}(cx)c^4d^3x+\frac{3e\,\mathrm{arcsech}(cx)c^4d^2x^2}{2}+e^2\,\mathrm{arcsech}(cx)c^4d\,x^3+\frac{e^3\,\mathrm{arcsech}(cx)c^4x^4}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}}{}\right.}{}$
default	$\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\mathrm{arcsech}(cx)c^4d^4}{4e}+\mathrm{arcsech}(cx)c^4d^3x+\frac{3e\,\mathrm{arcsech}(cx)c^4d^2x^2}{2}+e^2\,\mathrm{arcsech}(cx)c^4d\,x^3+\frac{e^3\,\mathrm{arcsech}(cx)c^4x^4}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}}{}\right.}{}$

```
[In] int((e*x+d)^3*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*(e*x+d)^4/e+b/c*(1/4*c*e^3*arcsech(c*x)*x^4+c*e^2*arcsech(c*x)*x^3*d+
3/2*c*e*arcsech(c*x)*x^2*d^2+arcsech(c*x)*x*c*d^3+1/4*c/e*arcsech(c*x)*d^4+
1/12/c^2/e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-3*c^4*d^4*arctanh(1
/(-c^2*x^2+1)^(1/2))+12*c^3*d^3*e*arcsin(c*x)-e^4*(-c^2*x^2+1)^(1/2)*c^2*x^
2-6*c^2*d*e^3*x*(-c^2*x^2+1)^(1/2)-18*c^2*d^2*e^2*(-c^2*x^2+1)^(1/2)+6*c*d*
e^3*arcsin(c*x)-2*e^4*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(144) = 288$.

Time = 0.36 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.36

$$\int (d+ex)^3\left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx$$

$$=\frac{3\,ac^3e^3x^4+12\,ac^3de^2x^3+18\,ac^3d^2ex^2+12\,ac^3d^3x-12\left(2\,bc^2d^3+bde^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-3\left(4\,bc\right.}{}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*e^3*x^4 + 12*a*c^3*d*e^2*x^3 + 18*a*c^3*d^2*e*x^2 + 12*a*c^3*
d^3*x - 12*(2*b*c^2*d^3 + b*d*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) - 1)/(c*x)) - 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*e + 4*b*c^3*d*e^2 + b*c^3*e^3
)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 3*(b*c^3*e^3*x^4 + 4*b*
c^3*d*e^2*x^3 + 6*b*c^3*d^2*e*x^2 + 4*b*c^3*d^3*x - 4*b*c^3*d^3 - 6*b*c^3*d
^2*e - 4*b*c^3*d*e^2 - b*c^3*e^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
 1)/(c*x)) - (b*c^2*e^3*x^3 + 6*b*c^2*d*e^2*x^2 + 2*(9*b*c^2*d^2*e + b*e^3)
*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

Sympy [F]

$$\int (d+ex)^3 \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int (a+b\operatorname{asech}(cx))\,(d+ex)^3\,dx$$

```
[In] integrate((e*x+d)**3*(a+b*asech(c*x)),x)
[Out] Integral((a + b*asech(c*x))*(d + e*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int (d+ex)^3 \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx \\
&= \frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{2}\left(x^2\operatorname{arsech}(cx) - \frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bd^2e \\
&\quad + \frac{1}{2}\left(2\,x^3\operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bde^2 \\
&\quad + \frac{1}{12}\left(3\,x^4\operatorname{arsech}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} - 3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)be^3 \\
&\quad + ad^3x + \frac{\left(cx\operatorname{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd^3}{c}
\end{aligned}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arcsech(c*x) - x*s
qrt(1/(c^2*x^2) - 1)/c)*b*d^2*e + 1/2*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^
2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/
c)*b*d*e^2 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*e^3 + a*d^3*x + (c*x*arcsech(c*x) - arcta
n(sqrt(1/(c^2*x^2) - 1)))*b*d^3/c
```

Giac [F]

$$\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex+d)^3 (b \operatorname{arsech}(cx) + a) \, dx$$

```
[In] integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^3*(b*arcsech(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(a+b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^3 \, dx$$

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^3,x)
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^3, x)
```

3.75 $\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 201

$$\begin{aligned}\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = &-\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} \\ &-\frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} \\ &+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e} \\ &+\frac{b(6c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3} \\ &-\frac{bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{3e}\end{aligned}$$

```
[Out] 1/3*(e*x+d)^3*(a+b*arcsech(c*x))/e+1/6*b*(6*c^2*d^2+e^2)*arcsin(c*x)*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)/c^3-1/3*b*d^3*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)/e-b*d*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+
1)^(1/2)/c^2-1/6*b*e^2*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)
/c^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6423, 1823, 858, 222, 272, 65, 214}

$$\int (d+ex)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right) \, dx = \frac{(d+ex)^3 \left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)\left(6c^2d^2+e^2\right)}{6c^3} - \frac{bd^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{3e} - \frac{bde\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{6c^2}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*d*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/c^2) - (b*e^
2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(6*c^2) + ((d + e
*x)^3*(a + b*ArcSech[c*x]))/(3*e) + (b*(6*c^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3) - (b*d^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
+ c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(3*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^3}{x\sqrt{1-c^2x^2}}\,dx}{3e}\\
&= -\frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2}+\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}\\
&\quad -\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-2c^2d^3-e\left(6c^2d^2+e^2\right)x-6c^2de^2x^2}{x\sqrt{1-c^2x^2}}\,dx}{6c^2e}\\
&= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2}-\frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2}\\
&\quad +\frac{(d+ex)^3\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{2c^4d^3+c^2e\left(6c^2d^2+e^2\right)x}{x\sqrt{1-c^2x^2}}\,dx}{6c^4e}
\end{aligned}
$$

$$= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{\left(bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-c^2x^2}}\,dx}{3e} + \frac{\left(b(6c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx}{6c^2}$$

$$= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3} + \frac{\left(bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{6e}$$

$$= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3} - \frac{\left(bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{3c^2e}$$

$$= -\frac{bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + \frac{(d+ex)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3} - \frac{bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{3e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int (d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$= \frac{-bce\sqrt{\frac{1-cx}{1+cx}}(1+cx)(6d+ex) + 2ac^3x(3d^2+3dex+e^2x^2) + 2bc^3x(3d^2+3dex+e^2x^2)\operatorname{sech}^{-1}(cx) + ib(6}{6c^3}$$

[In] Integrate[(d + e*x)^2*(a + b*ArcSech[c*x]),x]

[Out] (-(b*c*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(6*d + e*x)) + 2*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcSech[c*x] + I*b*(6*c^2*d^2 + e^2)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(6*c^3)

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.99

method	result
parts	$\frac{a(ex+d)^3}{3e}+\frac{b\left(\frac{c\,e^2\operatorname{arcsech}(cx)x^3}{3}+ce\operatorname{arcsech}(cx)x^2d+\operatorname{arcsech}(cx)xc\,d^2+\frac{c\operatorname{arcsech}(cx)d^3}{3e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(-2c^3d^3\operatorname{arc}\cdots\right.}{\cdots}\right.}{c}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{b\left(\frac{\operatorname{arcsech}(cx)c^3d^3}{3e}+\operatorname{arcsech}(cx)c^3d^2x+e\operatorname{arcsech}(cx)c^3d\,x^2+\frac{e^2\operatorname{arcsech}(cx)c^3x^3}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(-2c^3d^3\operatorname{arctan}\cdots\right.}{\cdots}\right.}{c^2}}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{b\left(\frac{\operatorname{arcsech}(cx)c^3d^3}{3e}+\operatorname{arcsech}(cx)c^3d^2x+e\operatorname{arcsech}(cx)c^3d\,x^2+\frac{e^2\operatorname{arcsech}(cx)c^3x^3}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(-2c^3d^3\operatorname{arctan}\cdots\right.}{\cdots}\right.}{c^2}}{c}$

[In] int((e*x+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/3*a*(e*x+d)^3/e+b/c*(1/3*c*e^2*arcsech(c*x)*x^3+c*e*arcsech(c*x)*x^2*d+arcsech(c*x)*x*c*d^2+1/3*c/e*arcsech(c*x)*d^3+1/6/c/e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-2*c^3*d^3*arctanh(1/(-c^2*x^2+1)^(1/2))+6*c^2*d^2*e*arcsin(c*x)-6*c*d*e^2*(-c^2*x^2+1)^(1/2)-e^3*c*x*(-c^2*x^2+1)^(1/2)+e^3*arcsin(c*x))/(-c^2*x^2+1)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(107) = 214.

Time = 0.33 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.39

$$\int (d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$=\frac{2\,ac^3e^2x^3+6\,ac^3dex^2+6\,ac^3d^2x-2\left(6\,bc^2d^2+be^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-2\left(3\,bc^3d^2+3\,bc^3de+bc^3e^2\right)\cdots}{\cdots}$$

[In] integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] 1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x - 2*(6*b*c^2*d^2 + b*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 2*(3*b*c^3*d

```
^2 + 3*b*c^3*d*e + b*c^3*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/
x) + 2*(b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b
*c^3*d*e - b*c^3*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) -
 (b*c^2*e^2*x^2 + 6*b*c^2*d*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

Sympy [F]

$$\int (d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (a+b\,\text{asech}\,(cx))\,(d+ex)^2\,dx$$

```
[In] integrate((e*x+d)**2*(a+b*asech(c*x)),x)
[Out] Integral((a + b*asech(c*x))*(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int (d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx\\&=\frac{1}{3}ae^2x^3+adex^2+\left(x^2\,\text{arsech}\,(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bde\\&\quad+\frac{1}{6}\left(2\,x^3\,\text{arsech}\,(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be^2\\&\quad+ad^2x+\frac{\left(cx\,\text{arsech}\,(cx)-\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd^2}{c}\end{aligned}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*
b*d*e + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2)
- 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*e^2 + a*d^2*x + (c*x*
arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d^2/c
```

Giac [F]

$$\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (ex+d)^2 (b \operatorname{arsech}(cx) + a) \, dx$$

```
[In] integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^2*(b*arcsech(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(a+b \operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^2 \, dx$$

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^2,x)
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^2, x)
```

3.76 $\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 142

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=-\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2}+\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}+\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}-\frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2e}$$

```
[Out] 1/2*(e*x+d)^2*(a+b*arcsech(c*x))/e+b*d*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1
)^(1/2)/c-1/2*b*d^2*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(
1/2)/e-1/2*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6423, 1823, 858, 222, 272, 65, 214}

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{(d+ex)^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e}+\frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{c}-\frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2e}-\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{2c^2}$$

```
[In] Int[(d + e*x)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/2*(b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/c^2 + ((d +
 e*x)^2*(a + b*ArcSech[c*x]))/(2*e) + (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*
x]*ArcSin[c*x])/c - (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[
1 - c^2*x^2]])/(2*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^2}{x\sqrt{1-c^2x^2}}\,dx}{2e}$$

$$= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} + \frac{(d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-c^2d^2-2c^2dex}{x\sqrt{1-c^2x^2}}\,dx}{2c^2e}$$

$$= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} + \frac{(d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e} + \left(bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx + \frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-c^2x^2}}\,dx}{2e}$$

$$= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} + \frac{(d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e} + \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c} + \frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{4e}$$

$$= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} + \frac{(d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e} + \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c} - \frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{2c^2e}$$

$$= -\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2c^2} + \frac{(d+ex)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e} + \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c} - \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2e}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int (d+ex)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = adx+\frac{1}{2}aex^2+be\left(-\frac{1}{2c^2}-\frac{x}{2c}\right)\sqrt{\frac{1-cx}{1+cx}}+bdx\operatorname{sech}^{-1}(cx)+\frac{1}{2}bex^2\operatorname{sech}^{-1}(cx)+\frac{2bd\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcSech[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + b*e*(-1/2*1/c^2 - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)]
+ b*d*x*ArcSech[c*x] + (b*e*x^2*ArcSech[c*x])/2 + (2*b*d*Sqrt[(1 - c*x)/(1
+ c*x)]*Sqrt[1 - c^2*x^2]*ArcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^2*x)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

method	result	size
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b\left(\frac{c\,\operatorname{arcsech}(cx)e\,x^2}{2}+\operatorname{arcsech}(cx)dxc+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(2dc\arcsin(cx)-e\sqrt{-c^2x^2+1}\right)}{2\sqrt{-c^2x^2+1}}\right)}{c}$	107
derivativedivides	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\operatorname{arcsech}(cx)d\,c^2x+\frac{\operatorname{arcsech}(cx)e\,c^2x^2}{2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(2dc\arcsin(cx)-e\sqrt{-c^2x^2+1}\right)}{2\sqrt{-c^2x^2+1}}\right)}{c}}{c}$	125
default	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\operatorname{arcsech}(cx)d\,c^2x+\frac{\operatorname{arcsech}(cx)e\,c^2x^2}{2}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(2dc\arcsin(cx)-e\sqrt{-c^2x^2+1}\right)}{2\sqrt{-c^2x^2+1}}\right)}{c}}{c}$	125

```
[In] int((e*x+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arcsech(c*x)*e*x^2+arcsech(c*x)*d*x*c+1/2*(-(c
*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(2*d*c*arcsin(c*x)-e*(-c^2*x^2+1)^(1
/2))/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(72) = 144$.

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.25

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{acex^2+2\,acdx-bex\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-4\,bd\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-(2\,bcd+bce)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)+(bce}{2\,c}$$

```
[In] integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c*e*x^2 + 2*a*c*d*x - b*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*b*d*a
rctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - (2*b*c*d + b*c*e)*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c*e*x^2 + 2*b*c*d*x - 2
*b*c*d - b*c*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

Sympy [F]

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int (a+b\,\text{asech}\,(cx))\,(d+ex)\,\,dx$$

```
[In] integrate((e*x+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{1}{2}\,aex^2+\frac{1}{2}\left(x^2\,\text{arsech}\,(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)be + adx + \frac{\left(cx\,\text{arsech}\,(cx)-\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd}{c}$$

```
[In] integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*b*e + a*d*
x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d/c
```

Giac [F]

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (ex+d)(b\,\text{arsech}\,(cx)+a)\,dx$$

```
[In] integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arcsech(c*x) + a), x)
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int (d+ex)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{a\,x\,(2\,d+e\,x)}{2}+\frac{b\,d\,\text{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}+\frac{b\,e\,x^2\,\text{acosh}\left(\frac{1}{c\,x}\right)}{2}+b\,d\,x\,\text{acosh}\left(\frac{1}{c\,x}\right)-\frac{b\,e\,x\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{2\,c}$$

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x),x)
```

```
[Out] (a*x*(2*d + e*x))/2 + (b*d*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))
))/c + (b*e*x^2*acosh(1/(c*x)))/2 + b*d*x*acosh(1/(c*x)) - (b*e*x*(1/(c*x)
- 1)^(1/2)*(1/(c*x) + 1)^(1/2))/(2*c)
```

3.77 $\int \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \left(a+b\text{sech}^{-1}(cx)\right) \, dx=ax+bx\text{sech}^{-1}(cx)+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}$$

```
[Out] a*x+b*x*arcsech(c*x)+b*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6412, 222}

$$\int \left(a+b\text{sech}^{-1}(cx)\right) \, dx=ax+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{c}+bx\text{sech}^{-1}(cx)$$

```
[In] Int[a + b*ArcSech[c*x],x]
```

```
[Out] a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])
/c
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6412

```
Int[ArcSech[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSech[c*x], x] + Dist[Sqrt[1
 + c*x]*Sqrt[1/(1 + c*x)], Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + b\int \operatorname{sech}^{-1}(cx)\,dx\\ &= ax + bx\operatorname{sech}^{-1}(cx) + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx\\ &= ax + bx\operatorname{sech}^{-1}(cx) + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \left(a + b\operatorname{sech}^{-1}(cx)\right)\,dx = ax + bx\operatorname{sech}^{-1}(cx) + \frac{2b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x}$$

```
[In] Integrate[a + b*ArcSech[c*x],x]
[Out] a*x + b*x*ArcSech[c*x] + (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*A
rcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^2*x)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

method	result	size
default	$ax + bx\ \operatorname{arcsech}(cx) - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\ \sqrt{1+\frac{1}{cx}}\right)}{c}$	42
parts	$ax + bx\ \operatorname{arcsech}(cx) - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\ \sqrt{1+\frac{1}{cx}}\right)}{c}$	42
derivativedivides	$\frac{acx+b\left(cx\ \operatorname{arcsech}(cx)-\arctan\left(\sqrt{-1+\frac{1}{cx}}\ \sqrt{1+\frac{1}{cx}}\right)\right)}{c}$	46

```
[In] int(a+b*arcsech(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b*x*arcsech(c*x)-b/c*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(20) = 40$.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.98

$$\int \left(a+b\text{sech}^{-1}(cx)\right) dx$$
$$= \frac{acx - bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2\,b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + (bcx-bc)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{c}$$

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="fricas")
[Out] (a*c*x - b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*arctan((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + (b*c*x - b*c)*log((c*x*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

Sympy [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right) dx = \int \left(a+b\operatorname{asech}(cx)\right) dx$$

```
[In] integrate(a+b*asech(c*x),x)
[Out] Integral(a + b*asech(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \left(a+b\text{sech}^{-1}(cx)\right) dx = ax + \frac{\left(cx\,\text{arsech}\,(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)b}{c}$$

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="maxima")
[Out] a*x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b/c
```

Giac [F]

$$\int \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int b\,\text{arsech}\,(cx)+a\,dx$$

```
[In] integrate(a+b*arcsech(c*x),x, algorithm="giac")
[Out] integrate(b*arcsech(c*x) + a, x)
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a+b\text{sech}^{-1}(cx)\right)\,dx=a\,x+b\,x\,\text{acosh}\left(\frac{1}{c\,x}\right)+\frac{b\,\text{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}$$

```
[In] int(a + b*acosh(1/(c*x)),x)
[Out] a*x + b*x*acosh(1/(c*x)) + (b*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/
2))))/c
```

3.78 $\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx = -\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} + \frac{b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)}{2e} - \frac{b\,\text{PolyLog}\left(2,-\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} - \frac{b\,\text{PolyLog}\left(2,-\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}$$

```
[Out] -(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e+(a
+b*arcsech(c*x))*ln(1+(e-(-c^2*d^2+e^2)^(1/2))/c/d/(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2)))/e+(a+b*arcsech(c*x))*ln(1+(e+(-c^2*d^2+e^2)^(1/2))/c/d/(1
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/e+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e-b*polylog(2,(-e+(-c^2*d^2+e^2)^(1/2))/c/d/(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/e-b*polylog(2,(-e-(-c^2*d^2+e^2)^(
1/2))/c/d/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/e
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6422, 2598}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx = \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}+1\right)}{e} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{-\text{sech}^{-1}(cx)}}{cd}+1\right)}{e} - \frac{\log\left(e^{-2\text{sech}^{-1}(cx)}+1\right)\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{b\,\text{PolyLog}\left(2,-\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} - \frac{b\,\text{PolyLog}\left(2,-\frac{\left(e+\sqrt{e^2-c^2d^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} + \frac{b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)}{2e}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e) + ((a + b*ArcSech[
c*x])*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/e + ((a +
 b*ArcSech[c*x])*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])]
)/e + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/(2*e) - (b*PolyLog[2, -((e - Sqr
t[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x]))])/e - (b*PolyLog[2, -((e + Sqrt[
-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x]))])/e
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rule 6422

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :>
Simp[(a + b*ArcSech[c*x])*(Log[1 + (e - Sqrt[(-c^2)*d^2 + e^2])/(c*d*E^ArcS
ech[c*x])]/e), x] + (Dist[b/e, Int[(Sqrt[(1 - c*x)/(1 + c*x)]*Log[1 + (e -
Sqrt[(-c^2)*d^2 + e^2])/(c*d*E^ArcSech[c*x])])/(x*(1 - c*x)), x], x] + Dist
[b/e, Int[(Sqrt[(1 - c*x)/(1 + c*x)]*Log[1 + (e + Sqrt[(-c^2)*d^2 + e^2])/(
c*d*E^ArcSech[c*x])])/(x*(1 - c*x)), x], x] - Dist[b/e, Int[(Sqrt[(1 - c*x)
```

```
/(1 + c*x)]*Log[1 + 1/E^(2*ArcSech[c*x])])/(x*(1 - c*x)), x], x] + Simp[(a
+ b*ArcSech[c*x])*(Log[1 + (e + Sqrt[(-c^2)*d^2 + e^2])/(c*d*E^ArcSech[c*x]
)]/e), x] - Simp[(a + b*ArcSech[c*x])*(Log[1 + 1/E^(2*ArcSech[c*x])]/e), x]
) /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e} \\ &\quad+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} \\ &\quad+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} \\ &\quad-\frac{b\int\frac{\sqrt{\frac{1-cx}{1+cx}}\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{x(1-cx)}dx}{e}+\frac{b\int\frac{\sqrt{\frac{1-cx}{1+cx}}\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{x(1-cx)}dx}{e} \\ &\quad+\frac{b\int\frac{\sqrt{\frac{1-cx}{1+cx}}\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{x(1-cx)}dx}{e} \\ &= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e} \\ &\quad+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} \\ &\quad+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} \\ &\quad+\frac{b\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)}{2e}-\frac{b\,\text{PolyLog}\left(2,-\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e} \\ &\quad-\frac{b\,\text{PolyLog}\left(2,-\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-\text{sech}^{-1}(cx)}}{cd}\right)}{e}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.72

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx = \frac{a\log(d+ex)}{e} + \frac{b\left(\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)-2\left(-4i\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\text{arctanh}\left(\frac{(-cd+e)\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)}{\sqrt{-c^2d^2+e^2}}\right)+\text{sech}^{-1}(cx\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSi
n[Sqrt[1 + e/(c*d)]/Sqrt[2]]*ArcTanh[((-(c*d) + e)*Tanh[ArcSech[c*x]/2])/Sq
rt[-(c^2*d^2) + e^2]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech
[c*x]*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + (2*I)*Ar
cSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E
^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^
ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]*Log[1 + (e + Sqrt[
-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + PolyLog[2, (-e + Sqrt[-(c^2*d^2)
 + e^2])/(c*d*E^ArcSech[c*x])] + PolyLog[2, -((e + Sqrt[-(c^2*d^2) + e^2])/
(c*d*E^ArcSech[c*x]))])))/(2*e)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.23

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\,\text{arcsech}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{b\,\text{arcsech}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1\right.}{e+\sqrt{-}}\right.}{e}$
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\text{arcsech}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{\text{arcsech}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{e+\sqrt{-c^2d^2+e}}\right.}{e}\right.}{}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\text{arcsech}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{\text{arcsech}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{e+\sqrt{-c^2d^2+e}}\right.}{e}\right.}{}$

```
[In] int((a+b*arcsech(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(e*x+d)/e+b/e*arcsech(c*x)*ln((-c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))+(-c^2*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))+b/e*arcsech(c*x)*l
n((c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+
(-c^2*d^2+e^2)^(1/2)))+b/e*dilog((-c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))+(-c^2*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))+b/e*dilog((c*d*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+(-c^2*d^2+e
^2)^(1/2)))-b/e*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
))-b/e*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*di
log(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*dilog(1-I*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{d+ex}\,dx$$

```
[In] integrate((a+b*asech(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x + d), x
) + a*log(e*x + d)/e
```

Giac [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{d + ex}\,dx = \int \frac{b\,\operatorname{arsech}\left(cx\right) + a}{ex + d}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{d + ex}\,dx = \int \frac{a + b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{d + e\,x}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x), x)
```

3.79 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2} dx = -\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{de}$$

```
[Out] (-a-b*arcsech(c*x))/e/(e*x+d)+b*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/
2)*(c*x+1)^(1/2)/d/e+b*arctan((c^2*d*x+e)/(c^2*d^2-e^2)^(1/2)/(-c^2*x^2+1)^
(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/(c^2*d^2-e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6423, 975, 272, 65, 214, 739, 210}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2} dx = -\frac{a+b\text{sech}^{-1}(cx)}{e(d+ex)} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arctan\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{de}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcSech[c*x])/(e*(d + e*x))) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*
x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(d*Sqrt[c
```

```
^2*d^2 - e^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2
*x^2]])/(d*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
```

```
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\operatorname{sech}^{-1}(cx)}{e(d+ex)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x(d+ex)\sqrt{1-c^2x^2}}\,dx}{e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e(d+ex)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \left(\frac{1}{dx\sqrt{1-c^2x^2}} - \frac{e}{d(d+ex)\sqrt{1-c^2x^2}}\right)dx}{e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e(d+ex)} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{(d+ex)\sqrt{1-c^2x^2}}\,dx}{d} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-c^2x^2}}\,dx}{de}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e(d+ex)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{-c^2d^2+e^2-x^2}\,dx, x, \frac{e+c^2dx}{\sqrt{1-c^2x^2}}\right)}{d} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{2de}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e(d+ex)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{c^2de}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{e(d+ex)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{de}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.51

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a}{e(d+ex)}-\frac{b\text{sech}^{-1}(cx)}{e(d+ex)}-\frac{b\log(x)}{de}+\frac{b\log(d+ex)}{d\sqrt{-c^2d^2+e^2}}+\frac{b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{de}-\frac{b\log\left(e+c^2dx+\sqrt{-c^2d^2+e^2}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{-c^2d^2+e^2}x\sqrt{\frac{1-cx}{1+cx}}\right)}{d\sqrt{-c^2d^2+e^2}}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcSech[c*x])/(e*(d + e*x)) - (b*Log[x])/(d*e) + (b
*Log[d + e*x])/(d*Sqrt[-(c^2*d^2) + e^2]) + (b*Log[1 + Sqrt[(1 - c*x)/(1 +
c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(d*e) - (b*Log[e + c^2*d*x + Sqrt[-
(c^2*d^2) + e^2]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[-(c^2*d^2) + e^2]*x*Sqr
t[(1 - c*x)/(1 + c*x)]])/(d*Sqrt[-(c^2*d^2) + e^2])
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.41

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^2\,\text{arcsech}(cx)}{(cex+cd)e}+\frac{c^2\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)-\ln\left(\frac{2\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\sqrt{-c^2x^2+1}\cdots}{cex+cd}\cdots\right.\right.}{e\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,d\sqrt{-c^2x^2+1}}\cdots\right.}{c}\cdots$
derivativedivides	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\text{arcsech}(cx)}{(cex+cd)e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)-\ln\left(\frac{2\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\sqrt{-c^2x^2+1}\,e+2c\cdots}{cex+cd}\cdots\right.\right.}{e\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,d\sqrt{-c^2x^2+1}}\cdots\right.}{c}\cdots$
default	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\text{arcsech}(cx)}{(cex+cd)e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)-\ln\left(\frac{2\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,\sqrt{-c^2x^2+1}\,e+2c\cdots}{cex+cd}\cdots\right.\right.}{e\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,d\sqrt{-c^2x^2+1}}\cdots\right.}{c}\cdots$

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arcsech(c*x)+c^2/e*(-(c*x-1)/c/x)^(1/2
)*x*((c*x+1)/c/x)^(1/2)*((-(c^2*d^2-e^2)/e^2)^(1/2)*arctanh(1/(-c^2*x^2+1)^
```

```
(1/2))-ln(2*((-(c^2*d^2-e^2)/e^2)^(1/2)*(-c^2*x^2+1)^(1/2)*e+d*c^2*x+e)/(c*
e*x+c*d)))/(-(c^2*d^2-e^2)/e^2)^(1/2)/d/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(99) = 198$.

Time = 0.30 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.93

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex)^2}\,dx$$

$$= \left[-\frac{ac^2d^3 - ade^2 + \sqrt{-c^2d^2 + e^2}(be^2x + bde)\log\left(\frac{c^2dex - (c^3d^2 - ce^2)x\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + e^2 - \sqrt{-c^2d^2+e^2}\left(c^2dx + cex\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right.}{ex+d}\right.}{c^2d^4e - d^2e^3 + (c}\right.$$

$$-\frac{ac^2d^3 - ade^2 - 2\sqrt{c^2d^2 - e^2}(be^2x + bde)\arctan\left(-\frac{\sqrt{c^2d^2-e^2}cdx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - \sqrt{c^2d^2-e^2}(ex+d)}{(c^2d^2-e^2)x}\right) + (bc^2d^3 - bde}{c^2d^4e - d^2e^3 + (c^2d^3e^2 - de}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-(a*c^2*d^3 - a*d*e^2 + sqrt(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*log((c^2*d*
e*x - (c^3*d^2 - c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2 - sqrt(-c^2*
d^2 + e^2)*(c^2*d*x + c*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e))/(e*x + d))
 + (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*log((c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) - 1)/x) + (b*c^2*d^3 - b*d*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x),
-(a*c^2*d^3 - a*d*e^2 - 2*sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*arctan(-(sq
rt(c^2*d^2 - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(c^2*d^2 - e^2
)*(e*x + d))/((c^2*d^2 - e^2)*x)) + (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b
*e^3)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^2*d^3 - b*d
*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e - d^2
*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{(d+ex)^2}\,dx$$

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**2,x)
[Out] Integral((a + b*asech(c*x))/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="maxima")
[Out] (c^2*integrate(x^2/(c^2*d^2*x^2 + (c^2*d^2*x^2 - d^2 + (c^2*d*e*x^2 - d*e)*
x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - d^2 + (c^2*d*e*x^2 - d*e)*x), x) + (x*log
(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - x*log(c) - x*log(x))/(d*e*x + d^2) - i
ntegrate(1/(c^2*d^2*x^2 - d^2 + (c^2*d*e*x^2 - d*e)*x), x))*b - a/(e^2*x +
d*e)
```

Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^2}\,dx=\int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^2}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^2,x)
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^2, x)
```

3.80 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 306

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3}\,dx &= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d\,(c^2d^2-e^2)\,(d+ex)} - \frac{a+b\text{sech}^{-1}(cx)}{2e(d+ex)^2} \\ &\quad + \frac{bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{2\,(c^2d^2-e^2)^{3/2}} \\ &\quad + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{2d^2\sqrt{c^2d^2-e^2}} \\ &\quad + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2d^2e}\end{aligned}$$

```
[Out] 1/2*(-a-b*arcsech(c*x))/e/(e*x+d)^2+1/2*b*c^2*arctan((c^2*d*x+e)/(c^2*d^2-e
^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)
^(3/2)+1/2*b*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^
2/e+1/2*b*arctan((c^2*d*x+e)/(c^2*d^2-e^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*d^2-e^2)^(1/2)+1/2*b*e*(1/(c*x+1))^(1/2)
*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6423, 975, 272, 65, 214, 745, 739, 210}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3}\,dx=-\frac{a+b\text{sech}^{-1}(cx)}{2e(d+ex)^2}+\frac{bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arctan\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{2\left(c^2d^2-e^2\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arctan\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{2d^2\sqrt{c^2d^2-e^2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2d^2e}+\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^3,x]
```

```
[Out] (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(2*d*(c^2*d^2 -
e^2)*(d + e*x)) - (a + b*ArcSech[c*x])/(2*e*(d + e*x)^2) + (b*c^2*Sqrt[(1 +
 c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1
- c^2*x^2])])/(2*(c^2*d^2 - e^2)^(3/2)) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
c*x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(2*d^2*
Sqrt[c^2*d^2 - e^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1
 - c^2*x^2]])/(2*d^2*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x(d+ex)^2\sqrt{1-c^2x^2}}\,dx}{2e} \\
&= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \left(\frac{1}{d^2x\sqrt{1-c^2x^2}} - \frac{e}{d(d+ex)^2\sqrt{1-c^2x^2}} - \frac{e}{d^2(d+ex)\sqrt{1-c^2x^2}}\right)dx}{2e} \\
&= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{(d+ex)\sqrt{1-c^2x^2}}\,dx}{2d^2} \\
&\quad + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{(d+ex)^2\sqrt{1-c^2x^2}}\,dx}{2d} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-c^2x^2}}\,dx}{2d^2e}
\end{aligned}
$$

$$= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{-c^2d^2+e^2-x^2}\,dx, x, \frac{e+c^2dx}{\sqrt{1-c^2x^2}}\right)}{2d^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{4d^2e} + \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{(d+ex)\sqrt{1-c^2x^2}}\,dx}{2\left(c^2d^2-e^2\right)}$$

$$= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{2d^2\sqrt{c^2d^2-e^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{2c^2d^2e} - \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{-c^2d^2+e^2-x^2}\,dx, x, \frac{e+c^2dx}{\sqrt{1-c^2x^2}}\right)}{2\left(c^2d^2-e^2\right)}$$

$$= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d\left(c^2d^2-e^2\right)(d+ex)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex)^2} + \frac{bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{2\left(c^2d^2-e^2\right)^{3/2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{2d^2\sqrt{c^2d^2-e^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2d^2e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2}+\frac{b\sqrt{\frac{1-cx}{1+cx}}(e+cex)}{d(cd-e)(cd+e)(d+ex)}-\frac{b\text{sech}^{-1}(cx)}{e(d+ex)^2}-\frac{b\log(x)}{d^2e}+\frac{b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d^2e}-\frac{ib(2c^2d^2-e^2)\log\left(\frac{4d^2e\sqrt{c^2d^2-e^2}\left(ie+ic^2dx+\sqrt{c^2d^2-e^2}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{c^2d^2-e^2}x\sqrt{\frac{1-cx}{1+cx}}\right)}{b(2c^2d^2-e^2)(d+ex)}\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}\right)$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(e + c*e*x))/(d*(c*d -
 e)*(c*d + e)*(d + e*x)) - (b*ArcSech[c*x])/(e*(d + e*x)^2) - (b*Log[x])/(d
^2*e) + (b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)
]])/(d^2*e) - (I*b*(2*c^2*d^2 - e^2)*Log[(4*d^2*e*Sqrt[c^2*d^2 - e^2]*(I*e
+ I*c^2*d*x + Sqrt[c^2*d^2 - e^2]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d^
2 - e^2]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/(b*(2*c^2*d^2 - e^2)*(d + e*x))])/(d
^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(267) = 534$.

Time = 1.88 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.94

method	result
parts	$-\frac{a}{2(ex+d)^2 e}+\frac{b\left(-\frac{c^3 \operatorname{arcsech}(cx)}{2(cex+cd)^2 e}+\frac{c^2\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3d^3\sqrt{-\frac{c^2d^2-e^2}{e^2}}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3 \cdots\right.}{\cdots}\right)}{\cdots}$
derivativedivides	$\frac{-\frac{a c^3}{2(cex+cd)^2 e}+b c^3\left(-\frac{\operatorname{arcsech}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3d^3\sqrt{-\frac{c^2d^2-e^2}{e^2}}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3d^2 \cdots\right.}{\cdots}\right)}{\cdots}$
default	$\frac{-\frac{a c^3}{2(cex+cd)^2 e}+b c^3\left(-\frac{\operatorname{arcsech}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3d^3\sqrt{-\frac{c^2d^2-e^2}{e^2}}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^3d^2 \cdots\right.}{\cdots}\right)}{\cdots}$

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arcsech(c*x)+1/2*c^2/e*(-(
c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^3*
d^3*(-(c^2*d^2-e^2)/e^2)^(1/2)+arctanh(1/(-c^2*x^2+1)^(1/2))*c^3*d^2*e*(-(c
^2*d^2-e^2)/e^2)^(1/2)*x-2*ln(2*((-(c^2*d^2-e^2)/e^2)^(1/2)*(-c^2*x^2+1)^(1
/2)*e+d*c^2*x+e)/(c*e*x+c*d))*c^3*d^3-2*ln(2*((-(c^2*d^2-e^2)/e^2)^(1/2)*(-
c^2*x^2+1)^(1/2)*e+d*c^2*x+e)/(c*e*x+c*d))*c^3*d^2*e*x-arctanh(1/(-c^2*x^2+
1)^(1/2))*c*d*e^2*(-(c^2*d^2-e^2)/e^2)^(1/2)-arctanh(1/(-c^2*x^2+1)^(1/2))*
e^3*(-(c^2*d^2-e^2)/e^2)^(1/2)*c*x+c*d*e^2*(-c^2*x^2+1)^(1/2)*(-(c^2*d^2-e^
2)/e^2)^(1/2)+ln(2*((-(c^2*d^2-e^2)/e^2)^(1/2)*(-c^2*x^2+1)^(1/2)*e+d*c^2*x
+e)/(c*e*x+c*d))*c*d*e^2+ln(2*((-(c^2*d^2-e^2)/e^2)^(1/2)*(-c^2*x^2+1)^(1/2
)*e+d*c^2*x+e)/(c*e*x+c*d))*e^3*c*x)/(-c^2*x^2+1)^(1/2)/d^2/(c*d+e)/(c*d-e)
/(c*e*x+c*d)/(-(c^2*d^2-e^2)/e^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(200) = 400.

Time = 0.37 (sec) , antiderivative size = 1212, normalized size of antiderivative = 3.96

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + (a + b)*d^2*e^4 - (b*c^2*d^2*e^4
 - b*e^6)*x^2 + (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d^2*e^3 - b*e^5)*x^2
+ 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(-c^2*d^2 + e^2)*log((c^2*d*e*x - (c
^3*d^2 - c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2 - sqrt(-c^2*d^2 + e^
2)*(c^2*d*x + c*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e))/(e*x + d)) - 2*(b*
c^2*d^3*e^3 - b*d*e^5)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^
4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3
 + b*d*e^5)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^4*d^6
 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1
)/(c*x)) - ((b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 - b*c*d^2*e^4)
*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 +
(c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^
4 + d^3*e^6)*x), -1/2*(a*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + (a + b)*d^2*e^4
- (b*c^2*d^2*e^4 - b*e^6)*x^2 - 2*(2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d^2
*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(c^2*d^2 - e^2)*ar
ctan(-(sqrt(c^2*d^2 - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(c^2*
d^2 - e^2)*(e*x + d))/((c^2*d^2 - e^2)*x)) - 2*(b*c^2*d^3*e^3 - b*d*e^5)*x
+ (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2*e
^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log((c*x*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^
2*e^4)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - ((b*c^3*d^3*e^
3 - b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 - b*c*d^2*e^4)*x)*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 - 2*c^2*d^4*
e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e^6)*x)]
```

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{(d+ex)^3}\,dx$$

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^3}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex + d)^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^3}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^3}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^3, x)
```

3.81 $\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 343

$$\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2}+\frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}-\frac{28bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{4b(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^3\sqrt{d+ex}}-\frac{4bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5e\sqrt{d+ex}}$$

```
[Out] 2/5*(e*x+d)^(5/2)*(a+b*arcsech(c*x))/e-28/15*b*d*EllipticE(1/2*(-c*x+1)^(1/
2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x+
d)^(1/2)/c/(c*(e*x+d)/(c*d+e))^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(
-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^
(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/c^3/(e*x+d)^(1/2)-4/5*b*d^3*EllipticPi(1/2*
(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/e/(e*x+d)^(1/2)-4/15*b*e*(1/(c*x+1))^(1/
2)*(c*x+1)^(1/2)*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6423, 972, 733, 430, 946, 174, 552, 551, 858, 435, 945, 1598}

$$\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \frac{2(d+ex)^{5/2} \left(a+b\text{sech}^{-1}(cx)\right)}{5e} - \frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^3\sqrt{d+ex}} - \frac{4bd^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5e\sqrt{d+ex}} - \frac{28bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex}}{15c^2}$$

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (-4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2])
/(15*c^2) + (2*(d + e*x)^(5/2)*(a + b*ArcSech[c*x]))/(5*e) - (28*b*d*Sqrt[(
1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/S
qrt[2]], (2*e)/(c*d + e)])/(15*c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*(2*c
^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d +
e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^3*Sqrt
[d + e*x]) - (4*b*d^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))
/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/
(5*e*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
```

```
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e}+\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^{5/2}}{x\sqrt{1-c^2x^2}}\,dx}{5e}$$

$$=\frac{2(d+ex)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e}+\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{d^3}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{3de^2x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e^3x^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{5e}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e} + \frac{1}{5}\left(6bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx + \frac{\left(2bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{5e} + \frac{1}{5}\left(6bde\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx + \frac{1}{5}\left(2be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e} + \frac{1}{5}\left(6bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx - \frac{1}{5}\left(6bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx + \frac{\left(2bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{5e} + \frac{\left(2be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{ex-2c^2dx^2}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15c^2} - \frac{\left(12bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c\sqrt{d+ex}}$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$
$$- \frac{12bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5c\sqrt{d+ex}}$$
$$- \frac{\left(4bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}dx,x,\sqrt{1-cx}\right)}{5e}$$
$$+ \frac{\left(2be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{e-2c^2dx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{15c^2}$$
$$- \frac{\left(12bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$
$$+ \frac{\left(12bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c\sqrt{d+ex}}$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}$$
$$- \frac{12bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{1}{15}\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx$$
$$+ \frac{\left(2b(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{15c^2}$$
$$- \frac{\left(4bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}dx,x,\sqrt{1-cx}\right)}{5e\sqrt{d+ex}}$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e} - \frac{12bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5e\sqrt{d+ex}} + \frac{\left(8bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} - \frac{\left(4b(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^3\sqrt{d+ex}}$$

$$= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d+ex)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e} - \frac{28bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b(2c^2d^2+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^3\sqrt{d+ex}} - \frac{4bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5e\sqrt{d+ex}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.84 (sec) , antiderivative size = 2653, normalized size of antiderivative = 7.73

$$\int (d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[d + e*x]*((-4*b*e)/(15*c^2) - (4*b*e*x)/(15*
c)) + Sqrt[d + e*x]*((2*a*d^2)/(5*e) + (4*a*d*x)/5 + (2*a*e*x^2)/5) + (2*b*
(d + e*x)^(5/2)*ArcSech[c*x])/(5*e) - (4*b*(7*c*d*e*Sqrt[(1 - c*x)/(1 + c*x
```

```
)]*(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x))) + ((7*I)*c
^2*d^2*e*(c*d + e)*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1
 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*(EllipticE[I*ArcSinh[Sq
rt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - EllipticF[I*ArcSinh[Sqrt[(
1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)]))/(c*d - e) - ((7*I)*c*d*e^2*(c*
d + e)*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*
d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)
/(1 + c*x)]], (c*d - e)/(c*d + e)] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1
+ c*x)]], (c*d - e)/(c*d + e)]))/(c*d - e) + (3*I)*c^3*d^3*Sqrt[1 + (1 - c*
x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c
*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/
(c*d + e)] - (2*I)*c^2*d^2*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1
- c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*EllipticF[I*A
rcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - I*e^3*Sqrt[1 + (1
 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(
1 + c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d
- e)/(c*d + e)] + ((3 + 3*I)*c^3*d^3*(-I + Sqrt[(1 - c*x)/(1 + c*x)])*(I +
Sqrt[(1 - c*x)/(1 + c*x)])*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d
*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqr
t[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[
((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*S
qrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*
(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) - e
] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] +
I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I
*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (1 - I)*Ellipti
cPi[(I*Sqrt[-(c*d) - e] - Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e
]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(
1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 +
 c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sq
rt[c*d - e])^2]))/Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 -
 c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*
x)/(1 + c*x)]))] + ((3 + 3*I)*c^3*d^3*(1 + I*Sqrt[(1 - c*x)/(1 + c*x)])*(I
+ Sqrt[(1 - c*x)/(1 + c*x)])*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c
*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + S
qrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqr
t[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e
*Sqrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e
)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) -
 e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e]
+ I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] +
 I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (1 + I)*Ellip
ticPi[((-I)*Sqrt[-(c*d) - e] + Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*
d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c
*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)
```

```
/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] -
 I*Sqrt[c*d - e])^2]))/Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt
[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1
 - c*x)/(1 + c*x)]))]]))/(15*c^3*e*(1 + (1 - c*x)/(1 + c*x))*Sqrt[(c*d + e +
 (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1
 + c*x))])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(302) = 604$.

Time = 12.72 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.38

method	result
derivativedivides	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsech}(cx)}{5}-\frac{2e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^2(ex+d)^{\frac{5}{2}}+9\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsech}(cx)}{5}-\frac{2e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^2(ex+d)^{\frac{5}{2}}+9\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$\frac{2a(ex+d)^{\frac{5}{2}}}{5e}+\frac{2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsech}(cx)}{5}-\frac{2e^2\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^2(ex+d)^{\frac{5}{2}}+9\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\cdots\right.}{\cdots}\right.}{\cdots}$

```
[In] int((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/5*a*(e*x+d)^(5/2)+b*(1/5*(e*x+d)^(5/2)*arcsech(c*x)-2/15/c*e^2*((-c*
(e*x+d)+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*((c/(c*d+e)
)^(1/2)*c^2*(e*x+d)^(5/2)+9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticF((e
*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*((-c*(e*x+d)+c*d+e)/
(c*d+e))^(1/2)*c^2*d^2-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+
d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*((-c*(e*x+d)+c*d+e)/(c*
d+e))^(1/2)*c^2*d^2-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)
^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))
*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*c^2*d^2-2*(c/(c*d+e))^(1/2)*c^2*d*(e*x+
d)^(3/2)-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c
*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*c*
d*e+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e)
)^(1/2),((c*d+e)/(c*d-e))^(1/2))*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*c*d*e+(
c/(c*d+e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*E
llipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*((-c*(e*x
+d)+c*d+e)/(c*d+e))^(1/2)*e^2-(c/(c*d+e))^(1/2)*e^2*(e*x+d)^(1/2))/(c/(c*d+
e))^(1/2)/(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)))
```

Fricas [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right)\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int (a+b\,\text{asech}\,(cx))\,(d+ex)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(a+b*asech(c*x)),x)
[Out] Integral((a + b*asech(c*x))*(d + e*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int (ex+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^(3/2)*(b*arcsech(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \left(d+e\,x\right)^{3/2} dx$$

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^(3/2),x)
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^(3/2), x)
```

3.82 $\int \sqrt{d+ex}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 279

$$\begin{aligned}
&\int \sqrt{d+ex}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx\\
&=\frac{2(d+ex)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e}-\frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c\sqrt{\frac{c(d+ex)}{cd+e}}}\\
&-\frac{4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c\sqrt{d+ex}}\\
&-\frac{4bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3e\sqrt{d+ex}}
\end{aligned}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arcsech(c*x))/e-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x+d)^(
1/2)/c/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/
2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c
*d+e))^(1/2)/c/(e*x+d)^(1/2)-4/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2
),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(
c*d+e))^(1/2)/e/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6423, 972, 733, 430, 946, 174, 552, 551, 858, 435}

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right) dx$$
$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} - \frac{4bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3e\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c\sqrt{d+ex}} - \frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c\sqrt{\frac{c(d+ex)}{cd+e}}}$$

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcSech[c*x]))/(3*e) - (4*b*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)])/(3*c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*d*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*Sqrt[d + e*x]) - (4*b*d^2*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}\,dx}{3e}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\left(\frac{2de}{\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{d^2}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e^2x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{3e}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{1}{3}\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx + \frac{\left(2bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3e} + \frac{1}{3}\left(2be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} + \frac{1}{3}\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx - \frac{1}{3}\left(2bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx + \frac{\left(2bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3e} - \frac{\left(8bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} - \frac{8bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c\sqrt{d+ex}} - \frac{\left(4bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}dx,x,\sqrt{1-cx}\right)}{3e} - \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} + \frac{\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c\sqrt{d+ex}} - \frac{\left(4bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}dx,x,\sqrt{1-cx}\right)}{3e\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c\sqrt{d+ex}} - \frac{4bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3e\sqrt{d+ex}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.04 (sec) , antiderivative size = 2938, normalized size of antiderivative = 10.53

$$\int \sqrt{d+ex}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\text{Result too large to show}$$

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcSech[c*x]),x]
```

```
[Out] ((2*a*d)/(3*e) + (2*a*x)/3)*Sqrt[d + e*x] + (2*b*(d + e*x)^(3/2)*ArcSech[c*
x])/(3*e) + (4*b*(-((e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[c*(1 + (1 - c*x)/(1 +
 c*x))]*Sqrt[c + (c*(1 - c*x))/(1 + c*x)]*Sqrt[(c*d + e + (c*d*(1 - c*x))/(
1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x))]])/(c*(1 +
 (1 - c*x)/(1 + c*x)))) + (Sqrt[c*(1 + (1 - c*x)/(1 + c*x))]*Sqrt[c + (c*(1
 - c*x))/(1 + c*x)]*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*(1 - c
*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))]*Sqrt[(c*d + e + (c*d*(1 - c*x))/
(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x))]*((I*c*d
*(-(c*d) - e)*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x)
)/((-(c*d) - e)*(1 + c*x))]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]]
, -((c*d - e)/(-(c*d) - e))] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)
]], -((c*d - e)/(-(c*d) - e))]))/((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x)
)*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) - (I*(-(c*d) - e)*e^2*Sqrt[
1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 +
c*x))]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d)
 - e))] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*
d) - e))]))/((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d -
e)*(1 - c*x))/(1 + c*x))]) - (I*c^2*d^2*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[
1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*EllipticF[I*ArcSinh[Sqr
t[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))])/Sqrt[c*(1 + (1 - c*x)/
(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))] + (I*c*d*e*Sqrt[1 +
 (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x
))]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e
))])/Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 +
 c*x))] + (I*c^2*d^2*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c
*x)/(1 + c*x)])^2*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 -
 c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*
x)/(1 + c*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d) - e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x
)/(1 + c*x)]))/((I + Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(
1 + c*x)]))]*Sqrt[(I*(Sqrt[-(c*d) - e]/Sqrt[c*d - e] + Sqrt[(1 - c*x)/(1 +
c*x)]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x
)]))]*((1 + I)*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*
(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I
+ Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sq
rt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (2*I)*EllipticPi[((-I)*(I + Sqrt[-(c
```

```
*d) - e]/Sqrt[c*d - e]))/(-I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt
[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((S
qrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sq
rt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2]
))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(
c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) - (I*c^2*d^2*(I + Sqrt[-(c*d)
- e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])^2*Sqrt[((Sqrt[-(c*d) -
 e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e]
+ I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d
) - e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x)/(1 + c*x)]))/((I + Sqrt[-(c*d) - e]/
Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(Sqrt[-(c*d) - e]
/Sqrt[c*d - e] + Sqrt[(1 - c*x)/(1 + c*x)]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*
d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*((-1 + I)*EllipticF[ArcSin[Sqrt[
((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sq
rt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqr
t[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2]
- (2*I)*EllipticPi[(I*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]))/(-I + Sqrt[-(c*
d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(
I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I +
 Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqr
t[-(c*d) - e] - I*Sqrt[c*d - e])^2]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])
*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x
))])))/(c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) -
(e*(1 - c*x))/(1 + c*x)))))/(3*c*e)
```

Maple [A] (verified)

Time = 11.26 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsech}(cx)}{3}-\frac{2e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(2\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)cd-\mathrm{El}\right.}{}\right.}{}$
default	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsech}(cx)}{3}-\frac{2e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(2\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)cd-\mathrm{El}\right.}{}\right.}{}$
parts	$\frac{2a(ex+d)^{\frac{3}{2}}}{3e}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsech}(cx)}{3}-\frac{2e^2\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\left(2\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)cd-\mathrm{E}\right.}{}\right.}{}$

```
[In] int((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arcsech(c*x)-2/3*e^2*((-c*(e*
x+d)+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*(2*EllipticF((
e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d-EllipticE((e*x+
d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d-EllipticPi((e*x+d)^
(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*
c*d-EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*e+El
lipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*e)*((-c*(e
*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)/(c/(c*d+e))^
(1/2)/(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(a+b\,\text{asech}\,(cx)\right)\sqrt{d+ex}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}(b\,\text{arsech}\,(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arcsech(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.83 $\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 187

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}$$

```
[Out] 2*(a+b*arcsech(c*x))*(e*x+d)^(1/2)/e-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/
2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c
*d+e))^(1/2)/c/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,
2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c*d+
e))^(1/2)/e/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used

$= \{6423, 958, 733, 430, 946, 174, 552, 551\}$

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcSech[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcSech[c*x]))/e - (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2
]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]) - (4*b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[
1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/S
qrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{d+ex}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e}+\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}\,dx}{e}\\
&= \frac{2\sqrt{d+ex}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e}+\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx\\
&\quad+\frac{\left(2bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{e}
\end{aligned}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e} + \frac{\left(2bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{e} - \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{c\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{e}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{e\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2, \arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.89 (sec) , antiderivative size = 1707, normalized size of antiderivative = 9.13

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2a\sqrt{d+ex}}{e} + \frac{2b\sqrt{d+ex}\text{sech}^{-1}(cx)}{e} - \frac{4ib\sqrt{\frac{cd+e+\frac{cd(1-cx)}{1+cx}-\frac{e(1-cx)}{1+cx}}{c+\frac{c(1-cx)}{1+cx}}}\left(2cd\sqrt{-\frac{i\left(\sqrt{-cd-e}\sqrt{cd-e}+cd\sqrt{\frac{1-cx}{1+cx}}-e\sqrt{\frac{1-cx}{1+cx}}\right)}{\left(-icd+\sqrt{-cd-e}\sqrt{cd-e}+ie\right)\left(-i+\sqrt{\frac{1-cx}{1+cx}}\right)}}\sqrt{-\frac{i\left(\sqrt{-cd-e}\sqrt{cd-e}-cd\sqrt{\frac{1-cx}{1+cx}}+e\sqrt{\frac{1-cx}{1+cx}}\right)}{\left(icd+\sqrt{-cd-e}\sqrt{cd-e}-ie\right)\left(-i+\sqrt{\frac{1-cx}{1+cx}}\right)}}(1$$

```
[In] Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*a*Sqrt[d + e*x])/e + (2*b*Sqrt[d + e*x]*ArcSech[c*x])/e - ((4*I)*b*Sqrt[
(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1
- c*x))/(1 + c*x))]*(2*c*d*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d
*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqr
t[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[
((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*S
qrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*
(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*EllipticF[ArcS
in[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)
]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))
]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d -
 e])^2] + (c*d - e)*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1
 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 -
c*x)/(1 + c*x)]))]*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1
 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqr
t[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] + (2*I)*c*d*Sqrt[((-I)*(Sqrt[
-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x
)/(1 + c*x)]))/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqr
t[(1 - c*x)/(1 + c*x)]))]*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*
Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(
c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 -
 c*x)/(1 + c*x))*(EllipticPi[(I*Sqrt[-(c*d) - e] - Sqrt[c*d - e])/(Sqrt[-(c
*d) - e] - I*Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d -
e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*
(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2
/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - EllipticPi[((-I)*Sqrt[-(c*d) - e
] + Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt
[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c
*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*
d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2])))/(e*
Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))
/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(
e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x))))
```

Maple [A] (verified)

Time = 11.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsech}(cx)-\frac{2c\,e^{2}\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)-\operatorname{Ellip}\right.}{\left(c^{2}(ex+d)^{2}-2c^{2}d(ex+d)+c\right.}\right.}{e}$
default	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsech}(cx)-\frac{2c\,e^{2}\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)-\operatorname{Ellip}\right.}{\left(c^{2}(ex+d)^{2}-2c^{2}d(ex+d)+c\right.}\right.}{e}$
parts	$\frac{2a\sqrt{ex+d}}{e}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arcsech}(cx)-\frac{2c\,e^{2}\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)-\operatorname{Ellipt}\right.}{\sqrt{\frac{c}{cd+e}}\left(c^{2}(ex+d)^{2}-2c^{2}d(\right.}\right.}{e}$

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*((e*x+d)^(1/2)*a+b*((e*x+d)^(1/2)*arcsech(c*x)-2*c*e^2*((-c*(e*x+d)+c*d
+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*(EllipticF((e*x+d)^(1/
2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(
c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2)))*((-c*(e*x
+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)/(c^2*(e*x+d)^2
-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/(c/(c*d+e))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex}}\,dx=\int \frac{a+b\operatorname{asech}(cx)}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{\sqrt{d + ex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{\sqrt{d + ex}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{ex + d}}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{\sqrt{d + ex}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{\sqrt{d + e\,x}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^(1/2), x)
```

3.84 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}$$

```
[Out] -2*(a+b*arcsech(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(
1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d
)/(c*d+e))^(1/2)/e/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6423, 946, 174, 552, 551}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}} - \frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSech[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqr
t[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}-\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{e}\\
&= -\frac{2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}-\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{e}
\end{aligned}
$$

$$= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}+\frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{e}$$

$$= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$+\frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{e\sqrt{d+ex}}$$

$$= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e\sqrt{d+ex}}+\frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.02 (sec) , antiderivative size = 1675, normalized size of antiderivative = 15.95

$$\int\frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2a}{e\sqrt{d+ex}}-\frac{2b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex}}$$

$$+\frac{4ib\left(2\sqrt{-\frac{i\left(\sqrt{-cd-e}\sqrt{cd-e}+cd\sqrt{\frac{1-cx}{1+cx}}-e\sqrt{\frac{1-cx}{1+cx}}\right)}{\left(-icd+\sqrt{-cd-e}\sqrt{cd-e}+ie\right)\left(-i+\sqrt{\frac{1-cx}{1+cx}}\right)}}\sqrt{-\frac{i\left(\sqrt{-cd-e}\sqrt{cd-e}-cd\sqrt{\frac{1-cx}{1+cx}}+e\sqrt{\frac{1-cx}{1+cx}}\right)}{\left(icd+\sqrt{-cd-e}\sqrt{cd-e}-ie\right)\left(-i+\sqrt{\frac{1-cx}{1+cx}}\right)}}\left(1+\frac{1-cx}{1+cx}\right)\operatorname{EllipticF}\left(\arcsin\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*a)/(e*Sqrt[d + e*x]) - (2*b*ArcSech[c*x])/(e*Sqrt[d + e*x]) + ((4*I)*b*
(2*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x
)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d -
 e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[((-I)*(Sqrt[-(c*d) - e]*
Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 + c*x)]
))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*x)/(1
+ c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) -
e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] +
 I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] +
I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] + Sqrt[((Sqrt[-(
c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d)
 - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[1 + (1 - c
*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 +
c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)
/(c*d + e)] + (2*I)*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(
```

```
1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)]))/(((-I)*c*d + Sqrt[-(c*d
) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[((-I)*(
Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1
- c*x)/(1 + c*x)]))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + S
qrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*(EllipticPi[(I*Sqrt[-
(c*d) - e] - Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e]), ArcSin[Sq
rt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/(
(Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (
Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^
2] - EllipticPi[((-I)*Sqrt[-(c*d) - e] + Sqrt[c*d - e])/(Sqrt[-(c*d) - e] -
 I*Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + S
qrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt
[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c
*d) - e] - I*Sqrt[c*d - e])^2]))/(e*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d -
 e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])
*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*(1 + (1 - c*x)/(1 + c*x))*Sqrt[(c*d + e
 + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/
(1 + c*x))])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(96) = 192$.

Time = 11.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.39

method	result
derivativedivides	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsech}(cx)}{\sqrt{ex+d}}-\frac{2c\,e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd+e}},\frac{cd+e}{cd},\frac{\sqrt{\frac{c}{cd-e}}}{\sqrt{\frac{c}{cd+e}}}\right)\sqrt{-c(e}}{d\sqrt{\frac{c}{cd+e}}\left(c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2\right)}\right.}{e}$
default	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsech}(cx)}{\sqrt{ex+d}}-\frac{2c\,e^2\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd+e}},\frac{cd+e}{cd},\frac{\sqrt{\frac{c}{cd-e}}}{\sqrt{\frac{c}{cd+e}}}\right)\sqrt{-c(e}}{d\sqrt{\frac{c}{cd+e}}\left(c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2\right)}\right.}{e}$
parts	$-\frac{2a}{\sqrt{ex+d}\,e}+\frac{2b\left(-\frac{\operatorname{arcsech}(cx)}{\sqrt{ex+d}}-\frac{2c\,e^2\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd+e}},\frac{cd+e}{cd},\frac{\sqrt{\frac{c}{cd-e}}}{\sqrt{\frac{c}{cd+e}}}\right)\sqrt{-}}{d\sqrt{\frac{c}{cd+e}}\left(c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2\right)}\right.}{e}$

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arcsech(c*x)-2*c*e^2*((-c*(e*x+d)
+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*EllipticPi((e*x+d)
^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))
*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)/d/(c
```

```
/(c*d+e))^(1/2)/(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)))
```

Fricas [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex)^{3/2}} \, dx = \int \frac{b \operatorname{arsech}(cx) + a}{(ex + d)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex)^{3/2}} \, dx = \int \frac{a + b \operatorname{asech}(cx)}{(d + ex)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**(3/2),x)
[Out] Integral((a + b*asech(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{3/2}}\, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex + d)^{\frac{3}{2}}}\, dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{3/2}}\, dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{3/2}}\, dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^(3/2),x)
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^(3/2), x)
```

3.85 $\int \frac{a+b\,\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 278

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\,\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3de\sqrt{d+ex}}$$

```
[Out] -2/3*(a+b*arcsech(c*x))/e/(e*x+d)^(3/2)-4/3*b*c*EllipticE(1/2*(-c*x+1)^(1/2
)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x+d
)^(1/2)/d/(c^2*d^2-e^2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*EllipticPi(1/2*(-c*
x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(
1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/d/e/(e*x+d)^(1/2)+4/3*b*e*(1/(c*x+1))^(1/2)*
(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules

used = {6423, 972, 759, 21, 733, 435, 946, 174, 552, 551}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{4bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3de\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d*(c^2*d^2
- e^2)*Sqrt[d + e*x]) - (2*(a + b*ArcSech[c*x]))/(3*e*(d + e*x)^(3/2)) - (4
*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*d*(c^2*d^2 - e^2)*Sqrt[(c*(d + e*x
))/(c*d + e)]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))
/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/
(3*d*e*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3e}$$

$$= -\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}+\frac{1}{dx\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{3e}$$

$$= -\frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3d} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3de}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3de} - \frac{\left(4bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3d\left(c^2d^2-e^2\right)}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\text{sech}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{3de} + \frac{\left(2bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{3d\left(c^2d^2-e^2\right)}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d^2-e^2)\sqrt{d+ex}} - \frac{2(a+b\operatorname{sech}^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{3de\sqrt{d+ex}} - \frac{\left(4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3d(c^2d^2-e^2)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d^2-e^2)\sqrt{d+ex}} - \frac{2(a+b\operatorname{sech}^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3de\sqrt{d+ex}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 25.13 (sec) , antiderivative size = 4527, normalized size of antiderivative = 16.28

$$\int\frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (-2*a)/(3*e*(d + e*x)^(3/2)) + Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[d + e*x]*((4*
b*c)/(3*d*(c^2*d^2 - e^2)) - (4*b)/(3*d*(c*d + e)*(d + e*x))) - (2*b*ArcSec
h[c*x])/(3*e*(d + e*x)^(3/2)) - (4*b*((e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[c*(
1 + (1 - c*x)/(1 + c*x))]*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*
x))/(1 + c*x)))/((1 + (1 - c*x)/(1 + c*x))*Sqrt[c + (c*(1 - c*x))/(1 + c*x)
]*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c +
 (c*(1 - c*x))/(1 + c*x))]]) - ((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))]*
Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*
(1 - c*x))/(1 + c*x))]*((I*(-(c*d) - e)*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqr
t[1 - ((c*d - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*(EllipticE[I*ArcSinh[
Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))] - EllipticF[I*ArcSin
h[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))]))/((c*d - e)*Sqrt[
```

```
c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) +
 (I*c*d*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-(c*
d) - e)*(1 + c*x))]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d
- e)/(-(c*d) - e))])/Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)
*(1 - c*x))/(1 + c*x))] + (I*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d
 - e)*(1 - c*x))/((-(c*d) - e)*(1 + c*x))]*EllipticF[I*ArcSinh[Sqrt[(1 - c*
x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))])/Sqrt[c*(1 + (1 - c*x)/(1 + c*x)
)*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))] - (I*c*d*(I + Sqrt[-(c*d) -
e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])^2*Sqrt[((Sqrt[-(c*d) - e
] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] +
I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d)
- e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x)/(1 + c*x)]))/((I + Sqrt[-(c*d) - e]/Sq
rt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(Sqrt[-(c*d) - e]/S
qrt[c*d - e] + Sqrt[(1 - c*x)/(1 + c*x)]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d
- e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*((1 + I)*EllipticF[ArcSin[Sqrt[((S
qrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[
-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-
(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (
2*I)*EllipticPi[((-I)*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]))/(-I + Sqrt[-(c*
d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(
I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I +
 Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqr
t[-(c*d) - e] - I*Sqrt[c*d - e])^2]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])
*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x
))]) - (I*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e])*e*(-I + Sqrt[(1 - c*x)/(1 +
c*x)])^2*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1
+ c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c
*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d) - e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x)/(1 + c*
x)]))/((I + Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]
))]*Sqrt[(I*(Sqrt[-(c*d) - e]/Sqrt[c*d - e] + Sqrt[(1 - c*x)/(1 + c*x)]))/(
(I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*((1
 + I)*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt
[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1
 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d)
 - e] - I*Sqrt[c*d - e])^2] - (2*I)*EllipticPi[((-I)*(I + Sqrt[-(c*d) - e]/
Sqrt[c*d - e]))/(-I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-
(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d
) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d)
 - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2]))/((I -
Sqrt[-(c*d) - e]/Sqrt[c*d - e])*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e +
 ((c*d - e)*(1 - c*x))/(1 + c*x))]) + (I*c*d*(I + Sqrt[-(c*d) - e]/Sqrt[c*d
 - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])^2*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[
c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d
- e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(-(Sqrt[-(c*d) - e]/Sqrt[c
*d - e]) + Sqrt[(1 - c*x)/(1 + c*x)]))/((I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]
```

```
)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(Sqrt[-(c*d) - e]/Sqrt[c*d - e
] + Sqrt[(1 - c*x)/(1 + c*x)]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*(-I +
 Sqrt[(1 - c*x)/(1 + c*x)]))]*((-1 + I)*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d)
 - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e
] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e]
 + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (2*I)*Ellip
ticPi[(I*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]))/(-I + Sqrt[-(c*d) - e]/Sqrt[
c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 -
 c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*
x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e]
 - I*Sqrt[c*d - e])^2]))/((I - Sqrt[-(c*d) - e]/Sqrt[c*d - e])*Sqrt[c*(1 +
(1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) + (I*(I
+ Sqrt[-(c*d) - e]/Sqrt[c*d - e])*e*(-I + Sqrt[(1 - c*x)/(1 + c*x)])^2*Sqrt
[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((S
qrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[
(I*(-(Sqrt[-(c*d) - e]/Sqrt[c*d - e]) + Sqrt[(1 - c*x)/(1 + c*x)]))/((I + S
qrt[-(c*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[(I*(
Sqrt[-(c*d) - e]/Sqrt[c*d - e] + Sqrt[(1 - c*x)/(1 + c*x)]))/((I - Sqrt[-(c
*d) - e]/Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*((-1 + I)*Ellipt
icF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(
1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 +
 c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sq
rt[c*d - e])^2] - (2*I)*EllipticPi[(I*(I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]))
/(-I + Sqrt[-(c*d) - e]/Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*
Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)]))/((Sqrt[-(c*d) - e] + I*Sqrt
[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[
c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2]))/((I - Sqrt[-(c*d) - e
]/Sqrt[c*d - e])*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1
- c*x))/(1 + c*x))])))/((1 + (1 - c*x)/(1 + c*x))*Sqrt[c + (c*(1 - c*x))/(1
 + c*x)]*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x
))/(c + (c*(1 - c*x))/(1 + c*x))]))))/(3*d*e*(c^2*d^2 - e^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(246) = 492$.

Time = 13.39 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.20

method	result
derivativedivides	$\frac{-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsech}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2c\,e^{2}\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^{2}d(ex+d)^{2}-\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsech}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2c\,e^{2}\sqrt{\frac{-c(ex+d)+cd+e}{cex}}\,x\sqrt{-\frac{-c(ex+d)+cd-e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^{2}d(ex+d)^{2}-\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$
parts	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}e}+\frac{2b\left(-\frac{\operatorname{arcsech}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2c\,e^{2}\sqrt{-\frac{c(ex+d)-cd-e}{cex}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cex}}\left(\sqrt{\frac{c}{cd+e}}\,c^{2}d(ex+d)^{2}-\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arcsech(c*x)+2/3*c*e^2*((-c
*(e*x+d)+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*((c/(c*d+e
))^(1/2)*c^2*d*(e*x+d)^2-((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*
d-e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d
-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)+c^2*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((
-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),
((c*d+e)/(c*d-e))^(1/2))*d^2*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/
2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^
(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*c^2*d^2*(e*x+d)^(1
/2)-2*(c/(c*d+e))^(1/2)*c^2*d^2*(e*x+d)+((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*
((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2
),((c*d+e)/(c*d-e))^(1/2))*c*d*e*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d+e)/(c*d+e))
^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e
))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d*e*(e*x+d)^(1/2)+(c/(c*d+e))^(1/2)*c^2
*d^3+((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*
EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/
(c/(c*d+e))^(1/2))*e^2*(e*x+d)^(1/2)-(c/(c*d+e))^(1/2)*d*e^2)/(c^2*(e*x+d)^
2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/d^2/(c/(c*d+e))^(1/2)/(c*d+e)/(c*d-e)/(e*x+d
)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*asech(c*x))/(e*x+d)**(5/2),x)

[Out] Integral((a + b*asech(c*x))/(d + e*x)**(5/2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more details)

Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex)^{5/2}}\, dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{5/2}}\, dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^(5/2),x)
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^(5/2), x)
```

3.86 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 609

$$\begin{aligned}\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx &= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} \\ &+ \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}} \\ &- \frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{16bc^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{\frac{c(d+ex)}{cd+e}}} \\ &- \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}} \\ &+ \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\left(c^2d^2-e^2\right)\sqrt{d+ex}} \\ &+ \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5d^2e\sqrt{d+ex}}\end{aligned}$$

```
[Out] -2/5*(a+b*arcsech(c*x))/e/(e*x+d)^(5/2)-16/15*b*c^3*EllipticE(1/2*(-c*x+1)^
(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e
*x+d)^(1/2)/(c^2*d^2-e^2)^2/(c*(e*x+d)/(c*d+e))^(1/2)-4/5*b*c*EllipticE(1/2
*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1
)^(1/2)*(e*x+d)^(1/2)/d^2/(c^2*d^2-e^2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/15*b*c*
EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/d/(c^2*d^2-e^2)/(e*x+d)^(1/2
```

```
)+4/5*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*
(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)/d^2/e/(e*x+d)^(1/
2)+4/15*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e
^2)/(e*x+d)^(3/2)+16/15*b*c^2*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1
)^(1/2)/(c^2*d^2-e^2)^2/(e*x+d)^(1/2)+4/5*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*d^2-e^2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6423, 972, 759, 849, 858, 733, 435, 430, 21, 946, 174, 552, 551}

$$\begin{aligned}\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = &-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}\\&+\frac{4bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\\&-\frac{4bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\\&-\frac{16bc^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{\frac{c(d+ex)}{cd+e}}}\\&+\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5d^2e\sqrt{d+ex}}\\&+\frac{16bc^2e\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}}\\&+\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}}+\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}}\end{aligned}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(15*d*(c^2*d^2
 - e^2)*(d + e*x)^(3/2)) + (16*b*c^2*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*S
qrt[1 - c^2*x^2])/(15*(c^2*d^2 - e^2)^2*Sqrt[d + e*x]) + (4*b*e*Sqrt[(1 + c
*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(5*d^2*(c^2*d^2 - e^2)*Sqrt[d +
e*x]) - (2*(a + b*ArcSech[c*x]))/(5*e*(d + e*x)^(5/2)) - (16*b*c^3*Sqrt[(1
+ c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqr
t[2]], (2*e)/(c*d + e)])/(15*(c^2*d^2 - e^2)^2*Sqrt[(c*(d + e*x))/(c*d + e)
]) - (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcS
```

```
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*(c^2*d^2 - e^2)*Sqrt[(c
*(d + e*x))/(c*d + e)]) + (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c
*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d
+ e)])/(15*d*(c^2*d^2 - e^2)*Sqrt[d + e*x]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqr
t[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*e*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6423

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSech[c*x])/(e*(m + 1))), x] + Dist[
b*(Sqrt[1 + c*x]/(e*(m + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x(d+ex)^{5/2}\sqrt{1-c^2x^2}}\,dx}{5e}\\
&= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}\\
&\quad-\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\left(-\frac{e}{d(d+ex)^{5/2}\sqrt{1-c^2x^2}}-\frac{e}{d^2(d+ex)^{3/2}\sqrt{1-c^2x^2}}+\frac{1}{d^2x\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{5e}\\
&= -\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}+\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{5d^2}\\
&\quad+\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{(d+ex)^{5/2}\sqrt{1-c^2x^2}}\,dx}{5d}\\
&\quad-\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{5d^2e}\\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}}+\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}}\\
&\quad-\frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{5d^2e}\\
&\quad-\frac{\left(4bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{5d^2\left(c^2d^2-e^2\right)}\\
&\quad-\frac{\left(4bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-\frac{3d}{2}+\frac{ex}{2}}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{15d\left(c^2d^2-e^2\right)}
\end{aligned}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{5d^2e} - \frac{\left(8bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\frac{1}{4}\left(-3c^2d^2-e^2\right)-c^2dex}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15d\left(c^2d^2-e^2\right)^2} + \frac{\left(2bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{5d^2\left(c^2d^2-e^2\right)}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} + \frac{\left(8bc^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{15\left(c^2d^2-e^2\right)^2} - \frac{\left(2bc^2(cd-e)(cd+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15d\left(c^2d^2-e^2\right)^2} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{5d^2e\sqrt{d+ex}} - \frac{\left(4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}}$$
$$+ \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$
$$- \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5d^2e\sqrt{d+ex}}$$
$$- \frac{\left(16bc^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$
$$+ \frac{\left(4bc(cd-e)(cd+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15d\left(c^2d^2-e^2\right)^2\sqrt{d+ex}}$$

$$= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d\left(c^2d^2-e^2\right)(d+ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15\left(c^2d^2-e^2\right)^2\sqrt{d+ex}}$$
$$+ \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$
$$- \frac{16bc^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\left(c^2d^2-e^2\right)\sqrt{d+ex}}$$
$$+ \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5d^2e\sqrt{d+ex}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.78 (sec) , antiderivative size = 8675, normalized size of antiderivative = 14.24

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex)^{7/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(7/2),x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1611 vs. 2(539) = 1078.

Time = 14.25 (sec) , antiderivative size = 1612, normalized size of antiderivative = 2.65

method	result	size
derivativedivides	Expression too large to display	1612
default	Expression too large to display	1612
parts	Expression too large to display	1634

```
[In] int((a+b*arcsech(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arcsech(c*x)-2/15*c*e^2*((-
c*(e*x+d)+c*d+e)/c/e/x)^(1/2)*x*(-(-c*(e*x+d)+c*d-e)/c/e/x)^(1/2)*(6*((-c*(
e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticF((
e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c^4*d^4*(e*x+d)^(3/
2)-7*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*
EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c^4*d^4*
(e*x+d)^(3/2)+3*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d
-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d
-e))^(1/2)/(c/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)-7*(c/(c*d+e))^(1/2)*c^4
*d^3*(e*x+d)^3-7*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*
d-e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/
2))*c^3*d^3*e*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+
d)+c*d-e)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)
/(c*d-e))^(1/2))*c^3*d^3*e*(e*x+d)^(3/2)+13*(c/(c*d+e))^(1/2)*c^4*d^4*(e*x+
d)^2-2*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2
)*EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c^2*d^
2*e^2*(e*x+d)^(3/2)+3*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e
)/(c*d-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e)
)^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)-6*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-
c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),
```

```
1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2
)-5*(c/(c*d+e))^(1/2)*c^4*d^5*(e*x+d)+3*(c/(c*d+e))^(1/2)*c^2*d*e^2*(e*x+d)
^3+3*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*
EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*c*d*e^3*
(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d
-e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2
))*c*d*e^3*(e*x+d)^(3/2)-(c/(c*d+e))^(1/2)*c^4*d^6-5*(c/(c*d+e))^(1/2)*c^2*
d^2*e^2*(e*x+d)^2+3*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/
(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*e^4*(e*x+d)^(3/2)+8*(c/(c*d+e))^(1/2)*c^2
*d^3*e^2*(e*x+d)+2*(c/(c*d+e))^(1/2)*c^2*d^4*e^2-3*(c/(c*d+e))^(1/2)*d*e^4*
(e*x+d)-(c/(c*d+e))^(1/2)*d^2*e^4)/(c^2*d^2-e^2)/(e*x+d)^(3/2)/(c*d-e)/(c*d
+e)/(c/(c*d+e))^(1/2)/d^3/(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)))
```

Fricas [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\int \frac{b\operatorname{arsech}(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*
e^2*x^2 + 4*d^3*e*x + d^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**(7/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex)^{7/2}}\,dx = \int \frac{b\,\operatorname{arsech}(cx) + a}{(ex + d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex)^{7/2}}\,dx = \int \frac{a + b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{7/2}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x)^(7/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x)^(7/2), x)
```

3.87 $\int (d+ex)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (d+ex)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx = \frac{(d+ex)^{1+m}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e(1+m)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\mathrm{Int}\left(\frac{(d+ex)^{1+m}}{x\sqrt{1-c^2x^2}},x\right)}{e(1+m)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arcsech(c*x))/e/(1+m)+b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
Unintegrable((e*x+d)^(1+m)/x/(-c^2*x^2+1)^(1/2),x)/e/(1+m)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d+ex)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx = \int (d+ex)^m \left(a+b\,\mathrm{sech}^{-1}(cx)\right) dx$$

```
[In] Int[(d + e*x)^m*(a + b*ArcSech[c*x]),x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcSech[c*x]))/(e*(1 + m)) + (b*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Defer[Int][(d + e*x)^(1 + m)/(x*Sqrt[1 - c^2*x^2]), x])/(
e*(1 + m))
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^{1+m}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e(1+m)} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(d+ex)^{1+m}}{x\sqrt{1-c^2x^2}}\,dx}{e(1+m)}$$

Mathematica [N/A]

Not integrable

Time = 21.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(d + e*x)^m*(a + b*ArcSech[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(e x +d \right)^{m} \left(a +b \,\operatorname{arcsech}\left(c x \right)\right)d x$$

```
[In] int((e*x+d)^m*(a+b*arcsech(c*x)),x)
[Out] int((e*x+d)^m*(a+b*arcsech(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\int { {\left(b \operatorname{arsech}\left(c x\right) + a\right)} {\left(e x + d\right)}^{m} \,d x}$$

```
[In] integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)*(e*x + d)^m, x)
```

Sympy [N/A]

Not integrable

Time = 13.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (d+ex)^m \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\int \left(a + b \operatorname{asech}{\left(c x \right)}\right) \left(d + e x\right)^{m}\, dx$$

```
[In] integrate((e*x+d)**m*(a+b*asech(c*x)),x)
[Out] Integral((a + b*asech(c*x))*(d + e*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 222, normalized size of antiderivative = 13.88

$$\int (d+ex)^m \left(a+b\operatorname{sech}^{-1}(cx)\right) \, dx = \int (b \operatorname{arsech}(cx)+a)(ex+d)^m \, dx$$

```
[In] integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] b*(((e*x + d)*(e*x + d)^m*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - (e*x + d)
*(e*x + d)^m*log(x))/(e*(m + 1)) - integrate((c^2*e*(m + 1)*x^3*log(c) - (e
*(m + 1)*log(c) - e)*x + d)*(e*x + d)^m/(c^2*e*(m + 1)*x^3 - e*(m + 1)*x),
x) + integrate((c^2*e*x^2 + c^2*d*x)*(e*x + d)^m/(c^2*e*(m + 1)*x^2 + (c^2*
e*(m + 1)*x^2 - e*(m + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1) - e*(m + 1)), x)) +
 (e*x + d)^(m + 1)*a/(e*(m + 1))
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (d+ex)^m \left(a+b\operatorname{sech}^{-1}(cx)\right) \, dx = \int (b \operatorname{arsech}(cx)+a)(ex+d)^m \, dx$$

```
[In] integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*(e*x + d)^m, x)
```

Mupad [N/A]

Not integrable

Time = 4.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (d+ex)^m \left(a+b\operatorname{sech}^{-1}(cx)\right) \, dx = \int \left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^m \,d x$$

```
[In] int((a + b*acosh(1/(c*x)))*(d + e*x)^m,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))*(d + e*x)^m, x)
```

3.88 $\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 229

$$\int x^4\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=-\frac{b(42c^2d+25e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{560c^6}-\frac{b(42c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4}-\frac{bex^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2}+\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{b(42c^2d+25e)\,\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{560c^7}$$

```
[Out] 1/5*d*x^5*(a+b*arcsech(c*x))+1/7*e*x^7*(a+b*arcsech(c*x))+1/560*b*(42*c^2*d
+25*e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^7-1/560*b*(42*c^2*d+25
*e)*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6-1/840*b*(42*c^
2*d+25*e)*x^3*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/42*b
*e*x^5*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 6436, 12, 470, 327, 222}

$$\int x^4\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)\left(42c^2d+25e\right)}{560c^7}-\frac{bex^5\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{42c^2}-\frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(42c^2d+25e)}{560c^6}-\frac{bx^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(42c^2d+25e)}{840c^4}$$

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/560*(b*(42*c^2*d + 25*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c
^2*x^2])/c^6 - (b*(42*c^2*d + 25*e)*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2])/(840*c^4) - (b*e*x^5*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2])/(42*c^2) + (d*x^5*(a + b*ArcSech[c*x]))/5 + (e*x^7*(a +
b*ArcSech[c*x]))/7 + (b*(42*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x
]*ArcSin[c*x])/(560*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^4(7d+5ex^2)}{35\sqrt{1-c^2x^2}}\,dx \\
&= \frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad+\frac{1}{35}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^4(7d+5ex^2)}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{bex^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2}+\frac{1}{5}dx^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad+\frac{1}{210}\left(b\left(42d+\frac{25e}{c^2}\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^4}{\sqrt{1-c^2x^2}}\,dx
\end{aligned}$$

$$= -\frac{b(42c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4} - \frac{bex^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} + \frac{1}{5}dx^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{\left(b\left(42d+\frac{25e}{c^2}\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{280c^2}$$

$$= -\frac{b(42c^2d+25e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{560c^6} - \frac{b(42c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4} - \frac{bex^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} + \frac{1}{5}dx^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{\left(b\left(42d+\frac{25e}{c^2}\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}dx}{560c^4}$$

$$= -\frac{b(42c^2d+25e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{560c^6} - \frac{b(42c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4} - \frac{bex^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} + \frac{1}{5}dx^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{b(42c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{560c^7}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.71

$$\int x^4\left(d+ex^2\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$= \frac{48ac^7x^5(7d+5ex^2) - bcx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(75e+2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4)\right)+48bc^7x^5(7d+5e}{1680c^7}$$

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) - b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(
75*e + 2*c^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*
(7*d + 5*e*x^2)*ArcSech[c*x] + (3*I)*b*(42*c^2*d + 25*e)*Log[(-2*I)*c*x + 2
*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(1680*c^7)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.93

method	result
parts	$a\left(\frac{1}{7}e\,x^{7}+\frac{1}{5}d\,x^{5}\right)+\frac{b\left(\frac{c^{5}\operatorname{arcsech}(cx)e\,x^{7}}{7}+\frac{\operatorname{arcsech}(cx)x^{5}c^{5}d}{5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(-40e\sqrt{-c^{2}x^{2}+1}\,c^{5}x^{5}-84c^{5}d\sqrt{-c^{2}x\right.}{}}{}\right)}{}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d\,c^{7}x^{5}}{5}+\frac{\operatorname{arcsech}(cx)e\,c^{7}x^{7}}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(84c^{5}d\sqrt{-c^{2}x^{2}+1}\,x^{3}+40e\sqrt{-c^{2}x^{2}+1}\,c^{5}x\right.}{}\right)}{c^{2}}}{c^{5}}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d\,c^{7}x^{5}}{5}+\frac{\operatorname{arcsech}(cx)e\,c^{7}x^{7}}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,\left(84c^{5}d\sqrt{-c^{2}x^{2}+1}\,x^{3}+40e\sqrt{-c^{2}x^{2}+1}\,c^{5}x\right.}{}\right)}{c^{2}}}{c^{5}}$

```
[In] int(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*e*x^7+1/5*d*x^5)+b/c^5*(1/7*c^5*arcsech(c*x)*e*x^7+1/5*arcsech(c*x)*
x^5*c^5*d+1/1680/c*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-40*e*(-c^2*
x^2+1)^(1/2)*c^5*x^5-84*c^5*d*(-c^2*x^2+1)^(1/2)*x^3-50*e*c^3*x^3*(-c^2*x^2
+1)^(1/2)-126*d*c^3*x*(-c^2*x^2+1)^(1/2)+126*d*c^2*arcsin(c*x)-75*e*c*x*(-c
^2*x^2+1)^(1/2)+75*e*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.13

$$\int x^4(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7ex^7+336\,ac^7dx^5-6\,(42\,bc^2d+25\,be)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-48\,(7\,bc^7d+5\,bc^7e)\log\left(\frac{cx\sqrt{-\frac{c^2x^2}{c^2x}}}{x}\right.}{}$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 - 6*(42*b*c^2*d + 25*b*e)*arctan(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 48*(7*b*c^7*d + 5*b*c^7*e
)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 48*(5*b*c^7*e*x^7 + 7*b
*c^7*d*x^5 - 7*b*c^7*d - 5*b*c^7*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
 + 1)/(c*x)) - (40*b*c^6*e*x^6 + 2*(42*b*c^6*d + 25*b*c^4*e)*x^4 + 3*(42*b*
c^4*d + 25*b*c^2*e)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

Sympy [F]

$$\int x^4(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int x^4(a+b\operatorname{asech}(cx))\left(d+ex^2\right)\,dx$$

```
[In] integrate(x**4*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**4*(a + b*asech(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07

$$\int x^4(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{1}{7}aex^7+\frac{1}{5}adx^5$$
$$+\frac{1}{40}\left(8x^5\operatorname{arsech}(cx)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}+\frac{3\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)bd$$
$$+\frac{1}{336}\left(48x^7\operatorname{arsech}(cx)-\frac{\frac{15\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+40\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}-1}}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3c^6\left(\frac{1}{c^2x^2}-1\right)^2+3c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^6}}{c}\right)be$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1
)^(3/2) + 5*sqrt(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2
*x^2) - 1) + c^4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*d + 1/336*(48
*x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40*(1/(c^2*x^2) - 1)^(3/
2) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2
) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arctan(sqrt(1/(c^2*x^2) - 1)
)/c^6)/c)*b*e
```

Giac [F]

$$\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int(ex^2+d)(b\,\text{arsech}\,(cx)+a)x^4\,dx$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^4\,\left(e\,x^2+d\right)\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^4*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
[Out] int(x^4*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.89 $\int x^2(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 174

$$\int x^2\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=-\frac{b(20c^2d+9e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4}-\frac{bex^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2}+\frac{1}{3}dx^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{b(20c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{120c^5}$$

```
[Out] 1/3*d*x^3*(a+b*arcsech(c*x))+1/5*e*x^5*(a+b*arcsech(c*x))+1/120*b*(20*c^2*d
+9*e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5-1/120*b*(20*c^2*d+9*e
)*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/20*b*e*x^3*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

= {14, 6436, 12, 470, 327, 222}

$$\int x^2\left(d+ex^2\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{1}{3}dx^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)\left(20c^2d+9e\right)}{120c^5}-\frac{bex^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{20c^2}-\frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(20c^2d+9e)}{120c^4}$$

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/120*(b*(20*c^2*d + 9*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^
2*x^2])/c^4 - (b*e*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]
)/(20*c^2) + (d*x^3*(a + b*ArcSech[c*x]))/3 + (e*x^5*(a + b*ArcSech[c*x]))/
5 + (b*(20*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(12
0*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}dx^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2(5d+3ex^2)}{15\sqrt{1-c^2x^2}}\,dx \\
&= \frac{1}{3}dx^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad+\frac{1}{15}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2(5d+3ex^2)}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{bex^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2}+\frac{1}{3}dx^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad+\frac{1}{60}\left(b\left(20d+\frac{9e}{c^2}\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{b(20c^2d+9e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4}-\frac{bex^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2} \\
&\quad+\frac{1}{3}dx^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad+\frac{\left(b\left(20d+\frac{9e}{c^2}\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx}{120c^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b(20c^2d+9e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4} \\
&\quad - \frac{bex^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2} + \frac{1}{3}dx^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right) \\
&\quad + \frac{1}{5}ex^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right) + \frac{b(20c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\arcsin(cx)}{120c^5}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

$$
\int x^2\left(d+ex^2\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx
$$

$$
= \frac{8ac^5x^3(5d+3ex^2) - bcx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(9e+c^2(20d+6ex^2)\right) + 8bc^5x^3(5d+3ex^2)\,\mathrm{sech}^{-1}(cx) + ib(20c^2d
}{120c^5}
$$

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (8*a*c^5*x^3*(5*d + 3*e*x^2) - b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(9
*e + c^2*(20*d + 6*e*x^2)) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcSech[c*x] + I*b
*(20*c^2*d + 9*e)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/
(120*c^5)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b\left(\frac{c^3\,\mathrm{arcsech}(cx)e\,x^5}{5}+\frac{\mathrm{arcsech}(cx)x^3c^3d}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(-6e\,c^3x^3\sqrt{-c^2x^2+1}-20d\,c^3x\sqrt{-c^2}\right.}{120c\sqrt{-}}\right.}{c^3}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\mathrm{arcsech}(cx)d\,c^5x^3}{3}+\frac{\mathrm{arcsech}(cx)e\,c^5x^5}{5}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(20d\,c^3x\sqrt{-c^2x^2+1}+6e\,c^3x^3\sqrt{-c^2x^2+1}-\right.}{120\sqrt{-c^2x^2+}}\right.}{c^2}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\mathrm{arcsech}(cx)d\,c^5x^3}{3}+\frac{\mathrm{arcsech}(cx)e\,c^5x^5}{5}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(20d\,c^3x\sqrt{-c^2x^2+1}+6e\,c^3x^3\sqrt{-c^2x^2+1}-\right.}{120\sqrt{-c^2x^2+}}\right.}{c^2}}{c^3}$

```
[In] int(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*e*x^5+1/3*d*x^3)+b/c^3*(1/5*c^3*arcsech(c*x)*e*x^5+1/3*arcsech(c*x)*
x^3*c^3*d+1/120/c*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-6*e*c^3*x^3*
```

```
(-c^2*x^2+1)^(1/2)-20*d*c^3*x*(-c^2*x^2+1)^(1/2)+20*d*c^2*arcsin(c*x)-9*e*c
*x*(-c^2*x^2+1)^(1/2)+9*e*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(100) = 200$.

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.37

$$\int x^2(d+ex^2)\left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx = \frac{24\,ac^5ex^5+40\,ac^5dx^3-2\,(20\,bc^2d+9\,be)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-8\,(5\,bc^5d+3\,bc^5e)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)}{12}$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 - 2*(20*b*c^2*d + 9*b*e)*arctan((c*x
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 8*(5*b*c^5*d + 3*b*c^5*e)*log
((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d
*x^3 - 5*b*c^5*d - 3*b*c^5*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x)) - (6*b*c^4*e*x^4 + (20*b*c^4*d + 9*b*c^2*e)*x^2)*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)))/c^5
```

Sympy [F]

$$\int x^2(d+ex^2)\left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx = \int x^2(a+b\,\mathrm{asech}\,(cx))\left(d+ex^2\right)\,dx$$

```
[In] integrate(x**2*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int x^2(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{5}\,aex^5+\frac{1}{3}\,adx^3+\frac{1}{6}\left(2\,x^3\,\text{arsech}\,(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bd$$
$$+\frac{1}{40}\left(8\,x^5\,\text{arsech}\,(cx)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}+\frac{3\,\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)be$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1
)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*d
 + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqrt(1/(c^2*x
^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) + 3*arc
tan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*e
```

Giac [F]

$$\int x^2(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (ex^2+d)(b\,\text{arsech}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.90 $\int (d+ex^2) (a+b\text{sech}^{-1}(cx)) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int (d+ex^2) (a+b\text{sech}^{-1}(cx)) \, dx = -\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + dx(a+b\text{sech}^{-1}(cx)) + \frac{1}{3}ex^3(a+b\text{sech}^{-1}(cx)) + \frac{b(6c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3}$$

```
[Out] d*x*(a+b*arcsech(c*x))+1/3*e*x^3*(a+b*arcsech(c*x))+1/6*b*(6*c^2*d+e)*arcsi
n(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3-1/6*b*e*x*(1/(c*x+1))^(1/2)*(c*x
+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6426, 12, 396, 222}

$$\int (d+ex^2) (a+b\text{sech}^{-1}(cx)) \, dx = dx(a+b\text{sech}^{-1}(cx)) + \frac{1}{3}ex^3(a+b\text{sech}^{-1}(cx)) + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)(6c^2d+e)}{6c^3} - \frac{bex\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{6c^2}$$

```
[In] Int[(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/6*(b*e*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/c^2 + d*x
*(a + b*ArcSech[c*x]) + (e*x^3*(a + b*ArcSech[c*x]))/3 + (b*(6*c^2*d + e)*S
qrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 6426

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x
] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt
[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ
[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= dx\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad +\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{3d+ex^2}{3\sqrt{1-c^2x^2}}\,dx \\
&= dx\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{3d+ex^2}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2}+dx\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad +\frac{1}{3}ex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{\left(b(6c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx}{6c^2}
\end{aligned}$$

$$= -\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + dx\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{b(6c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.69

$$\int \left(d+ex^2\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = adx + \frac{1}{3}aex^3 + be\sqrt{\frac{1-cx}{1+cx}}\left(-\frac{x}{6c^2}-\frac{x^2}{6c}\right) + bdx\operatorname{sech}^{-1}(cx) + \frac{1}{3}bex^3\operatorname{sech}^{-1}(cx) + \frac{2bd\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x} + \frac{ibe\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{6c^3}$$

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/6*x/c^2 - x^2/(6*c)
) + b*d*x*ArcSech[c*x] + (b*e*x^3*ArcSech[c*x])/3 + (2*b*d*Sqrt[(1 - c*x)/(
1 + c*x)]*Sqrt[1 - c^2*x^2]*ArcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^2*x
) + ((I/6)*b*e*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^3
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

method	result
parts	$a\left(\frac{1}{3}e\,x^3+dx\right)+\frac{b\Big(\frac{c\ \operatorname{arcsech}(cx)e\,x^3}{3}+\operatorname{arcsech}(cx)dxc+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\big(6d\,c^2\arcsin(cx)-ecx\sqrt{-c^2x^2+1}+e\arcsin(cx)}{6c\sqrt{-c^2x^2+1}}}{c}$
derivativedivides	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arcsech}(cx)d\,c^3x+\frac{\operatorname{arcsech}(cx)e\,c^3x^3}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(6d\,c^2\arcsin(cx)-ecx\sqrt{-c^2x^2+1}+e\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^2}}{c}$
default	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arcsech}(cx)d\,c^3x+\frac{\operatorname{arcsech}(cx)e\,c^3x^3}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(6d\,c^2\arcsin(cx)-ecx\sqrt{-c^2x^2+1}+e\arcsin(cx)\right)}{6\sqrt{-c^2x^2+1}}\right)}{c^2}}{c}$

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*e*x^3+d*x)+b/c*(1/3*c*arcsech(c*x)*e*x^3+arcsech(c*x)*d*x*c+1/6/c*(-
(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(6*d*c^2*arcsin(c*x)-e*c*x*(-c^2*x
^2+1)^(1/2)+e*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(64) = 128$.

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.87

$$\int \left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$= \frac{2\,ac^3ex^3 - bc^2ex^2\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + 6\,ac^3dx - 2\,(6\,bc^2d+be)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) - 2\,(3\,bc^3d+bc^3e)\log\left(\cdots\right.}{6\,c^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e*x^3 - b*c^2*e*x^2*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 6*a*c^3*d
*x - 2*(6*b*c^2*d + b*e)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c
*x)) - 2*(3*b*c^3*d + b*c^3*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)
/x) + 2*(b*c^3*e*x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*log((c*x*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c^3
```

Sympy [F]

$$\int \left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{asech}\,(cx)\right)\left(d+ex^2\right)\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$= \frac{1}{3}\,aex^3 + \frac{1}{6}\left(2\,x^3\,\text{arsech}\,(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be$$
$$+ adx + \frac{\left(cx\,\text{arsech}\,(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd}{c}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2
*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*e + a*d*x + (c*
x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d/c
```

Giac [F]

$$\int \left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)\left(b\,\text{arsech}\left(cx\right)+a\right)dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(e\,x^2+d\right)\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.91 $\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)}{x} + ex\left(a+b\,\text{sech}^{-1}(cx)\right) + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}$$

```
[Out] -d*(a+b*arcsech(c*x))/x+e*x*(a+b*arcsech(c*x))+b*e*arcsin(c*x)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)/c+b*d*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2
)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {14, 6436, 462, 222}

$$\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)}{x} + ex\left(a+b\,\text{sech}^{-1}(cx)\right) + \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{c} + \frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{x}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^2,x]
```

```
[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/x - (d*(a + b*Ar
cSech[c*x]))/x + e*x*(a + b*ArcSech[c*x]) + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[
1 + c*x]*ArcSin[c*x])/c
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}+ex\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-d+ex^2}{x^2\sqrt{1-c^2x^2}}\,dx \\
&= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x}-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} \\
&\quad+ex\left(a+b\operatorname{sech}^{-1}(cx)\right)+\left(be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx
\end{aligned}
$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} + ex\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = -\frac{ad}{x} + aex + bd\left(c+\frac{1}{x}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{bd\operatorname{sech}^{-1}(cx)}{x} + bex\operatorname{sech}^{-1}(cx) + \frac{2be\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\arctan\left(\frac{\sqrt{1-c^2x^2}}{1-cx}\right)}{c-c^2x}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x + b*d*(c + x^(-1))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*d*ArcS
ech[c*x])/x + b*e*x*ArcSech[c*x] + (2*b*e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1
- c^2*x^2]*ArcTan[Sqrt[1 - c^2*x^2]/(1 - c*x)])/(c - c^2*x)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

method	result	s
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(\frac{\operatorname{arcsech}(cx)ex}{c}-\frac{\operatorname{arcsech}(cx)d}{xc}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(\sqrt{-c^2x^2+1}\,c^2d+\arcsin(cx)ecx\right)}{c^2\sqrt{-c^2x^2+1}}\right)$	1
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsech}(cx)ex-\frac{\operatorname{arcsech}(cx)dc}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(\sqrt{-c^2x^2+1}\,c^2d+\arcsin(cx)ecx\right)}{\sqrt{-c^2x^2+1}}\right)}{c^2}\right)$	1
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsech}(cx)ex-\frac{\operatorname{arcsech}(cx)dc}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(\sqrt{-c^2x^2+1}\,c^2d+\arcsin(cx)ecx\right)}{\sqrt{-c^2x^2+1}}\right)}{c^2}\right)$	1

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(e*x-d/x)+b*c*(1/c*arcsech(c*x)*e*x-arcsech(c*x)*d/x/c+1/c^2*(-(c*x-1)/c/
x)^(1/2)*((c*x+1)/c/x)^(1/2)*((-c^2*x^2+1)^(1/2)*c^2*d+arcsin(c*x)*e*c*x)/(
-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(54) = 108$.

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.90

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx$$
$$= \frac{bc^2dx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + acex^2 - 2\,bex\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) - acd + (bcd-bce)x\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) + (bcex}{cx}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (b*c^2*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + a*c*e*x^2 - 2*b*e*x*arctan((c*x
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - a*c*d + (b*c*d - b*c*e)*x*log
((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c*e*x^2 - b*c*d + (b*c*d
- b*c*e)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)}{x^2}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**2,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx = \left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\operatorname{arsech}(cx)}{x}\right)bd + aex + \frac{\left(cx\operatorname{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)be}{c} - \frac{ad}{x}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d + a*e*x + (c*x*arcsech(c*x)
- arctan(sqrt(1/(c^2*x^2) - 1)))*b*e/c - a*d/x
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{(ex^2+d)(b\,\text{arsech}\,(cx)+a)}{x^2}\,dx$$

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = a\,e\,x-\frac{a\,d}{x}+b\,c\,d\left(\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}-\frac{\text{acosh}\left(\frac{1}{c\,x}\right)}{c\,x}\right)+\frac{b\,e\,\text{atan}\left(\frac{1}{\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}\right)}{c}+b\,e\,x\,\text{acosh}\left(\frac{1}{c\,x}\right)$$

[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^2,x)

[Out] a*e*x - (a*d)/x + b*c*d*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) - acosh(1/(c*x))/(c*x)) + (b*e*atan(1/((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))))/c + b*e*x*acosh(1/(c*x))

3.92 $\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3} + \frac{b(2c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x} - \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x}$$

```
[Out] -1/3*d*(a+b*arcsech(c*x))/x^3-e*(a+b*arcsech(c*x))/x+1/9*b*d*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+1/9*b*(2*c^2*d+9*e)*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6436, 12, 464, 270}

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(2c^2d+9e)}{9x} + \frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{9x^3}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x^3) + (b*(2*
c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x) -
(d*(a + b*ArcSech[c*x]))/(3*x^3) - (e*(a + b*ArcSech[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{x} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-d-3ex^2}{3x^4\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{d(a+b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{x} + \frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-d-3ex^2}{x^4\sqrt{1-c^2x^2}}\,dx$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3} - \frac{d(a+b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{x} + \frac{1}{9}\left(b(-2c^2d-9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{1-c^2x^2}}\,dx$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3} + \frac{b(2c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x} - \frac{d(a+b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{x}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex^2)\,(a+b\operatorname{sech}^{-1}(cx))}{x^4}\,dx = \frac{-3a(d+3ex^2)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\,(d+2c^2dx^2+9ex^2)-3b(d+3ex^2)\operatorname{sech}^{-1}(cx)}{9x^3}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^4,x]
[Out] (-3*a*(d + 3*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + 2*c^2*d*x^
2 + 9*e*x^2) - 3*b*(d + 3*e*x^2)*ArcSech[c*x])/(9*x^3)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result	size
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\operatorname{arcsech}(cx)e}{c^3x}-\frac{\operatorname{arcsech}(cx)d}{3x^3c^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(2c^4d\,x^2+9e\,c^2x^2+c^2d)}{9c^4x^2}\right)$	110
derivativedivides	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e}{cx}-\frac{\operatorname{arcsech}(cx)d}{3c\,x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(2c^4d\,x^2+9e\,c^2x^2+c^2d)}{9c^2x^2}\right)}{c^2}\right)$	123
default	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e}{cx}-\frac{\operatorname{arcsech}(cx)d}{3c\,x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(2c^4d\,x^2+9e\,c^2x^2+c^2d)}{9c^2x^2}\right)}{c^2}\right)$	123

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arcsech(c*x)*e/x-1/3*arcsech(c*x)*d/x^3/c^
3+1/9/c^4*(-(c*x-1)/c/x)^(1/2)/x^2*((c*x+1)/c/x)^(1/2)*(2*c^4*d*x^2+9*c^2*e
*x^2+c^2*d))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = -\frac{9\,aex^2+3\,ad+3\,(3\,bex^2+bd)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(bcdx+(2\,bc^3d+9\,bce)x^3)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{9\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*log((c*x*sqrt(-(c^2*x^2 - 1)/
(c^2*x^2)) + 1)/(c*x)) - (b*c*d*x + (2*b*c^3*d + 9*b*c*e)*x^3)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)))/x^3
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)}{x^4}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx = \left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\text{arsech}\,(cx)}{x}\right)be + \frac{1}{9}\,bd\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\,\text{arsech}\,(cx)}{x^3}\right)-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*e + 1/9*b*d*((c^4*(1/(c^2*x^2)
 - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3) - a*e/x
- 1/3*a*d/x^3
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{(ex^2+d)(b\,\operatorname{arsech}(cx)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^4, x)
```

3.93 $\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 183

$$\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} + \frac{b(12c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3} + \frac{2bc^2(12c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x} - \frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\,\text{sech}^{-1}(cx)\right)}{3x^3}$$

```
[Out] -1/5*d*(a+b*arcsech(c*x))/x^5-1/3*e*(a+b*arcsech(c*x))/x^3+1/25*b*d*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^5+1/225*b*(12*c^2*d+25*e)*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+2/225*b*c^2*(12*c^2*d+2
5*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

= {14, 6436, 12, 464, 277, 270}

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx = -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5} - \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3} + \frac{2bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(12c^2d+25e)}{225x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(12c^2d+25e)}{225x^3} + \frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{25x^5}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(25*x^5) + (b*(1
2*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*
x^3) + (2*b*c^2*(12*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1
 - c^2*x^2])/(225*x) - (d*(a + b*ArcSech[c*x]))/(5*x^5) - (e*(a + b*ArcSech
[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-3d-5ex^2}{15x^6\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3}+\frac{1}{15}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-3d-5ex^2}{x^6\sqrt{1-c^2x^2}}\,dx \\
&= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5}-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3} \\
&\quad+\frac{1}{75}\left(b\left(-12c^2d-25e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^4\sqrt{1-c^2x^2}}\,dx \\
&= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5}+\frac{b(12c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3} \\
&\quad-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3} \\
&\quad+\frac{1}{225}\left(2bc^2\left(-12c^2d-25e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^2\sqrt{1-c^2x^2}}\,dx
\end{aligned}$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} + \frac{b(12c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3} + \frac{2bc^2(12c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x} - \frac{d(a+b\operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{3x^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx = \frac{-15a(3d+5ex^2)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(25ex^2(1+2c^2x^2)+3d(3+4c^2x^2+8c^4x^4)\right)-15b(3d+5ex^2)\operatorname{sech}^{-1}(}{225x^5}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d + 5*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(25*e*x^2*(1
 + 2*c^2*x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*Arc
Sech[c*x])/(225*x^5)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

method	result
parts	$a\left(-\frac{e}{3x^3}-\frac{d}{5x^5}\right)+b\,c^5\left(-\frac{\operatorname{arcsech}(cx)e}{3c^5x^3}-\frac{\operatorname{arcsech}(cx)d}{5x^5c^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25}{225c^6x^4}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e}{3c^3x^3}-\frac{\operatorname{arcsech}(cx)d}{5c^3x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25e\,c^2x^2+9c^2\right.}{225c^4x^4}\right.}{c^2}\right.$
default	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e}{3c^3x^3}-\frac{\operatorname{arcsech}(cx)d}{5c^3x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25e\,c^2x^2+9c^2\right.}{225c^4x^4}\right.}{c^2}\right.$

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/3*e/x^3-1/5*d/x^5)+b*c^5*(-1/3/c^5*arcsech(c*x)*e/x^3-1/5*arcsech(c*x
)*d/x^5/c^5+1/225/c^6*(-(c*x-1)/c/x)^(1/2)/x^4*((c*x+1)/c/x)^(1/2)*(24*c^6*
d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+25*c^2*e*x^2+9*c^2*d))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = -\frac{75\,aex^2+45\,ad+15\,(5\,bex^2+3\,bd)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(2\,(12\,bc^5d+25\,bc^3e)x^5+9\,bcdx+(12\,bc^3d}{225\,x^5}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
[Out] -1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*log((c*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*(12*b*c^5*d + 25*b*c^3*e)*x^5 + 9*b*c*d*
x + (12*b*c^3*d + 25*b*c*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^5
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)}{x^6}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**6,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.72

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx &= \frac{1}{75}\,bd\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\,\text{arsech}\,(cx)}{x^5}\right)\\&\quad+\frac{1}{9}\,be\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\,\text{arsech}\,(cx)}{x^3}\right)-\frac{ae}{3\,x^3}-\frac{ad}{5\,x^5}\end{aligned}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
[Out] 1/75*b*d*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) +
 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) + 1/9*b*e*((c^4*(1/
(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3)
 - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(ex^2+d)(b\,\text{arsech}\,(cx)+a)}{x^6}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^6,x)
[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^6, x)
```

3.94 $\int \frac{(d+ex^2)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 238

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^8}\,dx = {} & \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} \\ & + \frac{b(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5} \\ & + \frac{4bc^2(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3675x^3} \\ & + \frac{8bc^4(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3675x} \\ & - \frac{d\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5x^5}\end{aligned}$$

```
[Out] -1/7*d*(a+b*arcsech(c*x))/x^7-1/5*e*(a+b*arcsech(c*x))/x^5+1/49*b*d*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^7+1/1225*b*(30*c^2*d+49*e)*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^5+4/3675*b*c^2*(30*c^2*d
+49*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+8/3675*b*c^4*
(30*c^2*d+49*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 6436, 12, 464, 277, 270}

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx = -\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(30c^2d+49e)}{1225x^5} + \frac{4bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(30c^2d+49e)}{3675x^3} + \frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{49x^7} + \frac{8bc^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(30c^2d+49e)}{3675x}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(49*x^7) + (b*(3
0*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(1225
*x^5) + (4*b*c^2*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[
1 - c^2*x^2])/(3675*x^3) + (8*b*c^4*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3675*x) - (d*(a + b*ArcSech[c*x]))/(7*x^7
) - (e*(a + b*ArcSech[c*x]))/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-5d-7ex^2}{35x^8\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}+\frac{1}{35}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-5d-7ex^2}{x^8\sqrt{1-c^2x^2}}\,dx \\
&= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7}-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5} \\
&\quad+\frac{1}{245}\left(b\left(-30c^2d-49e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^6\sqrt{1-c^2x^2}}\,dx
\end{aligned}$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} + \frac{b(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5} - \frac{d\big(a+b\text{sech}^{-1}(cx)\big)}{7x^7} - \frac{e\big(a+b\text{sech}^{-1}(cx)\big)}{5x^5} + \frac{\left(4bc^2(-30c^2d-49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^4\sqrt{1-c^2x^2}}\,dx}{1225}$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} + \frac{b(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5} + \frac{4bc^2(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3675x^3} - \frac{d\big(a+b\text{sech}^{-1}(cx)\big)}{7x^7} - \frac{e\big(a+b\text{sech}^{-1}(cx)\big)}{5x^5} + \frac{\left(8bc^4(-30c^2d-49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^2\sqrt{1-c^2x^2}}\,dx}{3675}$$

$$= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} + \frac{b(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5} + \frac{4bc^2(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3675x^3} + \frac{8bc^4(30c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3675x} - \frac{d\big(a+b\text{sech}^{-1}(cx)\big)}{7x^7} - \frac{e\big(a+b\text{sech}^{-1}(cx)\big)}{5x^5}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex^2)\,(a+b\text{sech}^{-1}(cx))}{x^8}\,dx$$

$$= \frac{-105a(5d+7ex^2)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(49ex^2(3+4c^2x^2+8c^4x^4)+15d(5+6c^2x^2+8c^4x^4+16c^6x^6)\right)-10}{3675x^7}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (-105*a*(5*d + 7*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(49*e*x^2*(
3 + 4*c^2*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6))
 - 105*b*(5*d + 7*e*x^2)*ArcSech[c*x])/(3675*x^7)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.62

method	result
parts	$a\left(-\frac{e}{5x^5}-\frac{d}{7x^7}\right)+b\,c^7\left(-\frac{\operatorname{arcsech}(cx)e}{5c^7x^5}-\frac{\operatorname{arcsech}(cx)d}{7x^7c^7}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x\right.}{3675c^8x}$
derivativedivides	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)d}{7c^5x^7}-\frac{\operatorname{arcsech}(cx)e}{5c^5x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x}{3675c^6x^6}\right.}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)d}{7c^5x^7}-\frac{\operatorname{arcsech}(cx)e}{5c^5x^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x}{3675c^6x^6}\right.}{c^2}\right.$

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/5*e/x^5-1/7*d/x^7)+b*c^7*(-1/5/c^7*arcsech(c*x)*e/x^5-1/7*arcsech(c*x
)*d/x^7/c^7+1/3675/c^8*(-(c*x-1)/c/x)^(1/2)/x^6*((c*x+1)/c/x)^(1/2)*(240*c^
8*d*x^6+392*c^6*e*x^6+120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e*x^
2+75*c^2*d))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx =$$
$$-\frac{735\,aex^2+525\,ad+105\,(7\,bex^2+5\,bd)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(8\,(30\,bc^7d+49\,bc^5e)x^7+4\,(30\,bc^5d+4}{3675\,x^7}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*log((c*x*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*(30*b*c^7*d + 49*b*c^5*e)*x^7 + 4*(3
0*b*c^5*d + 49*b*c^3*e)*x^5 + 75*b*c*d*x + 3*(30*b*c^3*d + 49*b*c*e)*x^3)*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^7
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(a+b\,\mathrm{asech}\,(cx))\,(d+ex^2)}{x^8}\,dx$$

[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**8,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**8, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx \\
&= \frac{1}{245}bd\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}+21\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+35\,c^8\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{35\,\mathrm{arsech}\,(cx)}{x^7}\right) \\
&\quad+\frac{1}{75}be\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10\,c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\,\mathrm{arsech}\,(cx)}{x^5}\right) \\
&\quad-\frac{ae}{5\,x^5}-\frac{ad}{7\,x^7}
\end{aligned}$$

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")

[Out] 1/245*b*d*((5*c^8*(1/(c^2*x^2) - 1)^(7/2) + 21*c^8*(1/(c^2*x^2) - 1)^(5/2) + 35*c^8*(1/(c^2*x^2) - 1)^(3/2) + 35*c^8*sqrt(1/(c^2*x^2) - 1))/c - 35*arcsech(c*x)/x^7) + 1/75*b*e*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) - 1/5*a*e/x^5 - 1/7*a*d/x^7

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arsech}\,(cx)+a)}{x^8}\,dx$$

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^8, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^8, x)
```

3.95 $\int x^5(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 232

$$\int x^5\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=-\frac{b(4c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{24c^8}+\frac{b(8c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\left(1-c^2x^2\right)^{3/2}}{72c^8}-\frac{b(4c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\left(1-c^2x^2\right)^{5/2}}{120c^8}+\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\left(1-c^2x^2\right)^{7/2}}{56c^8}+\frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right)$$

```
[Out] 1/6*d*x^6*(a+b*arcsech(c*x))+1/8*e*x^8*(a+b*arcsech(c*x))+1/72*b*(8*c^2*d+9
*e)*(-c^2*x^2+1)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/120*b*(4*c^2*d
+9*e)*(-c^2*x^2+1)^(5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8+1/56*b*e*(-c^2
*x^2+1)^(7/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/24*b*(4*c^2*d+3*e)*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^8
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6436, 12, 457, 78}

$$\int x^5(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)dx=\frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right)-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{5/2}(4c^2d+9e)}{120c^8}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}(8c^2d+9e)}{72c^8}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(4c^2d+3e)}{24c^8}+\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{7/2}}{56c^8}$$

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/24*(b*(4*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^
2])/c^8 + (b*(8*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^
2)^(3/2))/(72*c^8) - (b*(4*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
(1 - c^2*x^2)^(5/2))/(120*c^8) + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1
 - c^2*x^2)^(7/2))/(56*c^8) + (d*x^6*(a + b*ArcSech[c*x]))/6 + (e*x^8*(a +
b*ArcSech[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad +\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^5(4d+3ex^2)}{24\sqrt{1-c^2x^2}}\,dx \\
&= \frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad +\frac{1}{24}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^5(4d+3ex^2)}{\sqrt{1-c^2x^2}}\,dx \\
&= \frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad +\frac{1}{48}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{x^2(4d+3ex)}{\sqrt{1-c^2x}}\,dx,x,x^2\right) \\
&= \frac{1}{6}dx^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad +\frac{1}{48}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{4c^2d+3e}{c^6\sqrt{1-c^2x}}+\frac{(-8c^2d-9e)\sqrt{1-c^2x}}{c^6}\right.\right. \\
&\qquad \left.\left.+\frac{(4c^2d+9e)\left(1-c^2x\right)^{3/2}}{c^6}-\frac{3e(1-c^2x)^{5/2}}{c^6}\right)\,dx,x,x^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b(4c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{24c^8} \\
&\quad + \frac{b(8c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{72c^8} \\
&\quad - \frac{b(4c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{120c^8} + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{7/2}}{56c^8} \\
&\quad + \frac{1}{6}dx^6\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{8}ex^8\left(a+b\operatorname{sech}^{-1}(cx)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.54

$$
\begin{aligned}
&\int x^5\left(d+ex^2\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx \\
&= \frac{1}{24}ax^6\left(4d+3ex^2\right) \\
&\quad - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(144e+8c^2(28d+9ex^2)+2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6)\right)}{2520c^8} \\
&\quad + \frac{1}{24}bx^6\left(4d+3ex^2\right)\operatorname{sech}^{-1}(cx)
\end{aligned}
$$

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^6*(4*d + 3*e*x^2))/24 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(144*e
+ 8*c^2*(28*d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45
*e*x^6)))/(2520*c^8) + (b*x^6*(4*d + 3*e*x^2)*ArcSech[c*x])/24
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.60

method	result
parts	$a\left(\frac{1}{8}e\,x^8+\frac{1}{6}d\,x^6\right)+\frac{b\Big(\frac{c^6\,\mathrm{arcsech}(cx)e\,x^8}{8}+\frac{\mathrm{arcsech}(cx)d\,x^6c^6}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4\cdots}{2520c}\cdots}{c^6}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\Big(\frac{\mathrm{arcsech}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arcsech}(cx)e\,c^8x^8}{8}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2\cdots}{2520}\cdots}{c^2}}{c^6}$
default	$\frac{\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\Big(\frac{\mathrm{arcsech}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arcsech}(cx)e\,c^8x^8}{8}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2\cdots}{2520}\cdots}{c^2}}{c^6}$

```
[In] int(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/8*e*x^8+1/6*d*x^6)+b/c^6*(1/8*c^6*arcsech(c*x)*e*x^8+1/6*arcsech(c*x)*
d*x^6*c^6-1/2520/c*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(45*c^6*e*x^6
+84*c^6*d*x^4+54*c^4*e*x^4+112*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.72

$$\int x^5(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{315\,ac^7ex^8+420\,ac^7dx^6+105\,(3\,bc^7ex^8+4\,bc^7dx^6)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(45\,bc^6ex^7+6\,(14\,bc^6d+9\,bc^4e)}{2520\,c^7}$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*e*x^8 + 420*a*c^7*d*x^6 + 105*(3*b*c^7*e*x^8 + 4*b*c^7*d*
x^6)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (45*b*c^6*e*x^7
+ 6*(14*b*c^6*d + 9*b*c^4*e)*x^5 + 8*(14*b*c^4*d + 9*b*c^2*e)*x^3 + 16*(14*
b*c^2*d + 9*b*e)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.98

$$\int x^5(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \begin{cases} \frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\operatorname{asech}(cx)}{6}+\frac{bex^8\operatorname{asech}(cx)}{8}-\frac{bdx^4\sqrt{-c^2x^2+1}}{30c^2}-\frac{bex^6\sqrt{-c^2x^2+1}}{56c^2}-\frac{2bdx^2\sqrt{-c^2x^2+1}}{45c^4}-\frac{3bex^4\sqrt{-c^2x^2+1}}{140c^4}- \\ (a+\infty b)\left(\frac{dx^6}{6}+\frac{ex^8}{8}\right) \end{cases}$$

```
[In] integrate(x**5*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*asech(c*x)/6 + b*e*x**8*asech
(c*x)/8 - b*d*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - b*e*x**6*sqrt(-c**2*x**
2 + 1)/(56*c**2) - 2*b*d*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 3*b*e*x**4*s
qrt(-c**2*x**2 + 1)/(140*c**4) - 4*b*d*sqrt(-c**2*x**2 + 1)/(45*c**6) - b*e
*x**2*sqrt(-c**2*x**2 + 1)/(35*c**6) - 2*b*e*sqrt(-c**2*x**2 + 1)/(35*c**8)
, Ne(c, 0)), ((a + oo*b)*(d*x**6/6 + e*x**8/8), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.76

$$\int x^5(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{1}{8}\,aex^8+\frac{1}{6}\,adx^6 + \frac{1}{90}\left(15\,x^6\operatorname{arsech}(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)bd + \frac{1}{280}\left(35\,x^8\operatorname{arsech}(cx)+\frac{5\,c^6x^7\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}-21\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-35\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^7}\right.$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*
x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2
) - 1))/c^5)*b*d + 1/280*(35*x^8*arcsech(c*x) + (5*c^6*x^7*(1/(c^2*x^2) - 1
)^(7/2) - 21*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) - 1)
^(3/2) - 35*x*sqrt(1/(c^2*x^2) - 1))/c^7)*b*e
```

Giac [F]

$$\int x^5(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int (ex^2+d)(b\operatorname{arsech}(cx)+a)x^5\,dx$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int x^5\left(e\,x^2+d\right)\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^5*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^5*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.96 $\int x^3(d+ex^2)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 180

$$\int x^3\left(d+ex^2\right)\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\,dx=-\frac{b(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^6}+\frac{b(3c^2d+4e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\left(1-c^2x^2\right)^{3/2}}{36c^6}-\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\left(1-c^2x^2\right)^{5/2}}{30c^6}+\frac{1}{4}dx^4\left(a+b\,\mathrm{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\,\mathrm{sech}^{-1}(cx)\right)$$

```
[Out] 1/4*d*x^4*(a+b*arcsech(c*x))+1/6*e*x^6*(a+b*arcsech(c*x))+1/36*b*(3*c^2*d+4
*e)*(-c^2*x^2+1)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/30*b*e*(-c^2*x
^2+1)^(5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/12*b*(3*c^2*d+2*e)*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

$= \{14, 6436, 12, 457, 78\}$

$$\int x^3 (d+ex^2) \left(a+b\text{sech}^{-1}(cx)\right) dx = \frac{1}{4}dx^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{sech}^{-1}(cx)\right) + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}(3c^2d+4e)}{36c^6} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(3c^2d+2e)}{12c^6} - \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{30c^6}$$

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/12*(b*(3*c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^
2])/c^6 + (b*(3*c^2*d + 4*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^
2)^(3/2))/(36*c^6) - (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^
(5/2))/(30*c^6) + (d*x^4*(a + b*ArcSech[c*x]))/4 + (e*x^6*(a + b*ArcSech[c*
x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}dx^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3(3d+2ex^2)}{12\sqrt{1-c^2x^2}}\,dx \\
&= \frac{1}{4}dx^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad+\frac{1}{12}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3(3d+2ex^2)}{\sqrt{1-c^2x^2}}\,dx \\
&= \frac{1}{4}dx^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad+\frac{1}{24}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{x(3d+2ex)}{\sqrt{1-c^2x}}\,dx,x,x^2\right) \\
&= \frac{1}{4}dx^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad+\frac{1}{24}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\left(\frac{3c^2d+2e}{c^4\sqrt{1-c^2x}}+\frac{(-3c^2d-4e)\sqrt{1-c^2x}}{c^4}\right.\right. \\
&\qquad\left.\left.+\frac{2e(1-c^2x)^{3/2}}{c^4}\right)\,dx,x,x^2\right) \\
&= -\frac{b(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{12c^6}+\frac{b(3c^2d+4e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{36c^6} \\
&\quad-\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{30c^6}+\frac{1}{4}dx^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\operatorname{sech}^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.59

$$\int x^3(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{180}\left(15ax^4(3d+2ex^2)-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(16e+c^2(30d+8ex^2)+3c^4(5dx^2+2ex^4)\right)}{c^6}+15bx^4(3d+2ex^2)\operatorname{sech}^{-1}(cx)\right)$$

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (15*a*x^4*(3*d + 2*e*x^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(16*e +
c^2*(30*d + 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^6 + 15*b*x^4*(3*d + 2*
e*x^2)*ArcSech[c*x])/180
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.67

method	result
parts	$a\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b\left(\frac{c^4\,\mathrm{arcsech}(cx)e\,x^6}{6}+\frac{\mathrm{arcsech}(cx)x^4c^4d}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\left(6c^4e\,x^4+15c^4d\,x^2+8e\,c^2x^2+30c^2d+\right.}{180c}\right.}{c^4}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^2}{2}-\frac{\left(e\,c^2x^2+c^2d\right)^3}{3}\right)}{2c^2e^2}+\frac{b\left(-\frac{\mathrm{arcsech}(cx)c^6d^3}{12e^2}+\frac{\mathrm{arcsech}(cx)c^6d\,x^4}{4}+\frac{e\ \mathrm{arcsech}(cx)c^6x^6}{6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{}\right.}{}}{}$
default	$\frac{-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^2}{2}-\frac{\left(e\,c^2x^2+c^2d\right)^3}{3}\right)}{2c^2e^2}+\frac{b\left(-\frac{\mathrm{arcsech}(cx)c^6d^3}{12e^2}+\frac{\mathrm{arcsech}(cx)c^6d\,x^4}{4}+\frac{e\ \mathrm{arcsech}(cx)c^6x^6}{6}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}}{}\right.}{}}{}$

```
[In] int(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arcsech(c*x)*e*x^6+1/4*arcsech(c*x)*
x^4*c^4*d-1/180/c*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(6*c^4*e*x^4+1
5*c^4*d*x^2+8*c^2*e*x^2+30*c^2*d+16*e))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.82

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{30\,ac^5ex^6+45\,ac^5dx^4+15\,(2\,bc^5ex^6+3\,bc^5dx^4)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(6\,bc^4ex^5+(15\,bc^4d+8\,bc^2e)x^3+2}{180\,c^5}$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^5*e*x^6 + 45*a*c^5*d*x^4 + 15*(2*b*c^5*e*x^6 + 3*b*c^5*d*x^4)
*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (6*b*c^4*e*x^5 + (15
*b*c^4*d + 8*b*c^2*e)*x^3 + 2*(15*b*c^2*d + 8*b*e)*x)*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\begin{cases}\frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{asech}(cx)}{4}+\frac{bex^6\operatorname{asech}(cx)}{6}-\frac{bdx^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{bex^4\sqrt{-c^2x^2+1}}{30c^2}-\frac{bd\sqrt{-c^2x^2+1}}{6c^4}-\frac{2bex^2\sqrt{-c^2x^2+1}}{45c^4}-\frac{4be}{} \\ (a+\infty b)\left(\frac{dx^4}{4}+\frac{ex^6}{6}\right)\end{cases}$$

```
[In] integrate(x**3*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*asech(c*x)/4 + b*e*x**6*asech
(c*x)/6 - b*d*x**2*sqrt(-c**2*x**2 + 1)/(12*c**2) - b*e*x**4*sqrt(-c**2*x**
2 + 1)/(30*c**2) - b*d*sqrt(-c**2*x**2 + 1)/(6*c**4) - 2*b*e*x**2*sqrt(-c**
2*x**2 + 1)/(45*c**4) - 4*b*e*sqrt(-c**2*x**2 + 1)/(45*c**6), Ne(c, 0)), ((
a + oo*b)*(d*x**4/4 + e*x**6/6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.77

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3\,x^4\,\text{arsech}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)bd$$
$$+\frac{1}{90}\left(15\,x^6\,\text{arsech}\,(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)be$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2
) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d + 1/90*(15*x^6*arcsech(c
*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/
2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*e
```

Giac [F]

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (ex^2+d)(b\,\text{arsech}\,(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^3\left(e\,x^2+d\right)\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.97 $\int x(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right) dx$

Optimal result

Integrand size = 17, antiderivative size = 164

$$\int x(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right) dx = -\frac{b(2c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4} + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{12c^4} + \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e} - \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4e}$$

```
[Out] 1/4*(e*x^2+d)^2*(a+b*arcsech(c*x))/e+1/12*b*e*(-c^2*x^2+1)^(3/2)*(1/(c*x+1)
)^(1/2)*(c*x+1)^(1/2)/c^4-1/4*b*d^2*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)/e-1/4*b*(2*c^2*d+e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-
c^2*x^2+1)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used

= {6434, 531, 457, 90, 65, 214}

$$\int x(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e} - \frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(2c^2d+e)}{4c^4} + \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{12c^4}$$

```
[In] Int[x*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/4*(b*(2*c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])
/c^4 + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(12*c^4
) + ((d + e*x^2)^2*(a + b*ArcSech[c*x]))/(4*e) - (b*d^2*Sqrt[(1 + c*x)^(-1)
]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(4*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6434

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Dist[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^2}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{4e}$$

$$=\frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^2}{x\sqrt{1-c^2x^2}}\,dx}{4e}$$

$$=\frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{(d+ex)^2}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{8e}$$

$$=\frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{e(2c^2d+e)}{c^2\sqrt{1-c^2x}}+\frac{d^2}{x\sqrt{1-c^2x}}-\frac{e^2\sqrt{1-c^2x}}{c^2}\right)dx,x,x^2\right)}{8e}$$

$$=-\frac{b(2c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4}+\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{12c^4}+\frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}+\frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{8e}$$

$$=-\frac{b(2c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4}+\frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{12c^4}+\frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e}-\frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{4c^2e}$$

$$= -\frac{b(2c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4} + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{12c^4} + \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{4e} - \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4e}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.52

$$\int x\left(d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{1}{12}\left(3ax^2\left(2d+ex^2\right) - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(2e+c^2(6d+ex^2)\right)}{c^4} + 3bx^2\left(2d+ex^2\right)\text{sech}^{-1}(cx)\right)$$

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
[Out] (3*a*x^2*(2*d + e*x^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(2*e + c^2*
(6*d + e*x^2)))/c^4 + 3*b*x^2*(2*d + e*x^2)*ArcSech[c*x])/12
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.18

method	result
parts	$\frac{a\left(e\,x^2+d\right)^2}{4e}+\frac{b\left(\frac{c^2e\ \text{arcsech}(cx)x^4}{4}+\frac{\text{arcsech}(cx)x^2c^2d}{2}+\frac{c^2\,\text{arcsech}(cx)d^2}{4e}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(3c^4d^2\,\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)\cdots\right.}{12c\cdots}\right.}{c^2}$
derivativedivides	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^2}{4c^2e}+\frac{b\left(\frac{\text{arcsech}(cx)c^4d^2}{4e}+\frac{\text{arcsech}(cx)c^4d\,x^2}{2}+\frac{e\ \text{arcsech}(cx)c^4x^4}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(3c^4d^2\,\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\cdots\right.\right.}{12\cdots}\right.}{c^2}}{c^2}$
default	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^2}{4c^2e}+\frac{b\left(\frac{\text{arcsech}(cx)c^4d^2}{4e}+\frac{\text{arcsech}(cx)c^4d\,x^2}{2}+\frac{e\ \text{arcsech}(cx)c^4x^4}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\left(3c^4d^2\,\text{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\cdots\right.\right.}{12\cdots}\right.}{c^2}}{c^2}$

```
[In] int(x*(e*x^2+d)*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/4*a*(e*x^2+d)^2/e+b/c^2*(1/4*c^2*e*arcsech(c*x)*x^4+1/2*arcsech(c*x)*x^2*
c^2*d+1/4*c^2/e*arcsech(c*x)*d^2-1/12/c/e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c
```

```
/x)^(1/2)*(3*c^4*d^2*arctanh(1/(-c^2*x^2+1)^(1/2))+6*c^2*d*e*(-c^2*x^2+1)^(
1/2)+e^2*(-c^2*x^2+1)^(1/2)*c^2*x^2+2*e^2*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76

$$\int x(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^3ex^4+6\,ac^3dx^2+3\,(bc^3ex^4+2\,bc^3dx^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(bc^2ex^3+2\,(3\,bc^2d+be)x)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{12\,c^3}$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*e*x^4 + 6*a*c^3*d*x^2 + 3*(b*c^3*e*x^4 + 2*b*c^3*d*x^2)*log((
c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*e*x^3 + 2*(3*b*c^2*
d + b*e)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.77

$$\int x(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\begin{cases}\frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{asech}(cx)}{2}+\frac{bex^4\operatorname{asech}(cx)}{4}-\frac{bd\sqrt{-c^2x^2+1}}{2c^2}-\frac{bex^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{be\sqrt{-c^2x^2+1}}{6c^4} & \text{for } c\neq 0\\ (a+\infty b)\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*asech(c*x)/2 + b*e*x**4*asech
(c*x)/4 - b*d*sqrt(-c**2*x**2 + 1)/(2*c**2) - b*e*x**2*sqrt(-c**2*x**2 + 1)
/(12*c**2) - b*e*sqrt(-c**2*x**2 + 1)/(6*c**4), Ne(c, 0)), ((a + oo*b)*(d*x
**2/2 + e*x**4/4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.59

$$\int x(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{4}aex^4+\frac{1}{2}adx^2+\frac{1}{2}\left(x^2\,\text{arsech}\,(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bd$$
$$+\frac{1}{12}\left(3\,x^4\,\text{arsech}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)be$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)
/c)*b*d + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x
*sqrt(1/(c^2*x^2) - 1))/c^3)*b*e
```

Giac [F]

$$\int x(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int(ex^2+d)(b\,\text{arsech}\,(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x\,\left(e\,x^2+d\right)\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.98 $\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 296

$$
\begin{aligned}
\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx &= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c}+\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad+\frac{1}{2}ex^2\left(a+b\operatorname{sech}^{-1}(cx)\right) \\
&\quad-\frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad+\frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad-d\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\
&\quad+\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}
\end{aligned}
$$

```
[Out] 1/2*e*x^2*(a+b*arcsech(c*x))-d*(a+b*arcsech(c*x))*ln(1/x)+1/2*I*b*d*arccsc(
c*x)^2*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-b*d*arccsc(c*x)
*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(
1+1/c/x)^(1/2)+b*d*arccsc(c*x)*ln(1/x)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)
/(1+1/c/x)^(1/2)+1/2*I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^
2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/2*b*e*x*(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {6438, 14, 5958, 6874, 97, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = -d\log\left(\frac{1}{x}\right)\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\text{sech}^{-1}(cx)\right) + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\log\left(\frac{1}{x}\right)\csc^{-1}(cx)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{bex\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}{2c}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] -1/2*(b*e*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/c + ((I/2)*b*d*Sqrt[1 - 1
/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (e*x^2*
(a + b*ArcSech[c*x]))/2 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^
((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*d*Sqrt[1
- 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x
)]) - d*(a + b*ArcSech[c*x])*Log[x^(-1)] + ((I/2)*b*d*Sqrt[1 - 1/(c^2*x^2)]
*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
```

```
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}ex^2\left(a+b\text{sech}^{-1}(cx)\right)-d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{b\text{Subst}\left(\int \frac{-\frac{e}{2x^2}+d\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{2}ex^2\left(a+b\text{sech}^{-1}(cx)\right)-d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(-\frac{e}{2x^2\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}+\frac{d\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\right)dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{2}ex^2\left(a+b\text{sech}^{-1}(cx)\right)-d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd)\text{Subst}\left(\int \frac{\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{c}-\frac{(be)\text{Subst}\left(\int \frac{1}{x^2\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{1}{2}ex^2\big(a+b\,\text{sech}^{-1}(cx)\big) - d\big(a+b\,\text{sech}^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{\left(bd\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{1}{2}ex^2\big(a+b\,\text{sech}^{-1}(cx)\big) + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d\big(a+b\,\text{sech}^{-1}(cx)\big)\log\left(\frac{1}{x}\right) - \frac{\left(bd\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{1}{2}ex^2\big(a+b\,\text{sech}^{-1}(cx)\big) + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d\big(a+b\,\text{sech}^{-1}(cx)\big)\log\left(\frac{1}{x}\right) - \frac{\left(bd\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{2}ex^2\big(a+b\,\text{sech}^{-1}(cx)\big) + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d\big(a+b\,\text{sech}^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{\left(2ibd\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{2}ex^2\big(a+b\,\text{sech}^{-1}(cx)\big) - \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d\big(a+b\,\text{sech}^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{\left(bd\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$
\begin{aligned}
&= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{2}ex^2\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad - \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad - d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{\left(ibd\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{be\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{2}ex^2\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad - \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad - d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.36

$$
\begin{aligned}
\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx &= \frac{1}{2}aex^2 + be\left(-\frac{1}{2c^2}-\frac{x}{2c}\right)\sqrt{\frac{1-cx}{1+cx}} + \frac{1}{2}bex^2\text{sech}^{-1}(cx) \\
&\quad + ad\log(x) + \frac{1}{2}bd\left(-\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx)\right.\right. \\
&\quad \left.+ 2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\right) \\
&\quad \left.+ \text{PolyLog}\left(2, -e^{-2\text{sech}^{-1}(cx)}\right)\right)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] (a*e*x^2)/2 + b*e*(-1/2*1/c^2 - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)] + (b*e*x
^2*ArcSech[c*x])/2 + a*d*Log[x] + (b*d*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*Lo
g[1 + E^(-2*ArcSech[c*x])])) + PolyLog[2, -E^(-2*ArcSech[c*x])]))/2
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.54

method	result
parts	$\frac{ae\,x^2}{2}+ad\ln(x)+b\left(\frac{\operatorname{arcsech}(cx)^2 d}{2}+\frac{e\left(-\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2\operatorname{arcsech}(cx)+1\right)}{2c^2}-d\operatorname{arcsech}(cx)\,\mathrm{l}\right.$
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{c^2 d\operatorname{arcsech}(cx)^2}{2}+\frac{e\left(-\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2\operatorname{arcsech}(cx)+1\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\right.\right.\right.}{c^2}$
default	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{c^2 d\operatorname{arcsech}(cx)^2}{2}+\frac{e\left(-\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}+c^2x^2\operatorname{arcsech}(cx)+1\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\right.\right.\right.}{c^2}$

[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x,x,method=_RETURNVERBOSE)

[Out] 1/2*a*e*x^2+a*d*ln(x)+b*(1/2*arcsech(c*x)^2*d+1/2*e*(-(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)+c^2*x^2*arcsech(c*x)+1)/c^2-d*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*d*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))

Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)(b\operatorname{arsech}(cx)+a)}{x}\,dx$$

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="fricas")

[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))/x, x)

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)}{x}\,dx$$

[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x, x)

Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + a*d*log(x) + integrate(b*e*x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*
x) - 1) + 1/(c*x)) + b*d*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))
/x, x)
```

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x, x)
```

3.99 $\int \frac{(d+ex^2)\left(a+b\,\text{sech}^{-1}(cx)\right)}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 309

$$\int \frac{(d+ex^2)\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}+\frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{1}{4}bc^2d\text{sech}^{-1}(cx)-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2}-\frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-e\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

```
[Out] 1/4*b*c^2*d*arcsech(c*x)-1/2*d*(a+b*arcsech(c*x))/x^2-e*(a+b*arcsech(c*x))*
ln(1/x)+1/2*I*b*e*arccsc(c*x)^2*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c
/x)^(1/2)-b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)
^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+b*e*arccsc(c*x)*ln(1/x)*(1-1/c^2/x^
2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/2*I*b*e*polylog(2,(I/c/x+(1-1/c
^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/4*
b*c*d*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {6438, 14, 5958, 12, 6874, 92, 54, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2} - e\log\left(\frac{1}{x}\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx) + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} + \frac{be\sqrt{1-\frac{1}{c^2x^2}}\log\left(\frac{1}{x}\right)\csc^{-1}(cx)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} + \frac{bcd\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}{4x}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(4*x) + ((I/2)*b*e*Sqrt[1 - 1/
(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*c^2*d
*ArcSech[c*x])/4 - (d*(a + b*ArcSech[c*x]))/(2*x^2) - (b*e*Sqrt[1 - 1/(c^2*
x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[
1 + 1/(c*x)]) + (b*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-
1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - e*(a + b*ArcSech[c*x])*Log[x^(-1)] + ((I/
2)*b*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 +
1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
```

```
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$=-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{b\text{Subst}\left(\int\frac{dx^2+2e\log(x)}{2\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{b\text{Subst}\left(\int\frac{dx^2+2e\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right)}{2c}$$

$$=-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{b\text{Subst}\left(\int\left(\frac{dx^2}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}+\frac{2e\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\right)dx,x,\frac{1}{x}\right)}{2c}$$

$$=-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{(bd)\text{Subst}\left(\int\frac{x^2}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right)}{2c}+\frac{(be)\text{Subst}\left(\int\frac{\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{1}{4}(bcd)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right)+\frac{\left(be\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$=\frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}+\frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx)-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}+\frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\frac{\left(be\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$=\frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}+\frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx)-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}+\frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\frac{\left(be\sqrt{1-\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$=\frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}+\frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx)-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}+\frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{\left(2ibe\sqrt{1-\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$=\frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}+\frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx)-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}-\frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{\left(be\sqrt{1-\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$=\frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}+\frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx)-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}-\frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-e\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\frac{\left(ibe\sqrt{1-\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d\text{sech}^{-1}(cx) - \frac{d(a+b\text{sech}^{-1}(cx))}{2x^2} - \frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{be\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - e(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex^2)\,(a+b\text{sech}^{-1}(cx))}{x^3}\,dx = -\frac{ad}{2x^2} + bd\left(\frac{1}{4x^2}+\frac{c}{4x}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{bd\text{sech}^{-1}(cx)}{2x^2} - \frac{1}{4}bc^2d\log(x) + ae\log(x) + \frac{1}{4}bc^2d\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right) + \frac{1}{2}be\left(-\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx) + 2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)\right)$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] -1/2*(a*d)/x^2 + b*d*(1/(4*x^2) + c/(4*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*d
*ArcSech[c*x])/(2*x^2) - (b*c^2*d*Log[x])/4 + a*e*Log[x] + (b*c^2*d*Log[1 +
 Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/4 + (b*e*(-(Ar
cSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])])) + PolyLog[2, -E
^(-2*ArcSech[c*x])]))/2
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.54

method	result
parts	$-\frac{ad}{2x^2}+ae\ln\left(x\right)+\frac{be\,\mathrm{arcsech}(cx)^2}{2}+\frac{b\,c^2d\,\mathrm{arcsech}(cx)}{4}+\frac{bcd\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{4x}-\frac{bd\,\mathrm{arcsech}(cx)}{2x^2}-be\,\mathrm{ar}$
derivativedivides	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{be\,\mathrm{arcsech}(cx)^2}{2c^2}+\frac{bd\,\mathrm{arcsech}(cx)}{4}+\frac{bd\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{4cx}-\frac{b\,\mathrm{arcsech}(cx)d}{2c^2x^2}-\frac{be\,\mathrm{a}}{}\right.$
default	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{be\,\mathrm{arcsech}(cx)^2}{2c^2}+\frac{bd\,\mathrm{arcsech}(cx)}{4}+\frac{bd\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}}{4cx}-\frac{b\,\mathrm{arcsech}(cx)d}{2c^2x^2}-\frac{be\,\mathrm{a}}{}\right.$

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*d/x^2+a*e*ln(x)+1/2*b*e*arcsech(c*x)^2+1/4*b*c^2*d*arcsech(c*x)+1/4*
b*c*d/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-1/2*b*d/x^2*arcsech(c*x)-b
*e*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*b*e*po
lylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\mathrm{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(ex^2+d)(b\,\mathrm{arsech}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\mathrm{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(a+b\,\mathrm{asech}\,(cx))\,(d+ex^2)}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**3,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arsech}\,(cx)+a)}{x^3}\,dx$$

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")

[Out] -1/8*b*d*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) - 1))/c + 4*arcsech(c*x)/x^2) + b*e*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x) + a*e*log(x) - 1/2*a*d/x^2

Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arsech}\,(cx)+a)}{x^3}\,dx$$

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

[In] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^3,x)

[Out] int(((d + e*x^2)*(a + b*acosh(1/(c*x))))/x^3, x)

3.100 $\int x^2(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 275

$$
\begin{aligned}
&\int x^2(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx \\
&= -\frac{b(280c^4d^2+252c^2de+75e^2)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1680c^6} \\
&\quad -\frac{be(84c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4}-\frac{be^2x^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} \\
&\quad +\frac{1}{3}d^2x^3\left(a+b\text{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad +\frac{b(280c^4d^2+252c^2de+75e^2)\,\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{1680c^7}
\end{aligned}
$$

```
[Out] 1/3*d^2*x^3*(a+b*arcsech(c*x))+2/5*d*e*x^5*(a+b*arcsech(c*x))+1/7*e^2*x^7*(
a+b*arcsech(c*x))+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*arcsin(c*x)*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^7-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*
x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6-1/840*b*e*(84*c^2*
d+25*e)*x^3*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/42*b*e
^2*x^5*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {276, 6436, 12, 1281, 470, 327, 222}

$$\int x^2(d+ex^2)^2(a+b\text{sech}^{-1}(cx))\,dx$$
$$=\frac{1}{3}d^2x^3(a+b\text{sech}^{-1}(cx))+\frac{2}{5}dex^5(a+b\text{sech}^{-1}(cx))+\frac{1}{7}e^2x^7(a+b\text{sech}^{-1}(cx))$$
$$+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)\,(280c^4d^2+252c^2de+75e^2)}{1680c^7}$$
$$-\frac{be^2x^5\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{42c^2}-\frac{bex^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(84c^2d+25e)}{840c^4}$$
$$-\frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(280c^4d^2+252c^2de+75e^2)}{1680c^6}$$

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/1680*(b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt
[1 + c*x]*Sqrt[1 - c^2*x^2])/c^6 - (b*e*(84*c^2*d + 25*e)*x^3*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(840*c^4) - (b*e^2*x^5*Sqrt[(1 + c
*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(42*c^2) + (d^2*x^3*(a + b*ArcSe
ch[c*x]))/3 + (2*d*e*x^5*(a + b*ArcSech[c*x]))/5 + (e^2*x^7*(a + b*ArcSech[
c*x]))/7 + (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqr
t[1 + c*x]*ArcSin[c*x])/(1680*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}d^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2(35d^2+42dex^2+15e^2x^4)}{105\sqrt{1-c^2x^2}}\,dx\\
&= \frac{1}{3}d^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad+\frac{1}{105}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2(35d^2+42dex^2+15e^2x^4)}{\sqrt{1-c^2x^2}}\,dx
\end{aligned}$$

$$= -\frac{be^2x^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} + \frac{1}{3}d^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\operatorname{sech}^{-1}(cx)\right) - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2\left(-210c^2d^2-3e\left(84c^2d+25e\right)x^2\right)}{\sqrt{1-c^2x^2}}dx}{630c^2}$$

$$= -\frac{be(84c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4} - \frac{be^2x^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} + \frac{1}{3}d^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\operatorname{sech}^{-1}(cx)\right) + -\frac{\left(b(-840c^4d^2-9e(84c^2d+25e))\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{2520c^4}$$

$$= -\frac{b(280c^4d^2+252c^2de+75e^2)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1680c^6} - \frac{be(84c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4} - \frac{be^2x^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} + \frac{1}{3}d^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\operatorname{sech}^{-1}(cx)\right) + -\frac{\left(b(-840c^4d^2-9e(84c^2d+25e))\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}dx}{5040c^6}$$

$$= -\frac{b(280c^4d^2+252c^2de+75e^2)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1680c^6} - \frac{be(84c^2d+25e)\,x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4} - \frac{be^2x^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} + \frac{1}{3}d^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{b(280c^4d^2+252c^2de+75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{1680c^7}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.75

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

$$= \frac{16ac^7x^3(35d^2+42dex^2+15e^2x^4) - bcx\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(75e^2+2c^2e(126d+25ex^2)+8c^4(35d^2+21dex^2+\right.}{}$$

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (16*a*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - b*c*x*Sqrt[(1 - c*x)/(1
+ c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21
*d*e*x^2 + 5*e^2*x^4)) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*Ar
cSech[c*x] + I*b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(-2*I)*c*x + 2*Sq
rt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(1680*c^7)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.04

method	result
parts	$a\left(\frac{1}{7}e^{2}x^{7}+\frac{2}{5}de\,x^{5}+\frac{1}{3}x^{3}d^{2}\right)+\frac{b\left(\frac{c^{3}\operatorname{arcsech}(cx)e^{2}x^{7}}{7}+\frac{2c^{3}\operatorname{arcsech}(cx)de\,x^{5}}{5}+\frac{\operatorname{arcsech}(cx)d^{2}x^{3}c^{3}}{3}+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx}{c}}}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^{2}c^{7}x^{3}+\frac{2}{5}d\,c^{7}e\,x^{5}+\frac{1}{7}e^{2}c^{7}x^{7}\right)}{c^{4}}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d^{2}c^{7}x^{3}}{3}+\frac{2\operatorname{arcsech}(cx)d\,c^{7}e\,x^{5}}{5}+\frac{\operatorname{arcsech}(cx)e^{2}c^{7}x^{7}}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(28}{}\right.}{}}{}$
default	$\frac{\frac{a\left(\frac{1}{3}d^{2}c^{7}x^{3}+\frac{2}{5}d\,c^{7}e\,x^{5}+\frac{1}{7}e^{2}c^{7}x^{7}\right)}{c^{4}}+\frac{b\left(\frac{\operatorname{arcsech}(cx)d^{2}c^{7}x^{3}}{3}+\frac{2\operatorname{arcsech}(cx)d\,c^{7}e\,x^{5}}{5}+\frac{\operatorname{arcsech}(cx)e^{2}c^{7}x^{7}}{7}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(28}{}\right.}{}}{}$

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*x^3*d^2)+b/c^3*(1/7*c^3*arcsech(c*x)*e^2*x^7
+2/5*c^3*arcsech(c*x)*d*e*x^5+1/3*arcsech(c*x)*d^2*x^3*c^3+1/1680/c^3*(-(c*
x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-40*e^2*(-c^2*x^2+1)^(1/2)*c^5*x^5-1
68*c^5*d*e*(-c^2*x^2+1)^(1/2)*x^3-280*d^2*c^5*x*(-c^2*x^2+1)^(1/2)+280*d^2*
c^4*arcsin(c*x)-50*e^2*c^3*x^3*(-c^2*x^2+1)^(1/2)-252*d*c^3*e*x*(-c^2*x^2+1
)^(1/2)+252*d*c^2*e*arcsin(c*x)-75*e^2*c*x*(-c^2*x^2+1)^(1/2)+75*e^2*arcsin
(c*x))/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.24

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7e^2x^7+672\,ac^7dex^5+560\,ac^7d^2x^3-2\left(280\,bc^4d^2+252\,bc^2de+75\,be^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-1}{}$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 - 2*(280*
b*c^4*d^2 + 252*b*c^2*d*e + 75*b*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*
x^2)) - 1)/(c*x)) - 16*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*e^2)*log((c*
x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 16*(15*b*c^7*e^2*x^7 + 42*b*c^7*
d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c^7*d*e - 15*b*c^7*e^2)*lo
g((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (40*b*c^6*e^2*x^6 + 2*(
84*b*c^6*d*e + 25*b*c^4*e^2)*x^4 + (280*b*c^6*d^2 + 252*b*c^4*d*e + 75*b*c^
2*e^2)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

Sympy [F]

$$\int x^2(d+ex^2)^2\left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx=\int x^2(a+b\,\mathrm{asech}\,(cx))\left(d+ex^2\right)^2\,dx$$

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.19

$$\int x^2(d+ex^2)^2\left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{7}ae^2x^7+\frac{2}{5}adex^5+\frac{1}{3}ad^2x^3+\frac{1}{6}\left(2\,x^3\,\mathrm{arsech}\,(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bd^2$$
$$+\frac{1}{20}\left(8\,x^5\,\mathrm{arsech}\,(cx)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}+\frac{3\,\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)bde$$
$$+\frac{1}{336}\left(48\,x^7\,\mathrm{arsech}\,(cx)-\frac{\frac{15\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+40\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}-1}}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\,\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^6}}{c}\right)be^2$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/6*(2*x^3*arcsech(c*x) - (
sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^
2) - 1))/c^2)/c)*b*d^2 + 1/20*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(
3/2) + 5*sqrt(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^
2) - 1) + c^4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*d*e + 1/336*(48*
x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40*(1/(c^2*x^2) - 1)^(3/2
) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2)
 - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arctan(sqrt(1/(c^2*x^2) - 1))
/c^6)/c)*b*e^2
```

Giac [F]

$$\int x^2 (d + ex^2)^2 \left(a + b\text{sech}^{-1}(cx)\right) dx = \int (ex^2 + d)^2 (b \operatorname{arsech}(cx) + a)x^2 \, dx$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d + ex^2)^2 \left(a + b\text{sech}^{-1}(cx)\right) dx = \int x^2 \left(e\,x^2 + d\right)^2 \left(a + b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right) dx$$

```
[In] int(x^2*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.101 $\int (d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 204

$$\int (d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx = -\frac{be(40c^2d+9e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4} - \frac{be^2x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2} + d^2x\left(a+b\text{sech}^{-1}(cx)\right) + \frac{2}{3}dex^3\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\text{sech}^{-1}(cx)\right) + \frac{b(120c^4d^2+40c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{120c^5}$$

```
[Out] d^2*x*(a+b*arcsech(c*x))+2/3*d*e*x^3*(a+b*arcsech(c*x))+1/5*e^2*x^5*(a+b*ar
csech(c*x))+1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*arcsin(c*x)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)/c^5-1/120*b*e*(40*c^2*d+9*e)*x*(1/(c*x+1))^(1/2)*(c*x+1
)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4-1/20*b*e^2*x^3*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {200, 6426, 12, 1173, 396, 222}

$$\int \left(d+ex^2\right)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right) dx = d^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)\left(120c^4d^2+40c^2de+9e^2\right)}{120c^5}-\frac{be^2x^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{20c^2}-\frac{bex\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(40c^2d+9e)}{120c^4}$$

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/120*(b*e*(40*c^2*d + 9*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 -
c^2*x^2])/c^4 - (b*e^2*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*
x^2])/(20*c^2) + d^2*x*(a + b*ArcSech[c*x]) + (2*d*e*x^3*(a + b*ArcSech[c*x
]))/3 + (e^2*x^5*(a + b*ArcSech[c*x]))/5 + (b*(120*c^4*d^2 + 40*c^2*d*e + 9
*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(120*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 6426

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x
] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt
[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ
[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{15d^2+10dex^2+3e^2x^4}{15\sqrt{1-c^2x^2}}\,dx\\
&= d^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad+\frac{1}{15}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{15d^2+10dex^2+3e^2x^4}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{be^2x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2}+d^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad+\frac{1}{5}e^2x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-60c^2d^2-e\left(40c^2d+9e\right)x^2}{\sqrt{1-c^2x^2}}\,dx}{60c^2}\\
&= -\frac{be(40c^2d+9e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4}-\frac{be^2x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2}\\
&\quad+d^2x\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad-\frac{\left(b(-120c^4d^2-e(40c^2d+9e))\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx}{120c^4}
\end{aligned}
$$

$$= -\frac{be(40c^2d+9e)\,x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4} - \frac{be^2x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2} + d^2x\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{2}{3}dex^3\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{b(120c^4d^2+40c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{120c^5}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.85

$$\int \left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{8ac^5x(15d^2+10dex^2+3e^2x^4) - bcex\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(9e+c^2(40d+6ex^2)\right) + 8bc^5x(15d^2+10dex^2+3e^2 \cdots}{120c^5}$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (8*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*c*e*x*Sqrt[(1 - c*x)/(1 +
c*x)]*(1 + c*x)*(9*e + c^2*(40*d + 6*e*x^2)) + 8*b*c^5*x*(15*d^2 + 10*d*e*x
^2 + 3*e^2*x^4)*ArcSech[c*x] + I*b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(
-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/(120*c^5)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+x\,d^2\right)+\frac{b\left(\frac{c\,\operatorname{arcsech}(cx)e^2x^5}{5}+\frac{2c\,\operatorname{arcsech}(cx)de\,x^3}{3}+\operatorname{arcsech}(cx)xc\,d^2+\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,(1 \cdots}{\cdots}\right)}{\cdots}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\operatorname{arcsech}(cx)d^2c^5x+\frac{2\,\operatorname{arcsech}(cx)d\,c^5e\,x^3}{3}+\frac{\operatorname{arcsech}(cx)e^2c^5x^5}{5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(120d^2c^4 \cdots}{\cdots}\right)}{\cdots}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\operatorname{arcsech}(cx)d^2c^5x+\frac{2\,\operatorname{arcsech}(cx)d\,c^5e\,x^3}{3}+\frac{\operatorname{arcsech}(cx)e^2c^5x^5}{5}+\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(120d^2c^4 \cdots}{\cdots}\right)}{\cdots}}{c}$

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*e^2*x^5+2/3*d*e*x^3+x*d^2)+b/c*(1/5*c*arcsech(c*x)*e^2*x^5+2/3*c*arc
sech(c*x)*d*e*x^3+arcsech(c*x)*x*c*d^2+1/120/c^3*(-(c*x-1)/c/x)^(1/2)*x*((c
```

```
*x+1)/c/x)^(1/2)*(120*d^2*c^4*arcsin(c*x)-40*d*c^3*e*x*(-c^2*x^2+1)^(1/2)-6
*e^2*c^3*x^3*(-c^2*x^2+1)^(1/2)+40*d*c^2*e*arcsin(c*x)-9*e^2*c*x*(-c^2*x^2+
1)^(1/2)+9*e^2*arcsin(c*x))/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(130) = 260$.

Time = 0.37 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.50

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x-2\left(120\,bc^4d^2+40\,bc^2de+9\,be^2\right)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-8\left(15\,bc^5\right.}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x - 2*(120*b*c^4
*d^2 + 40*b*c^2*d*e + 9*b*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) -
 1)/(c*x)) - 8*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log((c*x*sqrt(-(
c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 8*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 1
5*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e - 3*b*c^5*e^2)*log((c*x*sqrt(-(
c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (6*b*c^4*e^2*x^4 + (40*b*c^4*d*e + 9*
b*c^2*e^2)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^5
```

Sympy [F]

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx = \int \left(a+b\,\text{asech}\,(cx)\right)\left(d+ex^2\right)^2\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.10

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$= \frac{1}{5}ae^2x^5+\frac{2}{3}adex^3+\frac{1}{3}\left(2\,x^3\operatorname{arsech}(cx)-\frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)bde$$
$$+\frac{1}{40}\left(8\,x^5\operatorname{arsech}(cx)-\frac{\frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}+\frac{3\,\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4}}{c}\right)be^2$$
$$+ad^2x+\frac{\left(cx\operatorname{arsech}(cx)-\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd^2}{c}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/3*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2)
 - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)
*b*d*e + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqrt(1/
(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) +
 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*e^2 + a*d^2*x + (c*x*arcsech(c*x
) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d^2/c
```

Giac [F]

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^2\left(b\operatorname{arsech}(cx)+a\right)dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int {\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
[Out] int((d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.102 $\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\text{sech}^{-1}(cx)\right) + \frac{be(12c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3}$$

```
[Out] -d^2*(a+b*arcsech(c*x))/x+2*d*e*x*(a+b*arcsech(c*x))+1/3*e^2*x^3*(a+b*arcse
ch(c*x))+1/6*b*e*(12*c^2*d+e)*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c
^3+b*d^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x-1/6*b*e^2*x*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {276, 6436, 12, 1279, 396, 222}

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^2} \, dx = -\frac{d^2(a+b\text{sech}^{-1}(cx))}{x} + 2dex(a+b\text{sech}^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\text{sech}^{-1}(cx)) + \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)(12c^2d+e)}{6c^3} + \frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{x} - \frac{be^2x\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{6c^2}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^2,x]
```

```
[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/x - (b*e^2*x*S
qrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(6*c^2) - (d^2*(a + b*
ArcSech[c*x]))/x + 2*d*e*x*(a + b*ArcSech[c*x]) + (e^2*x^3*(a + b*ArcSech[c
*x]))/3 + (b*e*(12*c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x
])/(6*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}+2dex\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-3d^2+6dex^2+e^2x^4}{3x^2\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}+2dex\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad+\frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-3d^2+6dex^2+e^2x^4}{x^2\sqrt{1-c^2x^2}}\,dx\\
&= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x}-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}+2dex\left(a+b\operatorname{sech}^{-1}(cx)\right)\\
&\quad+\frac{1}{3}e^2x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)-\frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-6de-e^2x^2}{\sqrt{1-c^2x^2}}\,dx
\end{aligned}
$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} - \frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{x} + 2dex(a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\operatorname{sech}^{-1}(cx)) + -\frac{\left(b(-12c^2de-e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}dx}{6c^2}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{be^2x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} - \frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{x} + 2dex(a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\operatorname{sech}^{-1}(cx)) + \frac{be(12c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{6c^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex^2)^2(a+b\operatorname{sech}^{-1}(cx))}{x^2}\,dx$$
$$= \frac{-bc\sqrt{\frac{1-cx}{1+cx}}(1+cx)(-6c^2d^2+e^2x^2)+2ac^3(-3d^2+6dex^2+e^2x^4)+2bc^3(-3d^2+6dex^2+e^2x^4)\operatorname{sech}^{-1}(cx}{6c^3x}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^2,x]
```

```
[Out] (-(b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(-6*c^2*d^2 + e^2*x^2)) + 2*a*c^
3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*A
rcSech[c*x] + I*b*e*(12*c^2*d + e)*x*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 +
 c*x)]*(1 + c*x)])/(6*c^3*x)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+bc\left(\frac{\operatorname{arcsech}(cx)e^2x^3}{3c}+\frac{2\operatorname{arcsech}(cx)dex}{c}-\frac{\operatorname{arcsech}(cx)d^2}{xc}-\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(\right.}{}\right.$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\operatorname{arcsech}(cx)c^3dex+\frac{\operatorname{arcsech}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arcsech}(cx)c^3d^2}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(6\sqrt{-\right.}{}\right.}{c^4}\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\operatorname{arcsech}(cx)c^3dex+\frac{\operatorname{arcsech}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arcsech}(cx)c^3d^2}{x}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(6\sqrt{-\right.}{}\right.}{c^4}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+b*c*(1/3/c*arcsech(c*x)*e^2*x^3+2/c*arcsech(c
*x)*d*e*x-arcsech(c*x)*d^2/x/c-1/6/c^4*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(
1/2)*(-6*(-c^2*x^2+1)^(1/2)*c^4*d^2-12*arcsin(c*x)*c^3*d*e*x+e^2*(-c^2*x^2+
1)^(1/2)*c^2*x^2-arcsin(c*x)*e^2*c*x)/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(107) = 214$.

Time = 0.32 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex^2)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^2}\,dx$$
$$= \frac{2\,ac^3e^2x^4+12\,ac^3dex^2-6\,ac^3d^2-2\,(12\,bc^2de+be^2)x\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)+2\,(3\,bc^3d^2-6\,bc^3de-bc}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^4 + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 2*(12*b*c^2*d*e + b
*e^2)*x*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 2*(3*b*c^3
*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) -
 1)/x) + 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 -
6*b*c^3*d*e - b*c^3*e^2)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c
*x)) + (6*b*c^4*d^2*x - b*c^2*e^2*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^3
*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^2}{x^2}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**2,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^2}\,dx = \frac{1}{3}\,ae^2x^3 + \left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\text{arsech}\,(cx)}{x}\right)bd^2 + \frac{1}{6}\left(2\,x^3\,\text{arsech}\,(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be^2 + 2\,adex + \frac{2\left(cx\,\text{arsech}\,(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bde}{c} - \frac{ad^2}{x}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
[Out] 1/3*a*e^2*x^3 + (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d^2 + 1/6*(2*x
^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + ar
ctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*e^2 + 2*a*d*e*x + 2*(c*x*arcsech(c*x)
 - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d*e/c - a*d^2/x
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arsech}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^2, x)
```

3.103 $\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{(d+ex^2)^2\,(a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3} + \frac{2bd(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{x} + e^2x\left(a+b\text{sech}^{-1}(cx)\right) + \frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\arcsin(cx)}{c}$$

```
[Out] -1/3*d^2*(a+b*arcsech(c*x))/x^3-2*d*e*(a+b*arcsech(c*x))/x+e^2*x*(a+b*arcse
ch(c*x))+b*e^2*arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c+1/9*b*d^2*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+2/9*b*d*(c^2*d+9*e)*(1/(
c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

= {276, 6436, 12, 1279, 462, 222}

$$\int \frac{(d+ex^2)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4} dx = -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} + e^2x\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{be^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arcsin(cx)}{c} + \frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{9x^3} + \frac{2bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(c^2d+9e)}{9x}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x^3) + (2*b
*d*(c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x
) - (d^2*(a + b*ArcSech[c*x]))/(3*x^3) - (2*d*e*(a + b*ArcSech[c*x]))/x + e
^2*x*(a + b*ArcSech[c*x]) + (b*e^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSi
n[c*x])/c
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{sech}^{-1}(cx)\right)\\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-d^2-6dex^2+3e^2x^4}{3x^4\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{sech}^{-1}(cx)\right)\\
&\quad+\frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-d^2-6dex^2+3e^2x^4}{x^4\sqrt{1-c^2x^2}}\,dx\\
&= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{x}\\
&\quad+e^2x\left(a+b\text{sech}^{-1}(cx)\right)-\frac{1}{9}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{2d(c^2d+9e)-9e^2x^2}{x^2\sqrt{1-c^2x^2}}\,dx\\
&= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3}+\frac{2bd(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x}\\
&\quad-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{x}\\
&\quad+e^2x\left(a+b\text{sech}^{-1}(cx)\right)+\left(be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx
\end{aligned}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x^3} + \frac{2bd(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{9x} - \frac{d^2(a+b\text{sech}^{-1}(cx))}{3x^3} - \frac{2de(a+b\text{sech}^{-1}(cx))}{x} + e^2x(a+b\text{sech}^{-1}(cx)) + \frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arcsin(cx)}{c}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex^2)^2(a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \frac{bcd\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+2c^2dx^2+18ex^2) - 3ac(d^2+6dex^2-3e^2x^4) - 3bc(d^2+6dex^2-3e^2x^4)\text{sech}^{-1}(cx}{9cx^3}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + 2*c^2*d*x^2 + 18*e*x^2) - 3
*a*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4) - 3*b*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*Ar
cSech[c*x] + (9*I)*b*e^2*x^3*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x)])/(9*c*x^3)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.14

method	result
parts	$a\left(e^2x - \frac{2de}{x} - \frac{d^2}{3x^3}\right) + b\,c^3\left(\frac{\text{arcsech}(cx)e^2x}{c^3} - \frac{2\,\text{arcsech}(cx)de}{c^3x} - \frac{\text{arcsech}(cx)d^2}{3x^3c^3} + \frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{\right.}}{}\right.$
derivativedivides	$c^3\left(\frac{a\left(e^2cx - \frac{c\,d^2}{3x^3} - \frac{2cde}{x}\right)}{c^4} + \frac{b\left(\text{arcsech}(cx)e^2cx - \frac{\text{arcsech}(cx)c\,d^2}{3x^3} - \frac{2\,\text{arcsech}(cx)cde}{x} + \frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{-c^2x^2+1}\,c^6d\right.}{}\right.}{c^4}\right.$
default	$c^3\left(\frac{a\left(e^2cx - \frac{c\,d^2}{3x^3} - \frac{2cde}{x}\right)}{c^4} + \frac{b\left(\text{arcsech}(cx)e^2cx - \frac{\text{arcsech}(cx)c\,d^2}{3x^3} - \frac{2\,\text{arcsech}(cx)cde}{x} + \frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{-c^2x^2+1}\,c^6d\right.}{}\right.}{c^4}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*c^3*(1/c^3*arcsech(c*x)*e^2*x-2/c^3*arcsech
(c*x)*d*e/x-1/3*arcsech(c*x)*d^2/x^3/c^3+1/9/c^6*(-(c*x-1)/c/x)^(1/2)/x^2*(
(c*x+1)/c/x)^(1/2)*(2*(-c^2*x^2+1)^(1/2)*c^6*d^2*x^2+(-c^2*x^2+1)^(1/2)*c^4
*d^2+18*(-c^2*x^2+1)^(1/2)*c^4*d*e*x^2+9*arcsin(c*x)*e^2*c^3*x^3)/(-c^2*x^2
+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(106) = 212$.

Time = 0.31 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\, dx = \frac{9\, ace^2x^4 - 18\, be^2x^3 \arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) - 18\, acdex^2 + 3\,(bcd^2 + 6\, bcde - 3\, bce^2)x^3 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*c*e^2*x^4 - 18*b*e^2*x^3*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) - 1)/(c*x)) - 18*a*c*d*e*x^2 + 3*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*lo
g((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 3*a*c*d^2 + 3*(3*b*c*e^2*x^
4 - 6*b*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*log((c
*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*d^2*x + 2*(b*c^4*d^2
 + 9*b*c^2*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c*x^3)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\, dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^2}{x^4}\, dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^4}\,dx$$
$$= 2\left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\text{arsech}\,(cx)}{x}\right)bde + ae^2x$$
$$+ \frac{1}{9}\,bd^2\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 3\,c^4\sqrt{\frac{1}{c^2x^2}-1}}{c} - \frac{3\,\text{arsech}\,(cx)}{x^3}\right)$$
$$+ \frac{\left(cx\,\text{arsech}\,(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)be^2}{c} - \frac{2\,ade}{x} - \frac{ad^2}{3\,x^3}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 2*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*((
c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*
x)/x^3) + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*e^2/c - 2*a*
d*e/x - 1/3*a*d^2/x^3
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arsech}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^4, x)
```

3.104 $\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 213

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx &= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5}+\frac{2bd(6c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3}\\ &\quad+\frac{b(24c^4d^2+100c^2de+225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x}\\ &\quad-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}\end{aligned}$$

```
[Out] -1/5*d^2*(a+b*arcsech(c*x))/x^5-2/3*d*e*(a+b*arcsech(c*x))/x^3-e^2*(a+b*arc
sech(c*x))/x+1/25*b*d^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/
x^5+2/225*b*d*(6*c^2*d+25*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(
1/2)/x^3+1/225*b*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{276, 6436, 12, 1279, 464, 270\}$

$$\int \frac{(d+ex^2)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx$$
$$= -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} + \frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{25x^5} + \frac{2bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(6c^2d+25e)}{225x^3} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(24c^4d^2+100c^2de+225e^2)}{225x}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(25*x^5) + (2*
b*d*(6*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/
(225*x^3) + (b*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[(1 + c*x)^(-1)]*Sq
rt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x) - (d^2*(a + b*ArcSech[c*x]))/(5*x^5)
 - (2*d*e*(a + b*ArcSech[c*x]))/(3*x^3) - (e^2*(a + b*ArcSech[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-3d^2-10dex^2-15e^2x^4}{15x^6\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} \\
&\quad+\frac{1}{15}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-3d^2-10dex^2-15e^2x^4}{x^6\sqrt{1-c^2x^2}}\,dx \\
&= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5}-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3} \\
&\quad-\frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}-\frac{1}{75}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{2d(6c^2d+25e)+75e^2x^2}{x^4\sqrt{1-c^2x^2}}\,dx \\
&= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5}+\frac{2bd(6c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3} \\
&\quad-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x} \\
&\quad-\frac{1}{225}\left(b\left(24c^4d^2+100c^2de+225e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x^2\sqrt{1-c^2x^2}}\,dx
\end{aligned}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} + \frac{2bd(6c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3} + \frac{b(24c^4d^2+100c^2de+225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x} - \frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{5x^5} - \frac{2de(a+b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e^2(a+b\operatorname{sech}^{-1}(cx))}{x}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)^2(a+b\operatorname{sech}^{-1}(cx))}{x^6}\,dx = \frac{-15a(3d^2+10dex^2+15e^2x^4)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(225e^2x^4+50dex^2(1+2c^2x^2)+3d^2(3+4c^2x^2+8c^4x^4)}{225x^5}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)*(225*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*
c^4*x^4)) - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcSech[c*x])/(225*x^5)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

method	result
parts	$a\left(-\frac{e^2}{x}-\frac{2de}{3x^3}-\frac{d^2}{5x^5}\right)+b\,c^5\left(-\frac{\operatorname{arcsech}(cx)e^2}{c^5x}-\frac{2\operatorname{arcsech}(cx)de}{3c^5x^3}-\frac{\operatorname{arcsech}(cx)d^2}{5x^5c^5}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,(2\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e^2}{cx}-\frac{\operatorname{arcsech}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arcsech}(cx)de}{3c\,x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(24c^8d^2x^4+100c^6de\right.}{}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)e^2}{cx}-\frac{\operatorname{arcsech}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arcsech}(cx)de}{3c\,x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(24c^8d^2x^4+100c^6de\right.}{}\right.}{c^4}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-e^2/x-2/3*d*e/x^3-1/5*d^2/x^5)+b*c^5*(-1/c^5*arcsech(c*x)*e^2/x-2/3/c^5
*arcsech(c*x)*d*e/x^3-1/5*arcsech(c*x)*d^2/x^5/c^5+1/225/c^8*(-(c*x-1)/c/x)
```

```
^(1/2)/x^4*((c*x+1)/c/x)^(1/2)*(24*c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x
^2+225*c^4*e^2*x^4+50*c^4*d*e*x^2+9*c^4*d^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^6} dx =$$

$$-\frac{225\,ae^2x^4 + 150\,adex^2 + 45\,ad^2 + 15\,(15\,be^2x^4 + 10\,bdex^2 + 3\,bd^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - ((24\,bc^5d^2 + 10}{225\,x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b*
d*e*x^2 + 3*b*d^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - ((
24*b*c^5*d^2 + 100*b*c^3*d*e + 225*b*c*e^2)*x^5 + 9*b*c*d^2*x + 2*(6*b*c^3*
d^2 + 25*b*c*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^5
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^6} dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^2}{x^6} dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**6,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^6} dx$$
$$= \left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\text{arsech}(cx)}{x}\right)be^2$$
$$+ \frac{1}{75}bd^2\left(\frac{3c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}} + 10c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 15c^6\sqrt{\frac{1}{c^2x^2}-1}}{c} - \frac{15\,\text{arsech}(cx)}{x^5}\right)$$
$$+ \frac{2}{9}bde\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 3c^4\sqrt{\frac{1}{c^2x^2}-1}}{c} - \frac{3\,\text{arsech}(cx)}{x^3}\right) - \frac{ae^2}{x} - \frac{2\,ade}{3\,x^3} - \frac{ad^2}{5\,x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(1/(c
^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^
2) - 1))/c - 15*arcsech(c*x)/x^5) + 2/9*b*d*e*((c^4*(1/(c^2*x^2) - 1)^(3/2)
 + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3) - a*e^2/x - 2/3*a*d
*e/x^3 - 1/5*a*d^2/x^5
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^6} dx = \int \frac{(ex^2+d)^2(b\,\text{arsech}(cx)+a)}{x^6} dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^6} dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6} dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^6, x)
```

3.105 $\int \frac{(d+ex^2)^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^8}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 281

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^8}\,dx &= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7}+\frac{2bd(15c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5}\\&\quad+\frac{b(360c^4d^2+1176c^2de+1225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{11025x^3}\\&\quad+\frac{2bc^2(360c^4d^2+1176c^2de+1225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{11025x}\\&\quad-\frac{d^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3x^3}\end{aligned}$$

```
[Out] -1/7*d^2*(a+b*arcsech(c*x))/x^7-2/5*d*e*(a+b*arcsech(c*x))/x^5-1/3*e^2*(a+b
*arcsech(c*x))/x^3+1/49*b*d^2*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^
(1/2)/x^7+2/1225*b*d*(15*c^2*d+49*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*
x^2+1)^(1/2)/x^5+1/11025*b*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^3+2/11025*b*c^2*(360*c^4*d^2+1176*
c^2*d*e+1225*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {276, 6436, 12, 1279, 464, 277, 270}

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^8} \, dx$$
$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} + \frac{bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{49x^7} + \frac{2bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(15c^2d+49e)}{1225x^5} + \frac{2bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(360c^4d^2+1176c^2de+1225e^2)}{11025x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(360c^4d^2+1176c^2de+1225e^2)}{11025x^3}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(49*x^7) + (2*
b*d*(15*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])
/(1225*x^5) + (b*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(11025*x^3) + (2*b*c^2*(360*c^4*d^2 + 1
176*c^2*d*e + 1225*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/(11025*x) - (d^2*(a + b*ArcSech[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcSech[c
*x]))/(5*x^5) - (e^2*(a + b*ArcSech[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3}\\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-15d^2-42dex^2-35e^2x^4}{105x^8\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3x^3}\\
&\quad+\frac{1}{105}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-15d^2-42dex^2-35e^2x^4}{x^8\sqrt{1-c^2x^2}}\,dx
\end{aligned}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{1}{735}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{6d(15c^2d+49e)+245e^2x^2}{x^6\sqrt{1-c^2x^2}}\,dx$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} + \frac{2bd(15c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{\left(b(360c^4d^2+1176c^2de+1225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^4\sqrt{1-c^2x^2}}\,dx}{3675}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} + \frac{2bd(15c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5} + \frac{b(360c^4d^2+1176c^2de+1225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{11025x^3} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3} - \frac{\left(2bc^2(360c^4d^2+1176c^2de+1225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{1-c^2x^2}}\,dx}{11025}$$

$$= \frac{bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49x^7} + \frac{2bd(15c^2d+49e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1225x^5} + \frac{b(360c^4d^2+1176c^2de+1225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{11025x^3} + \frac{2bc^2(360c^4d^2+1176c^2de+1225e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{11025x} - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{sech}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{3x^3}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx$$

$$= \frac{-105a(15d^2+42dex^2+35e^2x^4)+b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(1225e^2x^4(1+2c^2x^2)+294dex^2(3+4c^2x^2+8c^4x^4)\right.\cdots}{11025x^7}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^8,x]
```

[Out] (-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^4) + 45*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4)*ArcSech[c*x])/(11025*x^7)

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.74

method	result
parts	$a\left(-\frac{e^2}{3x^3}-\frac{2de}{5x^5}-\frac{d^2}{7x^7}\right)+b\,c^7\left(-\frac{\operatorname{arcsech}(cx)e^2}{3c^7x^3}-\frac{2\operatorname{arcsech}(cx)de}{5c^7x^5}-\frac{\operatorname{arcsech}(cx)d^2}{7x^7c^7}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,(7\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\operatorname{arcsech}(cx)de}{5c^3x^5}-\frac{\operatorname{arcsech}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arcsech}(cx)e^2}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(720c^{10}d^2x^6+23\right.}{}\right.}{}\right.$
default	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\operatorname{arcsech}(cx)de}{5c^3x^5}-\frac{\operatorname{arcsech}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arcsech}(cx)e^2}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(720c^{10}d^2x^6+23\right.}{}\right.}{}\right.$

[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x,method=_RETURNVERBOSE)

[Out] a*(-1/3*e^2/x^3-2/5*d*e/x^5-1/7*d^2/x^7)+b*c^7*(-1/3/c^7*arcsech(c*x)*e^2/x^3-2/5/c^7*arcsech(c*x)*d*e/x^5-1/7*arcsech(c*x)*d^2/x^7/c^7+1/11025/c^10*(-(c*x-1)/c/x)^(1/2)/x^6*((c*x+1)/c/x)^(1/2)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*c^4*d^2))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)^2\,(a+b\operatorname{sech}^{-1}(cx))}{x^8}\,dx = -\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\,(35\,be^2x^4+42\,bdex^2+15\,bd^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(2\,(360\,b}{}$$

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")

[Out] -1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4 + 42*b*d*e*x^2 + 15*b*d^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*

```
x)) - (2*(360*b*c^7*d^2 + 1176*b*c^5*d*e + 1225*b*c^3*e^2)*x^7 + (360*b*c^5
*d^2 + 1176*b*c^3*d*e + 1225*b*c*e^2)*x^5 + 225*b*c*d^2*x + 18*(15*b*c^3*d^
2 + 49*b*c*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^7
```

Sympy [F]

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)^2}{x^8}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**8,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**8, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx &= \frac{1}{245}bd^2\left(\frac{5c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}+21c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35c^8\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+35c^8\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{35\operatorname{arsech}(cx)}{x^7}\right)\\ &\quad+\frac{2}{75}bde\left(\frac{3c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+10c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15c^6\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{15\operatorname{arsech}(cx)}{x^5}\right)\\ &\quad+\frac{1}{9}be^2\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\operatorname{arsech}(cx)}{x^3}\right)-\frac{ae^2}{3x^3}-\frac{2ade}{5x^5}-\frac{ad^2}{7x^7}\end{aligned}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")
[Out] 1/245*b*d^2*((5*c^8*(1/(c^2*x^2) - 1)^(7/2) + 21*c^8*(1/(c^2*x^2) - 1)^(5/2
) + 35*c^8*(1/(c^2*x^2) - 1)^(3/2) + 35*c^8*sqrt(1/(c^2*x^2) - 1))/c - 35*a
rcsech(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(
c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5
) + 1/9*b*e^2*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/
c - 3*arcsech(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7
```

Giac [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx=\int \frac{(ex^2+d)^2(b\,\text{arsech}\,(cx)+a)}{x^8}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^8}\,dx=\int \frac{(e\,x^2+d)^2\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^8, x)
```

3.106 $\int x^3(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 278

$$\begin{aligned}
&\int x^3(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right) dx\\
&= -\frac{b(6c^4d^2+8c^2de+3e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{24c^8}\\
&\quad+\frac{b(6c^4d^2+16c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{72c^8}\\
&\quad-\frac{be(8c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{120c^8}+\frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{7/2}}{56c^8}\\
&\quad+\frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right)
\end{aligned}$$

```
[Out] 1/4*d^2*x^4*(a+b*arcsech(c*x))+1/3*d*e*x^6*(a+b*arcsech(c*x))+1/8*e^2*x^8*(
a+b*arcsech(c*x))+1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*(-c^2*x^2+1)^(3/2)*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/120*b*e*(8*c^2*d+9*e)*(-c^2*x^2+1)^(5/2
)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^8+1/56*b*e^2*(-c^2*x^2+1)^(7/2)*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)/c^8-1/24*b*(6*c^4*d^2+8*c^2*d*e+3*e^2)*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {272, 45, 6436, 12, 1265, 785}

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$
$$= \frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right)$$
$$-\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{5/2}\left(8c^2d+9e\right)}{120c^8}+\frac{be^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{7/2}}{56c^8}$$
$$+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}\left(6c^4d^2+16c^2de+9e^2\right)}{72c^8}$$
$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(6c^4d^2+8c^2de+3e^2)}{24c^8}$$

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/24*(b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]
*Sqrt[1 - c^2*x^2])/c^8 + (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(3/2))/(72*c^8) - (b*e*(8*c^2*d + 9*e)*
Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(5/2))/(120*c^8) + (b*e^2*
Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(7/2))/(56*c^8) + (d^2*x^4
*(a + b*ArcSech[c*x]))/4 + (d*e*x^6*(a + b*ArcSech[c*x]))/3 + (e^2*x^8*(a +
 b*ArcSech[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3(6d^2+8dex^2+3e^2x^4)}{24\sqrt{1-c^2x^2}}\,dx \\
&= \frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad+\frac{1}{24}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3(6d^2+8dex^2+3e^2x^4)}{\sqrt{1-c^2x^2}}\,dx \\
&= \frac{1}{4}d^2x^4\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{sech}^{-1}(cx)\right) \\
&\quad+\frac{1}{48}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{x(6d^2+8dex+3e^2x^2)}{\sqrt{1-c^2x}}\,dx,x,x^2\right)
\end{aligned}$$

$$= \frac{1}{4}d^2x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{48}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{6c^4d^2+8c^2de+3e^2}{c^6\sqrt{1-c^2x}}+\frac{(-6c^4d^2-16c^2de-9e^2)\sqrt{1-c^2x}}{c^6}+\frac{e(8c^2d+9e)(1-c^2x)^{3/2}}{c^6}-\frac{3e^2(1-c^2x)^{5/2}}{c^6}\right)dx,x,x^2\right)$$

$$= -\frac{b(6c^4d^2+8c^2de+3e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{24c^8}+\frac{b(6c^4d^2+16c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{72c^8}-\frac{be(8c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{120c^8}+\frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{7/2}}{56c^8}+\frac{1}{4}d^2x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{sech}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{sech}^{-1}(cx)\right)$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{1}{24}\left(6ad^2x^4+8adex^6+3ae^2x^8-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(144e^2+8c^2e(56d+9ex^2)+c^4(420d^2+224dex^2+54e^2x^4)+3c^6(70d^2x^2+56dex^4+1\right.}{105c^8}+bx^4\left(6d^2+8dex^2+3e^2x^4\right)\operatorname{sech}^{-1}(cx)\right)$$

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] (6*a*d^2*x^4 + 8*a*d*e*x^6 + 3*a*e^2*x^8 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1
+ c*x)*(144*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 5
4*e^2*x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/(105*c^8) + b*x
^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcSech[c*x])/24
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.71

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}x^4d^2\right)+\frac{b\left(\frac{c^4\operatorname{arcsech}(cx)e^2x^8}{8}+\frac{c^4\operatorname{arcsech}(cx)de\,x^6}{3}+\frac{\operatorname{arcsech}(cx)d^2x^4c^4}{4}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+}{cx}}}{}\right.}{}$
derivativedivides	$-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^3}{3}-\frac{\left(e\,c^2x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)c^8d^4}{24e^2}+\frac{\operatorname{arcsech}(cx)c^8d^2x^4}{4}+\frac{e\operatorname{arcsech}(cx)c^8d\,x^6}{3}+\frac{e^2\operatorname{arcsech}(cx)c^8x}{8}\right.}{}$
default	$-\frac{a\left(\frac{c^2d\left(e\,c^2x^2+c^2d\right)^3}{3}-\frac{\left(e\,c^2x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}+\frac{b\left(-\frac{\operatorname{arcsech}(cx)c^8d^4}{24e^2}+\frac{\operatorname{arcsech}(cx)c^8d^2x^4}{4}+\frac{e\operatorname{arcsech}(cx)c^8d\,x^6}{3}+\frac{e^2\operatorname{arcsech}(cx)c^8x}{8}\right.}{}$

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*x^4*d^2)+b/c^4*(1/8*c^4*arcsech(c*x)*e^2*x^8
+1/3*c^4*arcsech(c*x)*d*e*x^6+1/4*arcsech(c*x)*d^2*x^4*c^4-1/2520/c^3*(-(c*
x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c
^6*d^2*x^2+54*c^4*e^2*x^4+224*c^4*d*e*x^2+420*c^4*d^2+72*c^2*e^2*x^2+448*c^
2*d*e+144*e^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.82

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^7e^2x^8+840\,ac^7dex^6+630\,ac^7d^2x^4+105\left(3\,bc^7e^2x^8+8\,bc^7dex^6+6\,bc^7d^2x^4\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-}{}$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*e^2*x^8 + 840*a*c^7*d*e*x^6 + 630*a*c^7*d^2*x^4 + 105*(3*
b*c^7*e^2*x^8 + 8*b*c^7*d*e*x^6 + 6*b*c^7*d^2*x^4)*log((c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) + 1)/(c*x)) - (45*b*c^6*e^2*x^7 + 6*(28*b*c^6*d*e + 9*b*c^4
*e^2)*x^5 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^3 + 4*(105*b
*c^4*d^2 + 112*b*c^2*d*e + 36*b*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.19

$$\int x^3\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\begin{cases}\frac{ad^2x^4}{4}+\frac{adex^6}{3}+\frac{ae^2x^8}{8}+\frac{bd^2x^4\operatorname{asech}(cx)}{4}+\frac{bdex^6\operatorname{asech}(cx)}{3}+\frac{be^2x^8\operatorname{asech}(cx)}{8}-\frac{bd^2x^2\sqrt{-c^2x^2+1}}{12c^2}-\frac{bdex^4\sqrt{-c^2x^2+1}}{15c^2}-\frac{be^2x^6}{} \\ (a+\infty b)\left(\frac{d^2x^4}{4}+\frac{dex^6}{3}+\frac{e^2x^8}{8}\right)\end{cases}$$

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*asech
(c*x)/4 + b*d*e*x**6*asech(c*x)/3 + b*e**2*x**8*asech(c*x)/8 - b*d**2*x**2*
sqrt(-c**2*x**2 + 1)/(12*c**2) - b*d*e*x**4*sqrt(-c**2*x**2 + 1)/(15*c**2)
- b*e**2*x**6*sqrt(-c**2*x**2 + 1)/(56*c**2) - b*d**2*sqrt(-c**2*x**2 + 1)/
(6*c**4) - 4*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 3*b*e**2*x**4*sqrt
(-c**2*x**2 + 1)/(140*c**4) - 8*b*d*e*sqrt(-c**2*x**2 + 1)/(45*c**6) - b*e*
*2*x**2*sqrt(-c**2*x**2 + 1)/(35*c**6) - 2*b*e**2*sqrt(-c**2*x**2 + 1)/(35*
c**8), Ne(c, 0)), ((a + oo*b)*(d**2*x**4/4 + d*e*x**6/3 + e**2*x**8/8), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.88

$$\int x^3\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{1}{8}\,ae^2x^8+\frac{1}{3}\,adex^6+\frac{1}{4}\,ad^2x^4$$
$$+\frac{1}{12}\left(3\,x^4\operatorname{arsech}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)bd^2$$
$$+\frac{1}{45}\left(15\,x^6\operatorname{arsech}(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)bde$$
$$+\frac{1}{280}\left(35\,x^8\operatorname{arsech}(cx)+\frac{5\,c^6x^7\left(\frac{1}{c^2x^2}-1\right)^{\frac{7}{2}}-21\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}+35\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-35\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^7}\right.$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arcsech(c*x) +
(c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d^2 +
```

```
1/45*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3
*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*d*e + 1/280*(
35*x^8*arcsech(c*x) + (5*c^6*x^7*(1/(c^2*x^2) - 1)^(7/2) - 21*c^4*x^5*(1/(c
^2*x^2) - 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 35*x*sqrt(1/(c^2*
x^2) - 1))/c^7)*b*e^2
```

Giac [F]

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^2 (b \,\text{arsech}\,(cx)+a)x^3 \, dx$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int x^3 \left(e\,x^2+d\right)^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^3*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.107 $\int x(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 230

$$\int x\left(d+ex^2\right)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx=-\frac{b(3c^4d^2+3c^2de+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^6}+\frac{be(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{18c^6}-\frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{30c^6}+\frac{(d+ex^2)^3\left(a+b\text{sech}^{-1}(cx)\right)}{6e}-\frac{bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6e}$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arcsech(c*x))/e+1/18*b*e*(3*c^2*d+2*e)*(-c^2*x^2+1)^(3
/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/30*b*e^2*(-c^2*x^2+1)^(5/2)*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/c^6-1/6*b*d^3*arctanh((-c^2*x^2+1)^(1/2))*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)/e-1/6*b*(3*c^4*d^2+3*c^2*d*e+e^2)*(1/(c*x+1))^(1/
2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6434, 531, 457, 90, 65, 214}

$$\int x(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{(d+ex^2)^3\left(a+b\text{sech}^{-1}(cx)\right)}{6e} - \frac{bd^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6e} + \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}(3c^2d+2e)}{18c^6} - \frac{be^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{5/2}}{30c^6} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(3c^4d^2+3c^2de+e^2)}{6c^6}$$

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/6*(b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sq
rt[1 - c^2*x^2])/c^6 + (b*e*(3*c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c
*x]*(1 - c^2*x^2)^(3/2))/(18*c^6) - (b*e^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*
x]*(1 - c^2*x^2)^(5/2))/(30*c^6) + ((d + e*x^2)^3*(a + b*ArcSech[c*x]))/(6*
e) - (b*d^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/
(6*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6434

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Dist[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex^2)^3\left(a+b\,\text{sech}^{-1}(cx)\right)}{6e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^3}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{6e}\\
&= \frac{(d+ex^2)^3\left(a+b\,\text{sech}^{-1}(cx)\right)}{6e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^3}{x\sqrt{1-c^2x^2}}\,dx}{6e}\\
&= \frac{(d+ex^2)^3\left(a+b\,\text{sech}^{-1}(cx)\right)}{6e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{(d+ex)^3}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{12e}\\
&= \frac{(d+ex^2)^3\left(a+b\,\text{sech}^{-1}(cx)\right)}{6e}\\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{e(3c^4d^2+3c^2de+e^2)}{c^4\sqrt{1-c^2x}}+\frac{d^3}{x\sqrt{1-c^2x}}-\frac{e^2(3c^2d+2e)\sqrt{1-c^2x}}{c^4}+\frac{e^3(1-c^2x)^{3/2}}{c^4}\right)dx,x\right.}{12e}
\end{aligned}$$

$$= -\frac{b(3c^4d^2+3c^2de+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^6} + \frac{be(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{18c^6} - \frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{30c^6} + \frac{(d+ex^2)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6e} + \frac{\left(bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{12e}$$

$$= -\frac{b(3c^4d^2+3c^2de+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^6} + \frac{be(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{18c^6} - \frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{30c^6} + \frac{(d+ex^2)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6e} - \frac{\left(bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{6c^2e}$$

$$= -\frac{b(3c^4d^2+3c^2de+e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^6} + \frac{be(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{18c^6} - \frac{be^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{5/2}}{30c^6} + \frac{(d+ex^2)^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6e} - \frac{bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6e}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.60

$$\int x\left(d+ex^2\right)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{1}{6}ax^2\left(3d^2+3dex^2+e^2x^4\right) - \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(8e^2+2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4)\right)}{90c^6} + \frac{1}{6}bx^2\left(3d^2+3dex^2+e^2x^4\right)\operatorname{sech}^{-1}(cx)$$

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
[Out] (a*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4))/6 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
 c*x)*(8*e^2 + 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x
^4)))/(90*c^6) + (b*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcSech[c*x])/6
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.23

method	result
parts	$\frac{a(e\,x^2+d)^3}{6e}+\frac{b\left(\frac{c^2e^2\,\mathrm{arcsech}(cx)x^6}{6}+\frac{c^2e\,\,\mathrm{arcsech}(cx)x^4d}{2}+\frac{\mathrm{arcsech}(cx)x^2c^2d^2}{2}+\frac{c^2\,\mathrm{arcsech}(cx)d^3}{6e}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,(15c^6d^3}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^3}{6c^4e}+\frac{b\left(\frac{\mathrm{arcsech}(cx)c^6d^3}{6e}+\frac{\mathrm{arcsech}(cx)c^6d^2x^2}{2}+\frac{e\,\,\mathrm{arcsech}(cx)c^6d\,x^4}{2}+\frac{e^2\,\mathrm{arcsech}(cx)c^6x^6}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(15c^6d}{}\right.}{}}{}$
default	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^3}{6c^4e}+\frac{b\left(\frac{\mathrm{arcsech}(cx)c^6d^3}{6e}+\frac{\mathrm{arcsech}(cx)c^6d^2x^2}{2}+\frac{e\,\,\mathrm{arcsech}(cx)c^6d\,x^4}{2}+\frac{e^2\,\mathrm{arcsech}(cx)c^6x^6}{6}-\frac{\sqrt{-\frac{cx-1}{cx}}\,cx\sqrt{\frac{cx+1}{cx}}\,(15c^6d}{}\right.}{}}{}$

```
[In] int(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*(e*x^2+d)^3/e+b/c^2*(1/6*c^2*e^2*arcsech(c*x)*x^6+1/2*c^2*e*arcsech(c
*x)*x^4*d+1/2*arcsech(c*x)*x^2*c^2*d^2+1/6*c^2/e*arcsech(c*x)*d^3-1/90/c^3/
e*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(15*c^6*d^3*arctanh(1/(-c^2*x^
2+1)^(1/2))+45*c^4*d^2*e*(-c^2*x^2+1)^(1/2)+15*c^4*d*e^2*(-c^2*x^2+1)^(1/2)
*x^2+3*e^3*(-c^2*x^2+1)^(1/2)*c^4*x^4+30*c^2*d*e^2*(-c^2*x^2+1)^(1/2)+4*e^3
*c^2*x^2*(-c^2*x^2+1)^(1/2)+8*e^3*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.83

$$\int x\left(d+ex^2\right)^2\left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{15\,ac^5e^2x^6+45\,ac^5dex^4+45\,ac^5d^2x^2+15\,(bc^5e^2x^6+3\,bc^5dex^4+3\,bc^5d^2x^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(3\,bc^4e^2}{90\,c^5}$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^5*e^2*x^6 + 45*a*c^5*d*e*x^4 + 45*a*c^5*d^2*x^2 + 15*(b*c^5*e^
2*x^6 + 3*b*c^5*d*e*x^4 + 3*b*c^5*d^2*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) + 1)/(c*x)) - (3*b*c^4*e^2*x^5 + (15*b*c^4*d*e + 4*b*c^2*e^2)*x^3 +
 (45*b*c^4*d^2 + 30*b*c^2*d*e + 8*b*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))
/c^5
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

$$\int x(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\begin{cases}\frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\operatorname{asech}(cx)}{2}+\frac{bdex^4\operatorname{asech}(cx)}{2}+\frac{be^2x^6\operatorname{asech}(cx)}{6}-\frac{bd^2\sqrt{-c^2x^2+1}}{2c^2}-\frac{bdex^2\sqrt{-c^2x^2+1}}{6c^2}-\frac{be^2x^4}{} \\ (a+\infty b)\left(\frac{d^2x^2}{2}+\frac{dex^4}{2}+\frac{e^2x^6}{6}\right)\end{cases}$$

```
[In] integrate(x*(e*x**2+d)**2*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*asech
(c*x)/2 + b*d*e*x**4*asech(c*x)/2 + b*e**2*x**6*asech(c*x)/6 - b*d**2*sqrt(
-c**2*x**2 + 1)/(2*c**2) - b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(6*c**2) - b*e**
2*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - b*d*e*sqrt(-c**2*x**2 + 1)/(3*c**4)
 - 2*b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 4*b*e**2*sqrt(-c**2*x**2
+ 1)/(45*c**6), Ne(c, 0)), ((a + oo*b)*(d**2*x**2/2 + d*e*x**4/2 + e**2*x**
6/6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.80

$$\int x(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}\,ae^2x^6+\frac{1}{2}\,adex^4+\frac{1}{2}\,ad^2x^2+\frac{1}{2}\left(x^2\operatorname{arsech}(cx)-\frac{x\sqrt{\frac{1}{c^2x^2}-1}}{c}\right)bd^2$$
$$+\frac{1}{6}\left(3\,x^4\operatorname{arsech}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^3}\right)bde$$
$$+\frac{1}{90}\left(15\,x^6\operatorname{arsech}(cx)-\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}-1}}{c^5}\right)be^2$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arcsech(c*x) - x*s
qrt(1/(c^2*x^2) - 1)/c)*b*d^2 + 1/6*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*
x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d*e + 1/90*(15*x^6*arcs
ech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1
)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*e^2
```

Giac [F]

$$\int x(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)^2(b\,\text{arsech}\,(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)^2\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.108 $\int \frac{(d+ex^2)^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 370

$$\begin{aligned}\int \frac{(d+ex^2)^2\,(a+b\,\mathrm{sech}^{-1}(cx))}{x}\,dx = &-\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3}\\ &-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}\\ &+\frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}\\ &+dex^2(a+b\,\mathrm{sech}^{-1}(cx))+\frac{1}{4}e^2x^4(a+b\,\mathrm{sech}^{-1}(cx))\\ &-\frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}\\ &+\frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}\\ &-d^2(a+b\,\mathrm{sech}^{-1}(cx))\log\left(\frac{1}{x}\right)\\ &+\frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}\end{aligned}$$

```
[Out] d*e*x^2*(a+b*arcsech(c*x))+1/4*e^2*x^4*(a+b*arcsech(c*x))-d^2*(a+b*arcsech(
c*x))*ln(1/x)+1/2*I*b*d^2*arccsc(c*x)^2*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2
```

```
)/(1+1/c/x)^(1/2)-b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-
1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+b*d^2*arccsc(c*x)*ln(1/x)
*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/2*I*b*d^2*polylog(2
,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c
/x)^(1/2)-1/6*b*e*(6*c^2*d+e)*x*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c^3-1/12*b
*e^2*x^3*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6438, 272, 45, 5958, 6874, 465, 97, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x} dx = &-d^2 \log\left(\frac{1}{x}\right)\left(a+b\text{sech}^{-1}(cx)\right) \\ &+ dex^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right) \\ &+\frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} \\ &+\frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} \\ &-\frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} \\ &+\frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\log\left(\frac{1}{x}\right)\csc^{-1}(cx)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} \\ &-\frac{bex\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}(6c^2d+e)}{6c^3} \\ &-\frac{be^2x^3\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}{12c}\end{aligned}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] -1/6*(b*e*(6*c^2*d + e)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/c^3 - (b*e^
2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x^3)/(12*c) + ((I/2)*b*d^2*Sqrt[1 -
1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + d*e*x^
2*(a + b*ArcSech[c*x]) + (e^2*x^4*(a + b*ArcSech[c*x]))/4 - (b*d^2*Sqrt[1 -
 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x
)]*Sqrt[1 + 1/(c*x)]) + (b*d^2*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)
```

```
])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - d^2*(a + b*ArcSech[c*x])*Log[x^
(-1)] + ((I/2)*b*d^2*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])
])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 465

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2*e*(m + 1
))), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
```

```
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)^2\,(a+b\text{arccosh}(\frac{x}{c}))}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= dex^2(a+b\text{sech}^{-1}(cx)) + \frac{1}{4}e^2x^4(a+b\text{sech}^{-1}(cx)) - d^2(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{e(e+4dx^2)}{4x^4}+d^2\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= dex^2(a+b\text{sech}^{-1}(cx)) + \frac{1}{4}e^2x^4(a+b\text{sech}^{-1}(cx)) - d^2(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(-\frac{e(e+4dx^2)}{4x^4\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}} + \frac{d^2\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\right)dx, x, \frac{1}{x}\right)}{c}$$

$$= dex^2(a+b\text{sech}^{-1}(cx)) + \frac{1}{4}e^2x^4(a+b\text{sech}^{-1}(cx)) - d^2(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{(bd^2)\text{Subst}\left(\int \frac{\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{c} - \frac{(be)\text{Subst}\left(\int \frac{e+4dx^2}{x^4\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$= -\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + dex^2(a+b\text{sech}^{-1}(cx)) + \frac{1}{4}e^2x^4(a+b\text{sech}^{-1}(cx)) - d^2(a+b\text{sech}^{-1}(cx))\log\left(\frac{1}{x}\right) - \frac{(be(6c^2d+e))\text{Subst}\left(\int \frac{1}{x^2\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{6c^3} + \frac{\left(bd^2\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int \frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + dex^2\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right) + \frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{\left(bd^2\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + dex^2\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right) + \frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{\left(bd^2\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + \frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + dex^2\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right) + \frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{\left(2ibd^2\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}$$
$$+ \frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + dex^2\left(a+b\text{sech}^{-1}(cx)\right)$$
$$+ \frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right) - \frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$+ \frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$
$$+ \frac{\left(bd^2\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c}$$
$$+ \frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + dex^2\left(a+b\text{sech}^{-1}(cx)\right)$$
$$+ \frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right) - \frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$+ \frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$
$$- \frac{\left(ibd^2\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + \frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + dex^2\left(a+b\text{sech}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{sech}^{-1}(cx)\right) - \frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{bd^2\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - d^2\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = adex^2 + \frac{1}{4}ae^2x^4 - \frac{bde\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{c^2} - \frac{be^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(2+c^2x^2\right)}{12c^4} + bdex^2\text{sech}^{-1}(cx) + \frac{1}{4}be^2x^4\text{sech}^{-1}(cx) - \frac{1}{2}bd^2\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx) + 2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\right) + ad^2\log(x) + \frac{1}{2}bd^2\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 - (b*d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/c^2
 - (b*e^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(2 + c^2*x^2))/(12*c^4) + b*d
*e*x^2*ArcSech[c*x] + (b*e^2*x^4*ArcSech[c*x])/4 - (b*d^2*ArcSech[c*x]*(Arc
Sech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])]))/2 + a*d^2*Log[x] + (b*d^2*Poly
Log[2, -E^(-2*ArcSech[c*x])])/2
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.75

method	result
parts	$a\left(\frac{e^2x^4}{4}+de\,x^2+d^2\ln(x)\right)+b\left(\frac{\operatorname{arcsech}(cx)^2d^2}{2}+\frac{e\left(12\operatorname{arcsech}(cx)c^4d\,x^2+3e\operatorname{arcsech}(cx)c^4x^4-12\sqrt{-\cdots}\right.}{\cdots}\right.$
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{b\left(\frac{c^4d^2\operatorname{arcsech}(cx)^2}{2}+\frac{e\left(12\operatorname{arcsech}(cx)c^4d\,x^2+3e\operatorname{arcsech}(cx)c^4x^4-12\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{b\left(\frac{c^4d^2\operatorname{arcsech}(cx)^2}{2}+\frac{e\left(12\operatorname{arcsech}(cx)c^4d\,x^2+3e\operatorname{arcsech}(cx)c^4x^4-12\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x,method=_RETURNVERBOSE)
[Out] a*(1/4*e^2*x^4+d*e*x^2+d^2*ln(x))+b*(1/2*arcsech(c*x)^2*d^2+1/12/c^4*e*(12*
arcsech(c*x)*c^4*d*x^2+3*e*arcsech(c*x)*c^4*x^4-12*(-(c*x-1)/c/x)^(1/2)*((c
*x+1)/c/x)^(1/2)*c^3*d*x-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e*c^3*x^3
-2*((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*e*c*x+12*c^2*d+2*e)-d^2*arcsech
(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*d^2*polylog(2,-(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)^2(b\operatorname{arsech}(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="fricas")
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arcsech(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)^2}{x}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + integrate(b*e^2*x^3*log(sqrt(1/(
c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x)) + 2*b*d*e*x*log(sqrt(1/(c*x) + 1)*sq
rt(1/(c*x) - 1) + 1/(c*x)) + b*d^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1)
+ 1/(c*x))/x, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x, x)
```

3.109 $\int \frac{(d+ex^2)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 373

$$\begin{aligned}\int \frac{(d+ex^2)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3} \, dx = {} & \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2c} \\ & + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d^2\operatorname{sech}^{-1}(cx) \\ & - \frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{sech}^{-1}(cx)\right) \\ & - \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\ & + \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\ & - 2de\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\ & + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}\end{aligned}$$

```
[Out] 1/4*b*c^2*d^2*arcsech(c*x)-1/2*d^2*(a+b*arcsech(c*x))/x^2+1/2*e^2*x^2*(a+b*
arcsech(c*x))-2*d*e*(a+b*arcsech(c*x))*ln(1/x)+I*b*d*e*arccsc(c*x)^2*(1-1/c
^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-2*b*d*e*arccsc(c*x)*ln(1-(I/
c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^
```

```
(1/2)+2*b*d*e*arccsc(c*x)*ln(1/x)*(1-1/c^2/x^2)^(1/2)/(-1+1/c/x)^(1/2)/(1+1
/c/x)^(1/2)+I*b*d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)*(1-1/c^2/x^2)^
(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/4*b*c*d^2*(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2)/x-1/2*b*e^2*x*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {6438, 272, 45, 5958, 12, 6874, 97, 92, 54, 2365, 2363, 4721, 3798, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx = & -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} \\ & -2de\log\left(\frac{1}{x}\right)\left(a+b\text{sech}^{-1}(cx)\right) \\ & +\frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right)+\frac{1}{4}bc^2d^2\text{sech}^{-1}(cx) \\ & +\frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} \\ & +\frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} \\ & -\frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} \\ & +\frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\log\left(\frac{1}{x}\right)\csc^{-1}(cx)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} \\ & +\frac{bcd^2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}{4x}-\frac{be^2x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}{2c}\end{aligned}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(4*x) - (b*e^2*Sqrt[-1 + 1/(
c*x)]*Sqrt[1 + 1/(c*x)]*x)/(2*c) + (I*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*
x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*c^2*d^2*ArcSech[c*x])/4 -
 (d^2*(a + b*ArcSech[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcSech[c*x]))/2 - (2
*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(S
qrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (2*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCs
c[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - 2*d*e*(a + b*A
```

```
rcSech[c*x])*Log[x^(-1)] + (I*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*
I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2365

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + e1*(e2/(d1*d2))*x^2]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + e1*(e2/(d1*d2
))*x^2], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5958

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)^2\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right) - 2de\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{e^2}{x^2}+d^2x^2+4de\log(x)}{2\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right) - 2de\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{e^2}{x^2}+d^2x^2+4de\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right) - 2de\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(-\frac{e^2}{x^2\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}} + \frac{d^2x^2}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}} + \frac{4de\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\right)dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\operatorname{sech}^{-1}(cx)\right) - 2de\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{\left(bd^2\right)\operatorname{Subst}\left(\int\frac{x^2}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right)}{2c} + \frac{(2bde)\operatorname{Subst}\left(\int\frac{\log(x)}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right)}{c}-\frac{\left(be^2\right)\operatorname{Subst}\left(\int\frac{1}{x^2\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right)}{2c}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c}-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)-2de\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{1}{4}\left(bcd^2\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right) + \frac{\left(2bde\sqrt{1-\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{1}{4}bc^2d^2\operatorname{sech}^{-1}(cx)-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-2de\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{\left(2bde\sqrt{1-\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x}-\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{1}{4}bc^2d^2\operatorname{sech}^{-1}(cx)-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-2de\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{\left(2bde\sqrt{1-\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d^2\operatorname{sech}^{-1}(cx) - \frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{sech}^{-1}(cx)\right) + \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - 2de\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{\left(4ibde\sqrt{1-\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d^2\operatorname{sech}^{-1}(cx) - \frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\operatorname{sech}^{-1}(cx)\right) - \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - 2de\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{\left(2bde\sqrt{1-\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d^2\text{sech}^{-1}(cx) - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right) - \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - 2de\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{\left(ibde\sqrt{1-\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bcd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{4x} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2c} + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)^2}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{1}{4}bc^2d^2\text{sech}^{-1}(cx) - \frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{sech}^{-1}(cx)\right) - \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{2bde\sqrt{1-\frac{1}{c^2x^2}}\csc^{-1}(cx)\log\left(\frac{1}{x}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - 2de\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx$$

$$= \frac{1}{4}\left(-\frac{2ad^2}{x^2}+2ae^2x^2-\frac{2be^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{c^2}-\frac{2bd^2\text{sech}^{-1}(cx)}{x^2}+2be^2x^2\text{sech}^{-1}(cx)\right.$$
$$+\frac{bd^2\sqrt{\frac{1-cx}{1+cx}}\left(\sqrt{1-cx}(1+cx)+2c^2x^2\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{x^2\sqrt{1-cx}}$$
$$-4bde\text{sech}^{-1}(cx)\left(\text{sech}^{-1}(cx)+2\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)\right)+8ade\log(x)$$
$$\left.+4bde\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)\right)$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*e^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x
))/c^2 - (2*b*d^2*ArcSech[c*x])/x^2 + 2*b*e^2*x^2*ArcSech[c*x] + (b*d^2*Sqr
t[(1 - c*x)/(1 + c*x)]*(Sqrt[1 - c*x]*(1 + c*x) + 2*c^2*x^2*Sqrt[1 + c*x]*A
rcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))/(x^2*Sqrt[1 - c*x]) - 4*b*d*e*ArcSech
[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])]) + 8*a*d*e*Log[x] + 4*
b*d*e*PolyLog[2, -E^(-2*ArcSech[c*x])])/4
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.67

method	result
parts	$a\left(\frac{e^2x^2}{2}-\frac{d^2}{2x^2}+2de\ln(x)\right)+bde\,\text{arcsech}(cx)^2+\frac{b\,c^2d^2\,\text{arcsech}(cx)}{4}+\frac{bc\,d^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{4x}-\frac{b}{}$
derivativedivides	$c^2\left(\frac{a\,x^2e^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}+\frac{bde\,\text{arcsech}(cx)^2}{c^2}+\frac{b\,d^2\,\text{arcsech}(cx)}{4}+\frac{b\,d^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{4cx}-\frac{b\,\text{arcsech}}{2c^2}\right.$
default	$c^2\left(\frac{a\,x^2e^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}+\frac{bde\,\text{arcsech}(cx)^2}{c^2}+\frac{b\,d^2\,\text{arcsech}(cx)}{4}+\frac{b\,d^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{4cx}-\frac{b\,\text{arcsech}}{2c^2}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e^2*x^2-1/2*d^2/x^2+2*d*e*ln(x))+b*d*e*arcsech(c*x)^2+1/4*b*c^2*d^2*
arcsech(c*x)+1/4*b*c*d^2/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-1/2*b*d
```

```
^2/x^2*arcsech(c*x)+1/2*b*e^2*x^2*arcsech(c*x)-1/2*b/c*e^2*(-(c*x-1)/c/x)^(
1/2)*x*((c*x+1)/c/x)^(1/2)+1/2*b/c^2*e^2-2*b*e*d*arcsech(c*x)*ln(1+(1/c/x+(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-b*e*d*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^3} dx = \int \frac{(ex^2+d)^2(b \operatorname{arsech}(cx)+a)}{x^3} dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arcsech(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^3} dx = \int \frac{(a+b \operatorname{asech}(cx))(d+ex^2)^2}{x^3} dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**3,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{sech}^{-1}(cx))}{x^3} dx = \int \frac{(ex^2+d)^2(b \operatorname{arsech}(cx)+a)}{x^3} dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
[Out] 1/2*a*e^2*x^2 - 1/8*b*d^2*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*
x^2) - 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(
1/(c^2*x^2) - 1) - 1))/c + 4*arcsech(c*x)/x^2) + 2*a*d*e*log(x) - 1/2*a*d^2
/x^2 + integrate(b*e^2*x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))
 + 2*b*d*e*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^3} \, dx = \int \frac{(ex^2+d)^2 (b \operatorname{arsech}(cx)+a)}{x^3} \, dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 \left(a+b\text{sech}^{-1}(cx)\right)}{x^3} \, dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3} \, dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^2*(a + b*acosh(1/(c*x))))/x^3, x)
```

3.110 $\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 519

$$\begin{aligned}
\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx &= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{e}-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} \\
&+\frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+\frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}
\end{aligned}$$

```
[Out] x*(a+b*arcsech(c*x))/e-b*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c/e+1/2*(
a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2
```

```
)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arcsech(c*x))*ln(1
+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(
1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3
/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*
(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*polylog(2,-c
*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/
2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*poly
log(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*
d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6438, 5959, 5883, 94, 211, 5909, 5962, 5681, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{d+ex^2}\,dx = {} & \frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ & -\frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e^{3/2}} \\ & +\frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}} \\ & -\frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2e^{3/2}} \\ & +\frac{x(a+b\operatorname{sech}^{-1}(cx))}{e}-\frac{b\arctan\left(\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\right)}{ce} \\ & -\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}} \\ & +\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}} \\ & -\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}} \\ & +\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}\end{aligned}$$

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

```
[Out] (x*(a + b*ArcSech[c*x]))/e - (b*ArcTan[Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]
])/(c*e) + (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x
])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSech[c*x
])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(
3/2)) + (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSech[c*x])*
Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2
)) - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c
^2*d + e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((c
*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*e^(3/2)) + (b*S
qrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(2*e^(3/2))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a + b\text{arccosh}\left(\frac{x}{c}\right)}{x^2\,(e + dx^2)}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{ex^2} - \frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x^2} dx, x, \frac{1}{x}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{e+dx^2} dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}} dx, x, \frac{1}{x}\right)}{ce} + \frac{d\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{e} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x} dx, x, \frac{1}{x}\right)}{2e^{3/2}} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x} dx, x, \frac{1}{x}\right)}{2e^{3/2}} - \frac{b\text{Subst}\left(\int \frac{1}{\frac{1}{c}+\frac{x^2}{c}} dx, x, \sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{c^2e}$$

$$= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} + \frac{d\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)} dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}} + \frac{d\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)} dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}}$$

$$= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} + \frac{d\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x} dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}} + \frac{d\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x} dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}} + \frac{d\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x} dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}} + \frac{d\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x} dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}}$$

$$= \frac{x(a+b\text{sech}^{-1}(cx))}{e} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce}$$
$$+ \frac{\sqrt{-d}(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$
$$- \frac{\sqrt{-d}(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$
$$+ \frac{\sqrt{-d}(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$
$$- \frac{\sqrt{-d}(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$
$$- \frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}}$$
$$+ \frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}}$$
$$- \frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}}$$
$$+ \frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2e^{3/2}}$$

$$= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce}$$
$$+ \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$
$$- \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$
$$+ \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$
$$- \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$
$$- \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^{3/2}}$$
$$+ \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^{3/2}}$$
$$- \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^{3/2}}$$
$$+ \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^{3/2}}$$

$$= \frac{x(a+b\operatorname{sech}^{-1}(cx))}{e} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} + \frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.77

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{d+ex^2}\,dx$$

$$= \frac{4ac\sqrt{e}x - 4ac\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + b\left(4\sqrt{e}\left(cx\operatorname{sech}^{-1}(cx) - 2\arctan\left(\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(cx)\right)\right)\right) - 2ic\sqrt{d}\left(-4i\right.\right.}{}$$

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

```
[Out] (4*a*c*Sqrt[e]*x - 4*a*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*(4*Sqrt[e]
*(c*x*ArcSech[c*x] - 2*ArcTan[Tanh[ArcSech[c*x]/2]]) - (2*I)*c*Sqrt[d]*((-4
*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d]
 + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E
^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Arc
Sech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1
 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2,
 (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2,
```

```
((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]) + (2*I)*c*S
qrt[d]*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((
(-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[
c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[
d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c
*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])]))))/(4*c*e^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 54.98 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.79

method	result
parts	$\frac{ax}{e} - \frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{b\,\mathrm{arcsech}(cx)x}{e} - \frac{2b\arctan\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce} + \frac{bcd\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2\right.\right.}\right.}{}$
derivativedivides	$\frac{\frac{ac^3x}{e} - \frac{ac^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + bc^2\left(\frac{\mathrm{arcsech}(cx)cx}{e} + \frac{dc^2\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2c^2d+4\_R1^2e\right.}{}\right.}{}\right.}{}$
default	$\frac{\frac{ac^3x}{e} - \frac{ac^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + bc^2\left(\frac{\mathrm{arcsech}(cx)cx}{e} + \frac{dc^2\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2c^2d+4\_R1^2e\right.}{}\right.}{}\right.}{}$

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/e*x-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*arcsech(c*x)/e*x-2*b/c/e*
arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))+1/8*b*c/e^2*d*sum((_R1^2*c^2
*d+4*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x
-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(
```

```
1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/8*
b*c/e^2*d*sum((_R1^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(
c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)
*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^2}{ex^2 + d}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{d + ex^2}\,dx = \int \frac{x^2(a + b\,\text{asech}\,(cx))}{d + ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d),x)
[Out] Integral(x**2*(a + b*asech(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{d + ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^2}{ex^2 + d}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{d + ex^2}\,dx = \int \frac{x^2\,\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2 + d}\,dx$$

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2),x)
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2), x)
```

3.111 $\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 459

$$\begin{aligned}
\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d+ex^2}\,dx &= -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e} \\
&+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e} \\
&+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e}
\end{aligned}$$

```
[Out] -(a+b*arcsech(c*x))^2/b/e-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))^2)/e+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsech
(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsech(c*
x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^
2*d+e)^(1/2)))/e+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)^2)/e+1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,-c*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/e+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(c^2*d+e)^(1/2)))/e
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

$= \{6438, 5959, 5882, 3799, 2221, 2317, 2438, 5962, 5681\}$

$$\int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{d+ex^2}\,dx = \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2e} - \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{be} - \frac{\log\left(e^{-2\operatorname{sech}^{-1}(cx)}+1\right)(a+b\operatorname{sech}^{-1}(cx))}{e} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e}$$

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

```
[Out] -((a + b*ArcSech[c*x])^2/(b*e)) - ((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSe
ch[c*x])])/e + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(S
qrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-
d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSech[c*
x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e)
 + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqr
t[c^2*d + e])])/(2*e) + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/(2*e) + (b*Pol
yLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e)
+ (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(
2*e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d +
e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^
2*d + e])])/(2*e)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{ex} - \frac{dx\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}\\
&= \frac{\text{Subst}\left(\int x\tanh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\text{sech}^{-1}(cx)\right)}{be}\\
&\quad + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{e}\\
&= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2be} + \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{sech}^{-1}(cx)\right)}{be}\\
&\quad - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e}\\
&= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2be} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e}\\
&\quad + \frac{\text{Subst}\left(\int \log\left(1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\text{sech}^{-1}(cx)\right)}{e}\\
&\quad - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2e}\\
&\quad + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2e}
\end{aligned}
$$

$$= -\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{be} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e}$$

$$-\frac{b\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\text{sech}^{-1}(cx)}{b}\right)}\right)}{2e}$$

$$-\frac{\sqrt{-d}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2e}$$

$$-\frac{\sqrt{-d}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2e}$$

$$+\frac{\sqrt{-d}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2e}$$

$$+\frac{\sqrt{-d}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2e}$$

$$= -\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{sech}^{-1}(cx)}{b}\right)}\right)}{2e}$$
$$- \frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e}$$
$$- \frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e}$$
$$- \frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e}$$
$$- \frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e}$$

$$
\begin{aligned}
&= -\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{be} - \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+e^{-2\mathrm{sech}^{-1}(cx)}\right)}{e} \\
&+ \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{b\,\mathrm{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{b}\right)}\right)}{2e} \\
&- \frac{b\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\mathrm{sech}^{-1}(cx)}\right)}{2e} \\
&- \frac{b\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\mathrm{sech}^{-1}(cx)}\right)}{2e} \\
&- \frac{b\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\mathrm{sech}^{-1}(cx)}\right)}{2e} \\
&- \frac{b\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\mathrm{sech}^{-1}(cx)}\right)}{2e}
\end{aligned}
$$

$$= -\frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)^2}{be} - \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+e^{-2\mathrm{sech}^{-1}(cx)}\right)}{e} + \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{b\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{b\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{b\,\mathrm{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{b}\right)}\right)}{2e}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.87

$$\int \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

$$= \frac{4ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\mathrm{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\mathrm{sech}^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)+4ib\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\mathrm{arctanh}\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)}{}\right.}{}$$

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
```

```
[Out] ((4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c
*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + (4*I)*b*ArcSin
[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e]
)*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - 2*b*ArcSech[c*x]*Log[1 + E^(-2*A
rcSech[c*x])] + b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*S
qrt[d]*E^ArcSech[c*x])] - (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])
] + b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Ar
cSech[c*x])] - (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcS
```

```
ech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])
] + (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(S
qrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcSech[c*x]*Log
[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*b*
ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + a*Log[d + e*x^2] + b*PolyLog[2
, -E^(-2*ArcSech[c*x])] - b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e
]))/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e
]))/(c*Sqrt[d]*E^ArcSech[c*x])])/(2*e)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.45 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.10

method	result
parts	$\frac{a\ln\left(e\,x^{2}+d\right)}{2e}-\frac{b\,\mathrm{arcsech}(cx)\ln\left(1+i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{e}-\frac{b\,\mathrm{arcsech}(cx)\ln\left(1-i\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\right)}{e}-$
derivativedivides	$\frac{\frac{a\,c^{2}\ln\left(e\,c^{2}x^{2}+c^{2}d\right)}{2e}+b\,c^{2}\left(\frac{c^{2}d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\,\_Z^{4}+\left(2c^{2}d+4e\right)\_Z^{2}+c^{2}d\right)}\frac{\left(\_R1^{2}+1\right)\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-}}{\_R}\right)\right.}{\_R}\right.}{4e}\right.}{}$
default	$\frac{\frac{a\,c^{2}\ln\left(e\,c^{2}x^{2}+c^{2}d\right)}{2e}+b\,c^{2}\left(\frac{c^{2}d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\,\_Z^{4}+\left(2c^{2}d+4e\right)\_Z^{2}+c^{2}d\right)}\frac{\left(\_R1^{2}+1\right)\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-}}{\_R}\right)\right.}{\_R}\right.}{4e}\right.}{}$

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/e*ln(e*x^2+d)-b/e*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2)))-b/e*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
)-b/e*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*dilog(1-I*(1/
c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4*b*c^2*d/e*sum((_R1^2+1)/(_R1^2*c
^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf
```

```
(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4*b/e*sum((_R1^2*c^2*d+c^2*d+4*e)/
(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R
1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{d+ex^2}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x*arcsech(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{d+ex^2}\,dx=\int \frac{x(a+b\operatorname{asech}(cx))}{d+ex^2}\,dx$$

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d),x)
[Out] Integral(x*(a + b*asech(c*x))/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{d+ex^2}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d
), x) + 1/2*a*log(e*x^2 + d)/e
```

Giac [F]

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x}{ex^2+d}\,dx$$

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2),x)

[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2), x)

3.112 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{d+ex^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 469

$$\begin{aligned}\int \frac{a+b\operatorname{sech}^{-1}(cx)}{d+ex^2}\,dx &= \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&\quad-\frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&\quad+\frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&\quad-\frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&\quad-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\\
&\quad-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[Out] 1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)
^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsech(c*x))
*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d
+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)
/e^(1/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog
(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
```

```
)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b
*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/
2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6428, 5909, 5962, 5681, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{d+ex^2}\,dx = {} & \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & -\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2\sqrt{-d}\sqrt{e}} \\ & +\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & -\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2\sqrt{-d}\sqrt{e}} \\ & -\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ & -\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c
^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-
d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a
 + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*
d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*
E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*Pol
yLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*Sqr
t[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[
c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (
c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e
])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6428

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a+b\,\text{arccosh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\,\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\,\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{e}}$$

$$= \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\,\mathrm{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\,\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\mathrm{sech}^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\,\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\mathrm{sech}^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\,\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\mathrm{sech}^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\,\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\mathrm{sech}^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{(a+b\,\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\,\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\,\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\,\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.81

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{d+ex^2}\,dx$$

$$= \frac{2a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - 4b\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\text{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right) + 4b\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\text{ar}}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2),x]
```

```
[Out] (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[
d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqr
t[c^2*d + e]] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTa
nh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - I*b*Ar
cSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] - 2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqr
t[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*ArcSech[c*x]*Log
[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*ArcSech[c*x]*Log[1 - (I*(Sqr
t[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*ArcSin[Sqrt[1 -
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(
```

```
c*Sqrt[d]*E^ArcSech[c*x])] - I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Ar
cSech[c*x])] - I*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d
]*E^ArcSech[c*x])] + I*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqr
t[d]*E^ArcSech[c*x])] + I*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(
c*Sqrt[d]*E^ArcSech[c*x])] - I*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])])/(2*Sqrt[d]*Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.62 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.64

method	result
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{bc\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)\right.}{\_R1^2c^2d+c^2d}\right.}{2}$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+}\right.\right.}{\_R1^2c^2d+}\right.}{2}\right.}{}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+}\right.\right.}{\_R1^2c^2d+}\right.}{2}\right.}{}$

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*b*c*sum(_R1/(_R1^2*c^2*d+c^2*d+2*
e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog
((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(
2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*b*c*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsec
h(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/
x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*
e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2}\, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{ex^2 + d}\, dx$$

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e*x^2 + d), x)

Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2}\, dx = \int \frac{a + b\,\text{asech}\,(cx)}{d + ex^2}\, dx$$

[In] integrate((a+b*asech(c*x))/(e*x**2+d),x)

[Out] Integral((a + b*asech(c*x))/(d + e*x**2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2}\, dx = \text{Exception raised: ValueError}$$

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2}\, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{ex^2 + d}\, dx$$

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{d + ex^2} \, dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d} \, dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2),x)
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2), x)
```

3.113 $\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 417

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)}\,dx = \frac{(a+b\text{sech}^{-1}(cx))^2}{2bd} - \frac{(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$

```
[Out] 1/2*(a+b*arcsech(c*x))^2/b/d-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*a
rcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcs
ech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2
```

```
*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6438, 5959, 5962, 5681, 2221, 2317, 2438}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = -\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2d} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2d} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}$$

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (a + b*ArcSech[c*x])^2/(2*b*d) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])
*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) -
((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c
^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - (b*PolyLog[2, -((c*Sqrt[-d]*E^Arc
Sech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]
*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - (b*PolyLog[2, -((c*S
qrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d) - (b*PolyLog[2
, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$+\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$+\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}dx, x, \text{sech}^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2d}$$

$$=\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}+\frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2d}+\frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2d}+\frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2d}+\frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2d}$$

$$=\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 841, normalized size of antiderivative = 2.02

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\,(d+ex^2)}\,dx =$$

$$-\frac{b\operatorname{sech}^{-1}(cx)^2+4ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)+4ib\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{a}}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] -1/2*(b*ArcSech[c*x]^2 + (4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/S
qrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*
d + e]] + (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh
[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + b*ArcSec
h[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]
- (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqr
t[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcSech[c*x]*Log[1
 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*b*A
rcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcSech[c*x]*Log[1 - (I*(Sqr
t[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*b*ArcSin[Sqrt[
1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d
]*E^ArcSech[c*x])] - 2*a*Log[x] + a*Log[d + e*x^2] - b*PolyLog[2, ((-I)*(-S
qrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, (I*(-
Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, ((-I
)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - b*PolyLog[2, (
I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/d
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.06 (sec) , antiderivative size = 2081, normalized size of antiderivative = 4.99

method	result	size
parts	Expression too large to display	2081
derivativedivides	Expression too large to display	2108
default	Expression too large to display	2108

[In] int((a+b*arcsech(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] -1/2*a/d*ln(e*x^2+d)+a/d*ln(x)+b*(1/2/d*arcsech(c*x)^2-1/4*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/e/(c^2*d+e)/d*arcsech(c*x)^2+1/8*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/e/(c^2*d+e)/d*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c^2/d^2*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+1/4*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*c^2*arcsech(c*x)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c^4/d^3*e*arcsech(c*x)^2-1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c^4/d^3*e*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d*arcsech(c*x)^2+1/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c^2/d^2*arcsech(c*x)^2-1/4*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c^2/d^2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/4*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*c^2*arcsech(c*x)^2+1/8*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*c^2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^2*d+e)/c^2/d^2*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+1/4*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/e/(c^2*d+e)/d*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)-(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^2*d+e)/c^2/d^2*arcsech(c*x)^2+1/2*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^2*d+e)/c^2/d^2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2/d*sum((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e/(c^2*d+e)/d^3/c^4*arcsech(c*x)^2+1/2*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e/(c^2*d+e)/d^3/c^4*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))+(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e/(c^2*d+e)/d^3/c^4*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*

```
e))*arcsech(c*x)-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c^4/d^3*e*ln(1-d*c^2*(1/
c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))
*arcsech(c*x)+1/2*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d*arcsech(c*x)*ln(1-d*c^2*(
1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e
)))
```

Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{(ex^2+d)x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\text{asech}\left(cx\right)}{x\left(d+ex^2\right)}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d),x)
[Out] Integral((a + b*asech(c*x))/(x*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{(ex^2+d)x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="maxima")
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(sqrt(1/(c*x) + 1)*
sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^3 + d*x), x)
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2 + d\right)}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)),x)
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)), x)
```

3.114 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 523

$$
\begin{aligned}
\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx &= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d}-\frac{a}{dx}-\frac{b\text{sech}^{-1}(cx)}{dx} \\
&+\frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&-\frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+\frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&-\frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&-\frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+\frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&-\frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+\frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}
\end{aligned}
$$

```
[Out] -a/d/x-b*arcsech(c*x)/d/x+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3
/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*
(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*arcsech(c
*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2
)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*polylog(2,c*(1
/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2))
)*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*polylo
g(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e
)^(1/2)))*e^(1/2)/(-d)^(3/2)+b*c*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {6438, 5959, 5879, 75, 5909, 5962, 5681, 2221, 2317, 2438}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2(-d)^{3/2}} + \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2(-d)^{3/2}} - \frac{a}{dx} - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}} - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}} + \frac{bc\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}{d} - \frac{b\operatorname{sech}^{-1}(cx)}{dx}$$

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/d - a/(d*x) - (b*ArcSech[c*x])/(
d*x) + (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(S
qrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSech[c*x])*
Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(
3/2)) + (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSech[c*x])
*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^
(3/2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqr
t[c^2*d + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSe
ch[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[
2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*(-d)^(3/
2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2
*d + e])])/(2*(-d)^(3/2))
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{d}-\frac{e\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{d\,(e+dx^2)}\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}\\
&= -\frac{a}{dx}-\frac{b\text{Subst}\left(\int \text{arccosh}\left(\frac{x}{c}\right)\,dx, x, \frac{1}{x}\right)}{d}\\
&\quad+\frac{e\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d}\\
&= -\frac{a}{dx}-\frac{b\text{sech}^{-1}(cx)}{dx}+\frac{b\text{Subst}\left(\int \frac{x}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{cd}\\
&\quad+\frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d}+\frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d}
\end{aligned}
$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{2d}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{2d}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\text{sech}^{-1}(cx)}{dx}$$
$$+ \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$- \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$+ \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$- \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$- \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$+ \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$- \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$+ \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$
\begin{aligned}
&= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\operatorname{sech}^{-1}(cx)}{dx} \\
&+ \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&- \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+ \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&- \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&- \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2(-d)^{3/2}} \\
&+ \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2(-d)^{3/2}} \\
&- \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2(-d)^{3/2}} \\
&+ \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2(-d)^{3/2}}
\end{aligned}
$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b\mathrm{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e}(a+b\mathrm{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}(a+b\mathrm{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{\sqrt{e}(a+b\mathrm{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}(a+b\mathrm{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\mathrm{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.28 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.78

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx$$

$$= \frac{-4a\sqrt{d} - 4a\sqrt{e}x\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + b\left(4\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx) - 4\sqrt{d}\mathrm{sech}^{-1}(cx) - 2i\sqrt{e}x\left(-4i\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{}}{c\sqrt{}}}}{\sqrt{2}}\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] (-4*a*Sqrt[d] - 4*a*Sqrt[e]*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*(4*Sqrt[d]*Sq
rt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 4*Sqrt[d]*ArcSech[c*x] - (2*I)*Sqrt[e]*
x*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*S
qrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log
[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d +
 e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe
ch[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]
*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + Poly
```

Log[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyL
og[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]) + (2*
I)*Sqrt[e]*x*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcT
anh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + Ar
cSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(-Sqrt[e]
 + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c
*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d +
 e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe
ch[c*x])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Arc
Sech[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe
ch[c*x])])))/(4*d^(3/2)*x)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 52.02 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.71

method	result
parts	$-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{a}{dx}+\frac{bc\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{d}-\frac{b\,\mathrm{arcsech}(cx)}{dx}+\frac{bce\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right.}\right.}{}$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{d}-\frac{b\,\mathrm{arcsech}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+\right.}\right.}{}\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{d}-\frac{b\,\mathrm{arcsech}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+\right.}\right.}{}\right.$

[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] -a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/d/x+b*c/d*(-(c*x-1)/c/x)^(1/2)
*((c*x+1)/c/x)^(1/2)-b*arcsech(c*x)/d/x+1/2*b*c*e/d*sum(_R1/(_R1^2*c^2*d+c^
2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)
+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*
_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*b*c*e/d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*

```
e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog
((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(
2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)x^2}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{x^2(d+ex^2)}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**2/(e*x**2+d),x)
[Out] Integral((a + b*asech(c*x))/(x**2*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2 (d + ex^2)} \, dx = \int \frac{b \,\text{arsech}\,(cx) + a}{(ex^2 + d)x^2} \, dx$$

[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2 (d + ex^2)} \, dx = \int \frac{a + b \,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^2 \,(e\,x^2 + d)} \, dx$$

[In] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)),x)

[Out] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)), x)

3.115 $\int \frac{x^5 \left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 631

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=-\frac{b\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\,x}{2ce^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)^2}{be^3}-\frac{bd\sqrt{-1+\frac{1}{c^2x^2}}\,\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e^3}-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(cx)}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{bd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

```
[Out] 1/2*d*(a+b*arcsech(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arcsech(c*x))/e^2+2*d*(
a+b*arcsech(c*x))^2/b/e^3+2*d*(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2)/e^3-d*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arc
sech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1
```

```
/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsec
h(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))^2)/e^3-b*d*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)
^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-b*d*polylog(2
,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^
(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*b*d*arctanh((c^2*d+e)^(1/2)/c/x/e^(
1/2)/(-1+1/c^2/x^2)^(1/2))*(-1+1/c^2/x^2)^(1/2)/e^(5/2)/(c^2*d+e)^(1/2)/(-1
+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/2*b*x*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/c/e^
2
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules

used = {6438, 5959, 5883, 97, 5882, 3799, 2221, 2317, 2438, 5957, 533, 385, 214, 5962, 5681}

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{e^3}-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}-\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{e^3}+\frac{2d\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^3}+\frac{2d\log\left(e^{-2\operatorname{sech}^{-1}(cx)}+1\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3}+\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e^2}-\frac{bd\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{2e^{5/2}\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\sqrt{c^2d+e}}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^3}-\frac{bx\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}{2ce^2}$$

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(b*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(c*e^2) + (d*(a + b*ArcSech
[c*x]))/(2*e^2*(e + d/x^2)) + (x^2*(a + b*ArcSech[c*x]))/(2*e^2) + (2*d*(a
+ b*ArcSech[c*x])^2)/(b*e^3) - (b*d*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2
*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]*S
qrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (2*d*(a + b*ArcSech[c*x])*Log[1 + E^
```

```
(-2*ArcSech[c*x])])/e^3 - (d*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^Arc
Sech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcSech[c*x])*Log[
1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a +
 b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d
+ e])])/e^3 - (d*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (b*d*PolyLog[2, -E^(-2*ArcSech[c*x])])/e
^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d +
e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^
2*d + e])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] +
Sqrt[c^2*d + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt
[e] + Sqrt[c^2*d + e])])/e^3
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
```

```
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x^3\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{e^2x^3} - \frac{2d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e^3x} + \frac{d^2x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} + \frac{2d^2x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e^3\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}\right)\\
&= \frac{(2d)\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3}\\
&\quad - \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x^3}\,dx, x, \frac{1}{x}\right)}{e^2} - \frac{d^2\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e^2}
\end{aligned}$$

$$
\begin{aligned}
&=\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2}\\
&-\frac{(2d)\text{Subst}\left(\int x\tanh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx,x,a+b\text{sech}^{-1}(cx)\right)}{be^3}\\
&-\frac{(2d^2)\,\text{Subst}\left(\int\left(-\frac{\sqrt{-d}\left(a+b\,\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\,\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx,x,\frac{1}{x}\right)}{e^3}\\
&-\frac{b\text{Subst}\left(\int\frac{1}{x^2\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx,x,\frac{1}{x}\right)}{2ce^2}-\frac{(bd)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(e+dx^2)}\,dx,x,\frac{1}{x}\right)}{2ce^2}\\
&=-\frac{b\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2ce^2}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2}\\
&+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)^2}{be^3}-\frac{(-d)^{3/2}\text{Subst}\left(\int\frac{a+b\,\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{e^3}\\
&+\frac{(-d)^{3/2}\text{Subst}\left(\int\frac{a+b\,\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{e^3}\\
&-\frac{(4d)\text{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx,x,a+b\text{sech}^{-1}(cx)\right)}{be^3}\\
&-\frac{\left(bd\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x^2}{c^2}}(e+dx^2)}\,dx,x,\frac{1}{x}\right)}{2ce^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}
\end{aligned}
$$

$$= -\frac{b\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2ce^2} + \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e^2} + \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^3} + \frac{2d\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^3} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{e^3} + \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{e^3} - \frac{(2d)\text{Subst}\left(\int \log\left(1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{sech}^{-1}(cx)\right)}{e^3} - \frac{\left(bd\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int \frac{1}{e-\left(d+\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2ce^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{b\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2ce^2} + \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2e^2} + \frac{2d\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^3} - \frac{bd\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{2d\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^3} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{e^3} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{e^3} + \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{e^3} + \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{e^3} + \frac{(bd)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{sech}^{-1}(cx)}{b}\right)}\right)}{e^3}$$

$$= -\frac{b\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2ce^2} + \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{2e^2}$$

$$+ \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)^2}{be^3} - \frac{bd\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$+ \frac{2d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e^3}$$

$$- \frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{bd\,\text{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\text{sech}^{-1}(cx)}{b}\right)}\right)}{e^3}$$

$$+ \frac{(bd)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{e^3}$$

$$+ \frac{(bd)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{e^3}$$

$$+ \frac{(bd)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{e^3}$$

$$+ \frac{(bd)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{e^3}$$

$$= -\frac{b\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2ce^2} + \frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{2e^2}$$

$$+ \frac{2d\left(a+b\,\text{sech}^{-1}(cx)\right)^2}{be^3} - \frac{bd\sqrt{-1+\frac{1}{c^2x^2}}\,\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$+ \frac{2d\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e^3}$$

$$- \frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d\left(a+b\,\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{bd\,\text{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\,\text{sech}^{-1}(cx)}{b}\right)}\right)}{e^3}$$

$$+ \frac{(bd)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{e^3}$$

$$+ \frac{(bd)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{e^3}$$

$$+ \frac{(bd)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{e^3}$$

$$+ \frac{(bd)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{e^3}$$

$$= -\frac{b\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{2ce^2} + \frac{d(a+b\operatorname{sech}^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{2e^2}$$
$$+ \frac{2d(a+b\operatorname{sech}^{-1}(cx))^2}{be^3} - \frac{bd\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$+ \frac{2d(a+b\operatorname{sech}^{-1}(cx))\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^3}$$
$$- \frac{d(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} - \frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{bd\operatorname{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{sech}^{-1}(cx)}{b}\right)}\right)}{e^3}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.54 (sec) , antiderivative size = 1278, normalized size of antiderivative = 2.03

$$\int \frac{x^5(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx =$$

$$-\frac{-2aex^2+\frac{2ad^2}{d+ex^2}+4ad\log(d+ex^2)+b\left(\frac{2e\sqrt{\frac{1-cx}{1+cx}}}{c^2}+\frac{2ex\sqrt{\frac{1-cx}{1+cx}}}{c}-2ex^2\operatorname{sech}^{-1}(cx)+\frac{d^{3/2}\operatorname{sech}^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x}+\frac{d^{3/2}\operatorname{sec}}{\sqrt{d}+}\right.}{}$$

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*((2*e*S
qrt[(1 - c*x)/(1 + c*x)])/c^2 + (2*e*x*Sqrt[(1 - c*x)/(1 + c*x)])/c - 2*e*x
```

```
^2*ArcSech[c*x] + (d^(3/2)*ArcSech[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)
*ArcSech[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e
])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c
*x]/2])/Sqrt[c^2*d + e]] + (16*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*
d + e]] - 8*d*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] + 4*d*ArcSech[c*x]*
Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (8*I)
*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] -
Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 4*d*ArcSech[c*x]*Log[1 + (I
*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (8*I)*d*ArcSin
[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 4*d*ArcSech[c*x]*Log[1 - (I*(Sqrt[e
] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (8*I)*d*ArcSin[Sqrt[1 -
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] + 4*d*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c
^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (8*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e
])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]
*E^ArcSech[c*x])] + 2*d*Log[x] - 2*d*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*
x*Sqrt[(1 - c*x)/(1 + c*x)]] + (d*Sqrt[e]*Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[
(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d +
 e]))/(I*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + (d*Sqrt[e]*Log[(2*Sqrt[e]
*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*
d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + 4*d*P
olyLog[2, -E^(-2*ArcSech[c*x])] - 4*d*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 4*d*PolyLog[2, (I*(-Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 4*d*PolyLog[2, ((-I)*(Sqrt[e] +
Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 4*d*PolyLog[2, (I*(Sqrt[e]
+ Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]))/e^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.18 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.25

method	result
parts	$\frac{a x^2}{2e^2} - \frac{a d \ln(e x^2+d)}{e^3} - \frac{a d^2}{2e^3(e x^2+d)} + \frac{b\left(\frac{c^4\left(2 \operatorname{arcsech}(cx) c^4 d x^2 + e \operatorname{arcsech}(cx) c^4 x^4 - \sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\, c^3 d x - \sqrt{-\frac{cx-1}{cx}} \cdots\right.}{2\left(e c^2 x^2 + c^2 d\right) e^2} \cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{a c^6 x^2}{2e^2} - \frac{a c^6 d \ln\left(e c^2 x^2 + c^2 d\right)}{e^3} - \frac{a c^8 d^2}{2e^3\left(e c^2 x^2 + c^2 d\right)} + b c^4 \left(\frac{2 \operatorname{arcsech}(cx) c^4 d x^2 + e \operatorname{arcsech}(cx) c^4 x^4 - \sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\, c^3 d x - \sqrt{-\frac{cx-1}{cx}} \cdots}{2\left(e c^2 x^2 + c^2 d\right) e^2} \cdots\right.}{\cdots}$
default	$\frac{\frac{a c^6 x^2}{2e^2} - \frac{a c^6 d \ln\left(e c^2 x^2 + c^2 d\right)}{e^3} - \frac{a c^8 d^2}{2e^3\left(e c^2 x^2 + c^2 d\right)} + b c^4 \left(\frac{2 \operatorname{arcsech}(cx) c^4 d x^2 + e \operatorname{arcsech}(cx) c^4 x^4 - \sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\, c^3 d x - \sqrt{-\frac{cx-1}{cx}} \cdots}{2\left(e c^2 x^2 + c^2 d\right) e^2} \cdots\right.}{\cdots}$

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2/e^2-a*d/e^3*ln(e*x^2+d)-1/2*a*d^2/e^3/(e*x^2+d)+b/c^6*(1/2*c^4*(2
*arcsech(c*x)*c^4*d*x^2+e*arcsech(c*x)*c^4*x^4-(-(c*x-1)/c/x)^(1/2)*((c*x+1
)/c/x)^(1/2)*c^3*d*x-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e*c^3*x^3+c^2
*d+e*c^2*x^2)/(c^2*e*x^2+c^2*d)/e^2+1/2*(e*(c^2*d+e))^(1/2)/e^3/(c^2*d+e)*d
*c^6*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*
d+4*e)/(c^2*d*e+e^2)^(1/2))+2/e^3*d*c^6*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2)))+2/e^3*d*c^6*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2)))+2/e^3*d*c^6*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2)))+2/e^3*d*c^6*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2
)))-1/2/e^3*d*c^6*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcs
ech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/
c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+
4*e)*_Z^2+c^2*d))-1/2/e^3*d^2*c^8*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(ar
csech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-
1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*
d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^5 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx=\int \frac{\left(b \,\text{arsech}\,(cx)+a\right)x^5}{\left(ex^2+d\right)^2} \, dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^5*arcsech(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^5 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx=\int \frac{x^5 (a+b \,\text{asech}\,(cx))}{\left(d+ex^2\right)^2} \, dx$$

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**5*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^5 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx=\int \frac{\left(b \,\text{arsech}\,(cx)+a\right)x^5}{\left(ex^2+d\right)^2} \, dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] -1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*integ
rate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^2*x^4 + 2*d*
e*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{x^5 \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx=\int \frac{\left(b \,\text{arsech}\,(cx)+a\right)x^5}{\left(ex^2+d\right)^2} \, dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.116 $\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 580

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^2}+\frac{b\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^2}+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$

```
[Out] 1/2*(-a-b*arcsech(c*x))/e/(e+d/x^2)-(a+b*arcsech(c*x))^2/b/e^2-(a+b*arcsech
(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e^2+1/2*(a+b*arcs
ech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arc
sech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(
```

```
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*b*polyl
og(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)/e^2+1/2*b*polylog(2,-c*
(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))/e^2+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*b*polylog(
2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^
(1/2)))/e^2+1/2*b*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))
*(-1+1/c^2/x^2)^(1/2)/e^(3/2)/(c^2*d+e)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1
/2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules

used = {6438, 5959, 5882, 3799, 2221, 2317, 2438, 5957, 533, 385, 214, 5962, 5681}

$$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e^2} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2e^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(\frac{d}{x^2}+e\right)} - \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{be^2} - \frac{\log\left(e^{-2\operatorname{sech}^{-1}(cx)}+1\right)(a+b\operatorname{sech}^{-1}(cx))}{e^2} + \frac{b\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{2e^{3/2}\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\sqrt{c^2d+e}} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e^2}$$

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcSech[c*x])/(e*(e + d/x^2)) - (a + b*ArcSech[c*x])^2/(b*e^2)
+ (b*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/
(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c
*x)]) - ((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e^2 + ((a + b*A
rcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]
)])/(2*e^2) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sq
rt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[
-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSech
[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2
*e^2) + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/(2*e^2) + (b*PolyLog[2, -((c*S
```

```
qrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e^2) + (b*PolyLog
[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + (b*
PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*
e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]
)])/(2*e^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{e^2x} - \frac{dx\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{dx\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int x\tanh\left(\frac{a}{b}-\frac{x}{b}\right)dx, x, a+b\text{sech}^{-1}(cx)\right)}{be^2} + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e^2} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2ce}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2be^2} + \frac{2\text{Subst}\left(\int \frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\text{sech}^{-1}(cx)\right)}{be^2} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^2} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^2} + \frac{\left(b\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2ce\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$=-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2be^2}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e^2}$$
$$+\frac{\text{Subst}\left(\int\log\left(1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)dx,x,a+b\text{sech}^{-1}(cx)\right)}{e^2}$$
$$-\frac{\sqrt{-d}\text{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}dx,x,\text{sech}^{-1}(cx)\right)}{2e^2}$$
$$+\frac{\sqrt{-d}\text{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}dx,x,\text{sech}^{-1}(cx)\right)}{2e^2}$$
$$+\frac{\left(b\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{1}{e-\left(d+\frac{e}{c^2}\right)x^2}dx,x,\frac{1}{\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2ce\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$=-\frac{a+b\text{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{be^2}+\frac{b\sqrt{-1+\frac{1}{c^2x^2}}\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e^2}$$
$$-\frac{b\text{Subst}\left(\int\frac{\log(1+x)}{x}dx,x,e^{2\left(\frac{a}{b}-\frac{a+b\text{sech}^{-1}(cx)}{b}\right)}\right)}{2e^2}$$
$$-\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\text{sech}^{-1}(cx)\right)}{2e^2}$$
$$-\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\text{sech}^{-1}(cx)\right)}{2e^2}$$
$$+\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}dx,x,\text{sech}^{-1}(cx)\right)}{2e^2}$$
$$+\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}dx,x,\text{sech}^{-1}(cx)\right)}{2e^2}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^2} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^2}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$

$$+ \frac{b\operatorname{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{sech}^{-1}(cx)}{b}\right)}\right)}{2e^2}$$

$$- \frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^2}$$

$$- \frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^2}$$

$$- \frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^2}$$

$$- \frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^2}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^2} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^2}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$

$$+ \frac{b\operatorname{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{sech}^{-1}(cx)}{b}\right)}\right)}{2e^2}$$

$$- \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^2}$$

$$- \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^2}$$

$$- \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^2}$$

$$- \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^2}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^2} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^2}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{b\operatorname{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{sech}^{-1}(cx)}{b}\right)}\right)}{2e^2}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 1208, normalized size of antiderivative = 2.08

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$$

$$= \frac{\frac{2ad}{d+ex^2} + \frac{b\sqrt{d}\operatorname{sech}^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{b\sqrt{d}\operatorname{sech}^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} + 8ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right) + 8i}{}$$

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (
b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (8*I)*b*ArcSin[Sqrt[1 - (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[A
rcSech[c*x]/2])/Sqrt[c^2*d + e]] + (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sq
```

```
rt[c^2*d + e]] - 4*b*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] + 2*b*ArcSec
h[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]
- (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqr
t[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log
[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (4*I)*b
*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log[1 - (I*
(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[S
qrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d
+ e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] +
 Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*
Sqrt[d]*E^ArcSech[c*x])] + 2*b*Log[x] + 2*a*Log[d + e*x^2] - 2*b*Log[1 + Sq
rt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]] + (b*Sqrt[e]*Log[(
(2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[
e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e]
 + (b*Sqrt[e]*Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)
 + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*
x)])/Sqrt[c^2*d + e] + 2*b*PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*b*PolyLog[2
, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*Pol
yLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*
PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] -
 2*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]
)/(4*e^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.71 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.11

method	result
parts	$\frac{a\ln\left(e\,x^{2}+d\right)}{2e^{2}}+\frac{ad}{2e^{2}\left(e\,x^{2}+d\right)}-\frac{b\,c^{2}x^{2}\,\mathrm{arcsech}(cx)}{2\left(e\,c^{2}x^{2}+c^{2}d\right)e}-\frac{b\sqrt{e\left(c^{2}d+e\right)}\,\mathrm{arctanh}\left(\frac{2c^{2}d\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^{2}+2c^{2}d+4e}{4\sqrt{c^{2}de+e^{2}}}\right.}{2e^{2}\left(c^{2}d+e\right)}$
derivativedivides	$\frac{\frac{a\,c^{6}d}{2e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)}+\frac{a\,c^{4}\ln\left(e\,c^{2}x^{2}+c^{2}d\right)}{2e^{2}}+b\,c^{4}\left(-\frac{c^{2}x^{2}\,\mathrm{arcsech}(cx)}{2\left(e\,c^{2}x^{2}+c^{2}d\right)e}-\frac{\sqrt{e\left(c^{2}d+e\right)}\,\mathrm{arctanh}\left(\frac{2c^{2}d\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^{2}+2c^{2}d}{4\sqrt{c^{2}de+e^{2}}}\right.}{2e^{2}\left(c^{2}d+e\right)}\right.}{}$
default	$\frac{\frac{a\,c^{6}d}{2e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)}+\frac{a\,c^{4}\ln\left(e\,c^{2}x^{2}+c^{2}d\right)}{2e^{2}}+b\,c^{4}\left(-\frac{c^{2}x^{2}\,\mathrm{arcsech}(cx)}{2\left(e\,c^{2}x^{2}+c^{2}d\right)e}-\frac{\sqrt{e\left(c^{2}d+e\right)}\,\mathrm{arctanh}\left(\frac{2c^{2}d\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)^{2}+2c^{2}d}{4\sqrt{c^{2}de+e^{2}}}\right.}{2e^{2}\left(c^{2}d+e\right)}\right.}{}$

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/e^2*ln(e*x^2+d)+1/2*a*d/e^2/(e*x^2+d)-1/2*b*c^2*x^2*arcsech(c*x)/(c^2
*e*x^2+c^2*d)/e-1/2*b*(e*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arctanh(1/4*(2*c^2*
d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/
2))-b/e^2*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e
^2*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e^2*dilo
g(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e^2*dilog(1-I*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4*b/e^2*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2
*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=Root
Of(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4*b*c^2/e^2*d*sum((_R1^2+1)/(_R1
^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=Ro
otOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^3*arcsech(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^3(a+b\,\text{asech}\,(cx))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**3*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(sqrt
(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \left(a + b \operatorname{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \int \frac{x^3 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{\left(e x^2 + d\right)^2} \, dx$$

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.117 $\int \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 147

$$\int \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\mathrm{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2de}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}}$$

```
[Out] 1/2*(-a-b*arcsech(c*x))/e/(e*x^2+d)+1/2*b*arctanh((-c^2*x^2+1)^(1/2))*(1/(c
*x+1))^(1/2)*(c*x+1)^(1/2)/d/e-1/2*b*arctanh(e^(1/2)*(-c^2*x^2+1)^(1/2)/(c^
2*d+e)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/e^(1/2)/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6434, 531, 457, 88, 65, 214}

$$\int \frac{x\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\mathrm{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2de}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}}$$

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcSech[c*x])/(e*(d + e*x^2)) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
+ c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(2*d*e) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
 + c*x]*ArcTanh[(Sqrt[e]*Sqrt[1 - c^2*x^2])/Sqrt[c^2*d + e]])/(2*d*Sqrt[e]*
Sqrt[c^2*d + e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6434

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Dist[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\left(d+ex^2\right)}\,dx}{2e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-c^2x^2}\left(d+ex^2\right)}\,dx}{2e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}(d+ex)}\,dx,x,x^2\right)}{4e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-c^2x}(d+ex)}\,dx,x,x^2\right)}{4d} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{4de}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{2c^2d} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{2c^2de}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{2de} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.35

$$\int\frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx =$$

$$-\frac{\frac{2a}{d+ex^2}+\frac{2b\operatorname{sech}^{-1}(cx)}{d+ex^2}+\frac{2b\log(x)}{d}-\frac{2b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d}+\frac{b\sqrt{e}\log\left(\frac{4\left(\frac{ide+c^2d^{3/2}\sqrt{e}x}{\sqrt{c^2d+e}\left(\sqrt{d}+i\sqrt{e}x\right)}+\frac{de\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{-i\sqrt{d}\sqrt{e}+ex}\right)}{b}\right)}{d\sqrt{c^2d+e}}+\frac{b\sqrt{e}\cdots}{\cdots}}{4e}$$

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*((2*a)/(d + e*x^2) + (2*b*ArcSech[c*x])/(d + e*x^2) + (2*b*Log[x])/d -
 (2*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/d
 + (b*Sqrt[e]*Log[(4*((I*d*e + c^2*d^(3/2)*Sqrt[e]*x)/(Sqrt[c^2*d + e]*(Sqr
t[d] + I*Sqrt[e]*x)) + (d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/((-I)*Sqrt
[d]*Sqrt[e] + e*x)))/b])/(d*Sqrt[c^2*d + e]) + (b*Sqrt[e]*Log[(4*((d*e + I*
c^2*d^(3/2)*Sqrt[e]*x)/(Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x)) + (d*e*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(I*Sqrt[d]*Sqrt[e] + e*x)))/b])/(d*Sqrt[c
^2*d + e]))/e
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(124) = 248$.

Time = 5.52 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.14

method	result
parts	$-\frac{a}{2e(e x^2+d)}+\frac{b\left(-\frac{c^4 \operatorname{arcsech}(cx)}{2e\left(e c^2x^2+c^2d\right)}+\frac{c^3\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(-2\sqrt{\frac{c^2d+e}{e}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2d+\ln\left(-\frac{2\left(\sqrt{\frac{c^2d+e}{e}}\sqrt{-\cdots}\right.}{-\cdots}\right.\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
derivativedivides	$\frac{-\frac{a c^4}{2e\left(e c^2x^2+c^2d\right)}+b c^4\left(-\frac{\operatorname{arcsech}(cx)}{2e\left(e c^2x^2+c^2d\right)}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{\frac{c^2d+e}{e}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2d-\ln\left(\frac{2\sqrt{\frac{c^2d+e}{e}}\sqrt{-c^2x\cdots}}{cex\cdots}\right.\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	$\frac{-\frac{a c^4}{2e\left(e c^2x^2+c^2d\right)}+b c^4\left(-\frac{\operatorname{arcsech}(cx)}{2e\left(e c^2x^2+c^2d\right)}-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\left(2\sqrt{\frac{c^2d+e}{e}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)c^2d-\ln\left(\frac{2\sqrt{\frac{c^2d+e}{e}}\sqrt{-c^2x\cdots}}{cex\cdots}\right.\cdots\right.}{\cdots}\cdots\right.}{\cdots}$

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arcsech(c*x)+1/4*c^3
*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(-2*((c^2*d+e)/e)^(1/2)*arctanh
(1/(-c^2*x^2+1)^(1/2))*c^2*d+ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*
e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d+ln(2*(((c^2*d+e)
/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1
/2)))*c^2*d-2*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))*e+ln(-2*(((
c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^
```

```
2*d*e)^(1/2)))*e+ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^
(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*e)/(-c^2*x^2+1)^(1/2)/d/(e+(-c^2*d*e
)^(1/2))/(-e+(-c^2*d*e)^(1/2))/((c^2*d+e)/e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(89) = 178$.

Time = 0.31 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.10

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

$$= \left[-\frac{2\,ac^2d^2+2\,ade-\sqrt{c^2de+e^2}(bex^2+bd)\log\left(\frac{c^4d^2+4\,c^2de-\left(c^4de+2\,c^2e^2\right)x^2+4\,\left(c^3de+ce^2\right)x\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+4\,e^2+2\,\left(c^2ex^2-\right.}{ex^2+d}\right.}{4\,(c^2}\right.$$

$$-\frac{ac^2d^2+ade+\sqrt{-c^2de-e^2}(bex^2+bd)\arctan\left(\frac{\sqrt{-c^2de-e^2}\,cdx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-\sqrt{-c^2de-e^2}\,(ex^2+d)}{(c^2de+e^2)x^2}\right)+\left(bc^2d^2+bde\right.}{2\,\left(c^2d^3e+d^2e^2+\left(c^2d^2e^2+de^3\right)\right.}$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 + 2*a*d*e - sqrt(c^2*d*e + e^2)*(b*e*x^2 + b*d)*log((c^4
*d^2 + 4*c^2*d*e - (c^4*d*e + 2*c^2*e^2)*x^2 + 4*(c^3*d*e + c*e^2)*x*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 4*e^2 + 2*(c^2*e*x^2 - c^2*d - (c^3*d + 2*c*e)*x
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*e)*sqrt(c^2*d*e + e^2))/(e*x^2 + d)) +
2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) - 1)/x) + 2*(b*c^2*d^2 + b*d*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2), -
1/2*(a*c^2*d^2 + a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*arctan((sqrt(
-c^2*d*e - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)
*(e*x^2 + d))/((c^2*d*e + e^2)*x^2)) + (b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*
e^2)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^2*d^2 + b*
d*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*e + d^2*
e^2 + (c^2*d^2*e^2 + d*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{x(a+b\,\operatorname{asech}\,(cx))}{\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\operatorname{arsech}\,(cx)+a)x}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*(2*c^2*integrate(1/2*x^3/(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*e)*x^2 + (c^2*
d^2*x^2 + (c^2*d*e*x^2 - d*e)*x^2 - d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) - d^2
), x) + (x^2*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - x^2*log(c) - x^2*log(x
))/(d*e*x^2 + d^2) - 2*integrate(1/2*x/(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*e)*x
^2 - d^2), x))*b - 1/2*a/(e^2*x^2 + d*e)
```

Giac [F]

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\operatorname{arsech}\,(cx)+a)x}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.118 $\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 542

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^2}\,dx = -\frac{e(a+b\text{sech}^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{(a+b\text{sech}^{-1}(cx))^2}{2bd^2}+\frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}-\frac{(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}-\frac{(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

```
[Out] -1/2*e*(a+b*arcsech(c*x))/d^2/(e+d/x^2)+1/2*(a+b*arcsech(c*x))^2/b/d^2-1/2*
(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+
```

```
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2
*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1
/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/
2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,-c*(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
^2-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*b*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+
1/c^2/x^2)^(1/2))*e^(1/2)*(-1+1/c^2/x^2)^(1/2)/d^2/(c^2*d+e)^(1/2)/(-1+1/c/
x)^(1/2)/(1+1/c/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6438, 5959, 5957, 533, 385, 214, 5962, 5681, 2221, 2317, 2438}

$$
\begin{aligned}
\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = &-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2d^2}\\
&-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}\\
&-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2d^2}-\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2d^2\left(\frac{d}{x^2}+e\right)}\\
&+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{2d^2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\sqrt{c^2d+e}}\\
&-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}\\
&-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}
\end{aligned}
$$

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(e*(a + b*ArcSech[c*x]))/(d^2*(e + d/x^2)) + (a + b*ArcSech[c*x])^2/(2
*b*d^2) + (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt
```

```
[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(2*d^2*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*S
qrt[1 + 1/(c*x)]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x
])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSech[c*x])*Log[1 + (c
*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*A
rcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]
)])/(2*d^2) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sq
rt[e] + Sqrt[c^2*d + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^
ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sq
rt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d^2) - (b*PolyLog[
2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^3\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{ex\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{(e+dx^2)^2}\,dx,x,\frac{1}{x}\right)}{d}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{d} + \frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(e+dx^2)}\,dx,x,\frac{1}{x}\right)}{2cd^2}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{2(-d)^{3/2}} + \frac{\left(be\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x^2}{c^2}}(e+dx^2)}\,dx,x,\frac{1}{x}\right)}{2cd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx,x,\text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx,x,\text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}} + \frac{\left(be\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int \frac{1}{e-\left(d+\frac{e}{c^2}\right)x^2}\,dx,x,\frac{1}{\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2cd^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$
\begin{aligned}
&= -\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd^2} \\
&\quad+\frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad+\frac{\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2(-d)^{3/2}} \\
&\quad+\frac{\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2(-d)^{3/2}} \\
&\quad-\frac{\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2(-d)^{3/2}} \\
&\quad-\frac{\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2(-d)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd^2} \\
&\quad + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} \\
&\quad - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} \\
&\quad - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} \\
&\quad - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2} \\
&\quad - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2d^2} \\
&\quad + \frac{b\operatorname{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2d^2}
\end{aligned}
$$

$$= -\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd^2}$$

$$+ \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$+ \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2d^2}$$

$$+ \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2d^2}$$

$$+ \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2d^2}$$

$$+ \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2d^2}$$

$$= -\frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd^2}$$
$$+ \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 1189, normalized size of antiderivative = 2.19

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

$$= \frac{\frac{2ad}{d+ex^2} + \frac{b\sqrt{d}\operatorname{sech}^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{b\sqrt{d}\operatorname{sech}^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} - 2b\operatorname{sech}^{-1}(cx)^2 - 8ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\operatorname{se}\right.}{\sqrt{c^2d+e}}\right.}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] ((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (
b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] + I*Sqrt[e]*x) - 2*b*ArcSech[c*x]^2 - (8*I
)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt
[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - (8*I)*b*ArcSin[Sqrt
[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tan
h[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - 2*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e]
```

```
- Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c
*Sqrt[d]*E^ArcSech[c*x])] - 2*b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]
*E^ArcSech[c*x])] - 2*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])
)/(c*Sqrt[d]*E^ArcSech[c*x])] - (4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech
[c*x])] - 2*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[
d]*E^ArcSech[c*x])] - (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt
[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] +
4*a*Log[x] + 2*b*Log[x] - 2*a*Log[d + e*x^2] - 2*b*Log[1 + Sqrt[(1 - c*x)/(
1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]] + (b*Sqrt[e]*Log[((2*I)*Sqrt[e]*
(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x
)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + (b*Sqrt[e]*
Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*
Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*
d + e] + 2*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Ar
cSech[c*x])] + 2*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E
^ArcSech[c*x])] + 2*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt
[d]*E^ArcSech[c*x])] + 2*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sq
rt[d]*E^ArcSech[c*x])])/(4*d^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.00 (sec) , antiderivative size = 2226, normalized size of antiderivative = 4.11

method	result	size
parts	Expression too large to display	2226
derivativedivides	Expression too large to display	2275
default	Expression too large to display	2275

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d^2*ln(e*x^2+d)+1/2*a/d/(e*x^2+d)+a/d^2*ln(x)+b*(-(-c^2*d*(e*(c^2*d+
e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^2*d+e)/d^3/c^2*arcsec
h(c*x)^2+1/8*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1
/2)*e)/d^2/e/(c^2*d+e)*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)
+2*e)/d^4/c^4*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-
c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e-1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d
^3/c^2*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(
c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^4/c^
```

```
4*arcsech(c*x)^2*e-1/4*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^
2*d+e))^(1/2)*e)/d^2/e/(c^2*d+e)*arcsech(c*x)^2+1/2*(e*(c^2*d+e))^(1/2)/d^2
/(c^2*d+e)*arcsech(c*x)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/2*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^
2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^2*d+e)/d^3/c^2*polylog(2,d*c^2*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-
(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e/d^4/
(c^2*d+e)/c^4*arcsech(c*x)^2+1/2*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^
2-2*(e*(c^2*d+e))^(1/2)*e)*e/d^4/(c^2*d+e)/c^4*polylog(2,d*c^2*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))+1/8*(e*(
c^2*d+e))^(1/2)/d/e/(c^2*d+e)*c^2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/4*(-c^2*d*(e*(c^2*d
+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/d^2/e/(c^2*d+e)*ln(1-d*
c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2
)-2*e))*arcsech(c*x)-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^4/c^4*e*ln(1-d*c^2
*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2
*e))*arcsech(c*x)+(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e
))^(1/2)*e)/(c^2*d+e)/d^3/c^2*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)-1/2*x^2*c^2*arcse
ch(c*x)*e/(c^2*e*x^2+c^2*d)/d^2-1/4*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*c^2*a
rcsech(c*x)^2+(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(
1/2)*e)*e/d^4/(c^2*d+e)/c^4*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+1/4*(e*(c^2*d+e))^(
1/2)/d/e/(c^2*d+e)*c^2*arcsech(c*x)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/2/d^2*sum((_R1^2*c^2*d
+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*arcsech(c*
x)^2/d^2+1/4*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*polylog(2,d*c^2*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*(e*(
c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-1/2*(e*(c^2*d+e))^(1/2
)/d^2/(c^2*d+e)*arcsech(c*x)^2-1/4*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^3/c^
2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(
c^2*d+e))^(1/2)-2*e))+1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^3/c^2*arcsech
(c*x)^2)
```

Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\text{asech}\left(cx\right)}{x\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**2,x)
[Out] Integral((a + b*asech(c*x))/(x*(d + e*x**2)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^2} \,d x$$

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^2),x)
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

3.119 $\int \frac{x^4 \left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 840

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2}+\frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}+\frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}+\frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3b\sqrt{-d}\,\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\,\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3b\sqrt{-d}\,\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\,\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

```
[Out] x*(a+b*arcsech(c*x))/e^2-b*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/c/e^2+3
/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arcsech(c*x))*
ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+
e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/
```

```
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/
e^(5/2)-3/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*polylog(
2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)
^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/
c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*
polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(
c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2
)-1/4*d*(a+b*arcsech(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arcsech
(c*x))/e^2/(d/x+(-d)^(1/2)*e^(1/2))+1/2*b*d*arctan((1+1/c/x)^(1/2)*(c*d-(-d
)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/e^2
/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)+1/2*b*d*arct
an((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d
)^(1/2)*e^(1/2))^(1/2))/e^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*
e^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules

used = {6438, 5959, 5883, 94, 211, 5909, 5963, 95, 5962, 5681, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx &= \frac{x(a+b\operatorname{sech}^{-1}(cx))}{e^2}\\ &+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)(a+b\operatorname{sech}^{-1}(cx))}{4e^{5/2}}\\ &-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)(a+b\operatorname{sech}^{-1}(cx))}{4e^{5/2}}\\ &+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)(a+b\operatorname{sech}^{-1}(cx))}{4e^{5/2}}\\ &-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)(a+b\operatorname{sech}^{-1}(cx))}{4e^{5/2}}\\ &-\frac{d(a+b\operatorname{sech}^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d(a+b\operatorname{sech}^{-1}(cx))}{4e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}\\ &+\frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}\\ &+\frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}\\ &-\frac{b\arctan\left(\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}\\ &-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}\\ &+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}\\ &-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}\\ &+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}\end{aligned}$$

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(d*(a + b*ArcSech[c*x]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*Ar
cSech[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcSech[c*x]))/e^
2 + (b*d*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d
```

```
+ Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*S
qrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) + (b*d*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]
]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2
*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*ArcTan
[Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]])/(c*e^2) + (3*Sqrt[-d]*(a + b*ArcSec
h[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(
4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcSe
ch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/
(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSec
h[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2
, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*e^(5/2))
+ (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*
d + e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (
c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2))
```

<u>Rule 94</u>

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

<u>Rule 95</u>

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

<u>Rule 2317</u>

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{e^2x^2} - \frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{ce^2} + \frac{d\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} - \frac{b\text{Subst}\left(\int \frac{1}{\frac{1}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{c^2e^2} - \frac{d^2\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e^2} - \frac{d^2\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e^2} - \frac{d^2\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2e^2}$$

$$= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2} + \frac{d\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}} + \frac{d\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}} + \frac{(bd)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)}\,dx, x, \frac{1}{x}\right)}{4ce^2} - \frac{(bd)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)}\,dx, x, \frac{1}{x}\right)}{4ce^2} - \frac{d^2\text{Subst}\left(\int \left(-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{2e^2}$$

$$= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+\frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2}-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}$$

$$+\frac{d\text{Subst}\left(\int\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{4e^{5/2}}+\frac{d\text{Subst}\left(\int\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{4e^{5/2}}$$

$$+\frac{d\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}}$$

$$+\frac{d\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}}$$

$$+\frac{d\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}}$$

$$+\frac{d\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}}$$

$$+\frac{(bd)\text{Subst}\left(\int\frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx,x,\frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2ce^2}$$

$$-\frac{(bd)\text{Subst}\left(\int\frac{1}{-d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx,x,\frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2ce^2}$$

$$= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} + \frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$

$$+ \frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}$$

$$+ \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$- \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$+ \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$- \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$- \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}}$$

$$- \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{d\operatorname{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}dx, x, \operatorname{sech}^{-1}(cx)\right)}{4e^{5/2}}$$

$$+ \frac{d\operatorname{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}dx, x, \operatorname{sech}^{-1}(cx)\right)}{4e^{5/2}}$$

$$= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+\frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2}+\frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$

$$+\frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}$$

$$+\frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$-\frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$+\frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$-\frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$-\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^{5/2}}$$

$$+\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^{5/2}}$$

$$-\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^{5/2}}$$

$$+\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^{5/2}}$$

$$+\frac{d\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\operatorname{sech}^{-1}(cx)\right)}{4e^{5/2}}$$

$$+\frac{d\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\operatorname{sech}^{-1}(cx)\right)}{4e^{5/2}}$$

$$+\frac{d\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}dx,x,\operatorname{sech}^{-1}(cx)\right)}{4e^{5/2}}$$

$$= -\frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} + \frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$

$$+ \frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}$$

$$+ \frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

$$- \frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

$$+ \frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

$$- \frac{3\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

$$- \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$- \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$- \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{4e^{5/2}}$$

$$+ \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{4e^{5/2}}$$

$$- \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{4e^{5/2}}$$

$$+ \frac{\left(b\sqrt{-d}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{4e^{5/2}}$$

$$= -\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{e^2}+\frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$

$$+\frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}$$

$$+\frac{3\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

$$-\frac{3\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

$$+\frac{3\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

$$-\frac{3\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

$$-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$

$$-\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{4e^{5/2}}$$

$$+\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{4e^{5/2}}$$

$$-\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{4e^{5/2}}$$

$$+\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{4e^{5/2}}$$

$$= -\frac{d(a+b\operatorname{sech}^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\operatorname{sech}^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+ \frac{x(a+b\operatorname{sech}^{-1}(cx))}{e^2} + \frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$
$$+ \frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} - \frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}$$
$$+ \frac{3\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+ \frac{3\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} + \frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} + \frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 1270, normalized size of antiderivative = 1.51

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$= \frac{4a\sqrt{e}x + \frac{2ad\sqrt{e}x}{d+ex^2} + 4b\sqrt{e}x\operatorname{sech}^{-1}(cx) + \frac{bd\operatorname{sech}^{-1}(cx)}{-i\sqrt{d}+\sqrt{e}x} + \frac{bd\operatorname{sech}^{-1}(cx)}{i\sqrt{d}+\sqrt{e}x} - 6a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \frac{8b\sqrt{e}\arctan\left(\tanh\left(\frac{1}{2}\right.\right.}{c}}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (4*a*Sqrt[e]*x + (2*a*d*Sqrt[e]*x)/(d + e*x^2) + 4*b*Sqrt[e]*x*ArcSech[c*x]
 + (b*d*ArcSech[c*x])/((-I)*Sqrt[d] + Sqrt[e]*x) + (b*d*ArcSech[c*x])/(I*Sq
```

```
rt[d] + Sqrt[e]*x) - 6*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (8*b*Sqrt[e]
*ArcTan[Tanh[ArcSech[c*x]/2]])/c + 12*b*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x
]/2])/Sqrt[c^2*d + e]] - 12*b*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^
2*d + e]] + (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d +
 e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E
^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[d]*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*S
qrt[d]*E^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 - (I*(Sqrt[e]
+ Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[d]*ArcSin[Sqrt[1
 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])
)/(c*Sqrt[d]*E^ArcSech[c*x])] + (3*I)*b*Sqrt[d]*ArcSech[c*x]*Log[1 + (I*(Sq
rt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[d]*ArcSin[
Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d
 + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (I*b*Sqrt[d]*Sqrt[e]*Log[((2*I)*Sqrt[
e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*
d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + (I*b*Sqr
t[d]*Sqrt[e]*Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)
+ (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x
)])/Sqrt[c^2*d + e] + (3*I)*b*Sqrt[d]*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*PolyLog[2, (I*(-Sqr
t[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (3*I)*b*Sqrt[d]*Poly
Log[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (3*
I)*b*Sqrt[d]*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSec
h[c*x])])/(4*e^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 111.63 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.20

method	result	size
parts	Expression too large to display	1006
derivativedivides	Expression too large to display	1031
default	Expression too large to display	1031

```
[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/e^2*x-1/e^2*d*(-1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
)))+b/c^5*(1/2*x*c^5*arcsech(c*x)*(2*c^2*e*x^2+3*c^2*d)/e^2/(c^2*e*x^2+c^2*
d)-2/e^2*c^4*arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))-3/16/e^3*c^6*d*
```

```
sum((_R1^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_
R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*
d))+3/16/e^3*c^6*d*sum((_R1^2*c^2*d+4*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d
+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+di
log((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^
4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2)*e)*c*ar
ctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))
^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1
/2)*e)*c*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(
c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2-1/2*(-(c^2*d-2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)
^(1/2))/d^2/e^2-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)*c*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^2/e^2)
```

Fricas [F]

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^4*arcsech(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4(a+b\,\text{asech}\,(cx))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**4*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^2}\,dx$$

```
[In] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.120 $\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 786

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} - \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

```
[Out] 1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)
^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arcsech(c*x))
*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d
+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-
d)^(1/2)-1/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
```

```
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*polylog
(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b
*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/
2)+1/4*(a+b*arcsech(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arcsech(c*x
))/e/(d/x+(-d)^(1/2)*e^(1/2))-1/2*b*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*
e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/e/(c*d-(-d)
^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)-1/2*b*arctan((1+1/c/x)
^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1
/2))^(1/2))/e/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {6438, 5909, 5963, 95, 211, 5962, 5681, 2221, 2317, 2438}

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\text{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\text{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{4\sqrt{-d}e^{3/2}} + \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\arctan\left(\frac{\sqrt{\frac{1}{cx}+1}\sqrt{cd-\sqrt{-d}\sqrt{e}}}{\sqrt{\frac{1}{cx}-1}\sqrt{cd+\sqrt{-d}\sqrt{e}}}\right)}{2e\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} - \frac{b\arctan\left(\frac{\sqrt{\frac{1}{cx}+1}\sqrt{cd+\sqrt{-d}\sqrt{e}}}{\sqrt{\frac{1}{cx}-1}\sqrt{cd-\sqrt{-d}\sqrt{e}}}\right)}{2e\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcSech[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSech[c*x])/
(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sq
rt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqr
t[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) - (b*ArcTan[(Sqrt
[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*S
qrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sq
```

```
rt[e]]*e) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt
[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSech[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^
(3/2)) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e]
 + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSech[c*x])*Log[1 +
(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/
2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e
]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqr
t[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d
]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*
PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqr
t[-d]*e^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\right.\right.\\ &\qquad\left.\left.-\frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right)\end{aligned}$$

$$= \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}dx, x, \frac{1}{x}\right)}{4e} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}dx, x, \frac{1}{x}\right)}{4e} + \frac{d\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{-de-d^2x^2}dx, x, \frac{1}{x}\right)}{2e}$$

$$= \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)}dx, x, \frac{1}{x}\right)}{4ce} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)}dx, x, \frac{1}{x}\right)}{4ce} + \frac{d\text{Subst}\left(\int \left(-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{2e}$$

$$= \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}dx, x, \frac{1}{x}\right)}{4e^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}dx, x, \frac{1}{x}\right)}{4e^{3/2}} - \frac{b\text{Subst}\left(\int \frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2ce} + \frac{b\text{Subst}\left(\int \frac{1}{-d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2ce}$$

$$= \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} - \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}dx, x, \text{sech}^{-1}(cx)\right)}{4e^{3/2}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}dx, x, \text{sech}^{-1}(cx)\right)}{4e^{3/2}}$$

$$= \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}$$

$$- \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}$$

$$- \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4e^{3/2}}$$

$$- \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4e^{3/2}}$$

$$- \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4e^{3/2}}$$

$$- \frac{\operatorname{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4e^{3/2}}$$

$$= \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}$$
$$- \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}}$$

$$= \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}$$

$$- \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

$$- \frac{b\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}$$

$$+ \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}$$

$$- \frac{b\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}$$

$$+ \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}$$

$$=\frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\text{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}$$
$$-\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 1226, normalized size of antiderivative = 1.56

$$\int\frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$
$$=\frac{-\frac{2a\sqrt{e}x}{d+ex^2}+\frac{b\text{sech}^{-1}(cx)}{i\sqrt{d}-\sqrt{e}x}-\frac{b\text{sech}^{-1}(cx)}{i\sqrt{d}+\sqrt{e}x}+\frac{2a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{4b\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\text{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\text{sech}^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)}{\sqrt{d}}-}{}$$

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-2*a*Sqrt[e]*x)/(d + e*x^2) + (b*ArcSech[c*x])/(I*Sqrt[d] - Sqrt[e]*x) -
(b*ArcSech[c*x])/(I*Sqrt[d] + Sqrt[e]*x) + (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/Sqrt[d] - (4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[
(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d]
 + (4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqr
t[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d] - (I*b*ArcS
```

```
ech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])
])/Sqrt[d] - (2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 +
 (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[d] + (I*
b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSec
h[c*x])])/Sqrt[d] + (2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*
Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[
d] + (I*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E
^ArcSech[c*x])])/Sqrt[d] - (2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])
/Sqrt[d] - (I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqr
t[d]*E^ArcSech[c*x])])/Sqrt[d] + (2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c
*x])])/Sqrt[d] + (I*b*Sqrt[e]*Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqr
t[d] + Sqrt[e]*x)])/(Sqrt[d]*Sqrt[c^2*d + e]) - (I*b*Sqrt[e]*Log[(2*Sqrt[e]
*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*
d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)])/(Sqrt[d]*Sqrt[c^2*d + e
]) - (I*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe
ch[c*x])])/Sqrt[d] + (I*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sq
rt[d]*E^ArcSech[c*x])])/Sqrt[d] + (I*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[d] - (I*b*PolyLog[2, (I*(Sqrt[e
] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[d])/(4*e^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 46.70 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.16

method	result
parts	$-\frac{ax}{2e(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+\frac{b\left(-\frac{c^5\operatorname{arcsech}(cx)x}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{c^4\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\operatorname{arcse}\right.}{}\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a\,c^5x}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{\operatorname{arcsech}(cx)cx}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\operatorname{arcsec}\right.}{}}{}\right.}{}$
default	$\frac{-\frac{a\,c^5x}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{\operatorname{arcsech}(cx)cx}{2e\left(e\,c^2x^2+c^2d\right)}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(\operatorname{arcsec}\right.}{}}{}\right.}{}$

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/e*x/(e*x^2+d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c^3*(-1/
2*c^5*arcsech(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/4/e*c^4*sum(_R1/(_R1^2*c^2*d+c^2
*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+
dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_
Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(
1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/c/d^3/e-1/2*
(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c
^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2)*e)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(
1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d
^3/c+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))
^(1/2)+2*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/c/d^3/e-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^
(1/2)*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(
c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^3/c+1/4/e*c^4*sum(1/_R1/(_R1^2
*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=Root
Of(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\operatorname{arsech}(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2(a+b\operatorname{asech}(cx))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**2*(a + b*asech(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\operatorname{arsech}(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.121 $\int \frac{a+b\,\text{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 786

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx = -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}+\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

```
[Out] -1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsech(c*x)
)*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*
d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2
)/e^(1/2)+1/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylo
g(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+
e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*
b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(
```

```
1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1
/2)+1/4*(-a-b*arcsech(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arcsech(c*
x))/d/(d/x+(-d)^(1/2)*e^(1/2))+1/2*b*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)
*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/d/(c*d-(-d
)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)+1/2*b*arctan((1+1/c/x
)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(
1/2))^(1/2))/d/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2
)
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6428, 5959, 5909, 5963, 95, 211, 5962, 5681, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx = &-\frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\ &+\frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{4(-d)^{3/2}\sqrt{e}} \\ &-\frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\ &+\frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\ &+\frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\arctan\left(\frac{\sqrt{\frac{1}{cx}+1}\sqrt{cd-\sqrt{-d}\sqrt{e}}}{\sqrt{\frac{1}{cx}-1}\sqrt{cd+\sqrt{-d}\sqrt{e}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} \\ &+\frac{b\arctan\left(\frac{\sqrt{\frac{1}{cx}+1}\sqrt{cd+\sqrt{-d}\sqrt{e}}}{\sqrt{\frac{1}{cx}-1}\sqrt{cd-\sqrt{-d}\sqrt{e}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} \\ &+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\ &+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\end{aligned}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(a + b*ArcSech[c*x])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcSech[c*x
])/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]
*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*
d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) + (b*ArcTan[(S
qrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]
]*Sqrt[-1 + 1/(c*x)])])/(2*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-
d]*Sqrt[e]]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(S
qrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSech[c*x])*
Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(
3/2)*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSech[c*x])
*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^
(3/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqr
t[c^2*d + e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSe
ch[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[
2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*(-d)^(3/
2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2
*d + e])])/(4*(-d)^(3/2)*Sqrt[e])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6428

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d}$$
$$+\frac{e\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)\right)$$
$$-\frac{1}{4}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)$$
$$-\frac{1}{2}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)$$
$$-\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d\sqrt{e}}-\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d\sqrt{e}}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)}dx, x, \frac{1}{x}\right)}{4cd} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)}dx, x, \frac{1}{x}\right)}{4cd} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}dx, x, \text{sech}^{-1}(cx)\right)}{2d\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}dx, x, \text{sech}^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\text{Subst}\left(\int \frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2cd} - \frac{b\text{Subst}\left(\int \frac{1}{-d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2cd} + \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}dx, x, \frac{1}{x}\right)}{4d\sqrt{e}} + \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}dx, x, \frac{1}{x}\right)}{4d\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx, x, \text{sech}^{-1}(cx)\right)}{2d\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx, x, \text{sech}^{-1}(cx)\right)}{2d\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}dx, x, \text{sech}^{-1}(cx)\right)}{2d\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}dx, x, \text{sech}^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$+ \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \text{sech}^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{\text{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}dx, x, \text{sech}^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$+ \frac{\text{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}dx, x, \text{sech}^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$+\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$-\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+\frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$-\frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+\frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$-\frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\text{sech}^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+\frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\text{sech}^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$+\frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}dx,x,\text{sech}^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$+\frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}dx,x,\text{sech}^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$+\frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}dx,x,\text{sech}^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$+\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$-\frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+\frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$-\frac{b\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+\frac{b\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$
$$+\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$-\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}-\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$-\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+\frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\operatorname{sech}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$+ \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 1216, normalized size of antiderivative = 1.55

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx$$

$$= \frac{\frac{2a\sqrt{d}x}{d+ex^2} + \frac{b\sqrt{d}\operatorname{sech}^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex} + \frac{b\sqrt{d}\operatorname{sech}^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex} + \frac{2a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{4b\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\operatorname{arctanh}\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)}{\sqrt{e}}}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((2*a*Sqrt[d]*x)/(d + e*x^2) + (b*Sqrt[d]*ArcSech[c*x])/((-I)*Sqrt[d]*Sqrt[
e] + e*x) + (b*Sqrt[d]*ArcSech[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + (2*a*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[
d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqr
t[c^2*d + e]])/Sqrt[e] + (4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt
[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]
```

```
])/Sqrt[e] - (I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*S
qrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech
[c*x])])/Sqrt[e] + (I*b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] + (2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E
^ArcSech[c*x])])/Sqrt[e] + (I*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (2*b*ArcSin[Sqrt[1 - (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqr
t[d]*E^ArcSech[c*x])])/Sqrt[e] - (I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sq
rt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] + (2*b*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (I*b*Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d
 + e]))/(I*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + (I*b*Log[(2*Sqrt[e]*(I*
Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)
/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] - (I*b*Poly
Log[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqr
t[e] + (I*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSec
h[c*x])])/Sqrt[e] + (I*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*S
qrt[d]*E^ArcSech[c*x])])/Sqrt[e] - (I*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d
 + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/Sqrt[e])/(4*d^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 55.84 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.14

method	result
parts	$\frac{ax}{2d(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\,\mathrm{arcsech}(cx)x}{2d\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{arctanh}\left(\frac{cd\left(\right.}{\sqrt{\left(-\right.}}\right.}{2d^4c^3}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^3x}{2d\left(e\,c^2x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\mathrm{arcsech}(cx)x}{2cd\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{arctanh}\left(\right.}{2c^7d^4}\right.}{}$
default	$\frac{\frac{a\,c^3x}{2d\left(e\,c^2x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\mathrm{arcsech}(cx)x}{2cd\left(e\,c^2x^2+c^2d\right)}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{arctanh}\left(\right.}{2c^7d^4}\right.}{}$

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c^
3*arcsech(c*x)*x/d/(c^2*e*x^2+c^2*d)-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e
)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/c^
3+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/
2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2)*e)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^
2*d+e)/c^3-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2
*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/c^3+1/2*((c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*
d+e))^(1/2)*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)/c^3-1/4/d*c^2*sum(_R1/(_
R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=
RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4/d*c^2*sum(1/_R1/(_R1^2*c^2
*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))
/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c
^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^2}\, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)^2}\, dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^2}\, dx = \int \frac{a + b\,\text{asech}\,(cx)}{(d + ex^2)^2}\, dx$$

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**2,x)
[Out] Integral((a + b*asech(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^2}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^2}\, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)^2}\, dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^2}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^2,x)
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^2, x)
```

3.122 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 844

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2}-\frac{a}{d^2x}-\frac{b\text{sech}^{-1}(cx)}{d^2x}+\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}-\frac{3\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

```
[Out] -a/d^2/x-b*arcsech(c*x)/d^2/x-3/4*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d
)^(5/2)+3/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/
2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(a+b*arcse
ch(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arcsech(c*x))*ln(1+c*(1/c/x
```

```
+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^
(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)
)*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*polylog(2,
c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*po
lylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2
*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arcsech(c*x))/d^2/(-d/x+(-d)^(1
/2)*e^(1/2))-1/4*e*(a+b*arcsech(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))+b*c*(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2)/d^2-1/2*b*e*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(
1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/d^2/(c
*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)-1/2*b*e*arctan(
(1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(
1/2)*e^(1/2))^(1/2))/d^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(
1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules

used = {6438, 5959, 5879, 75, 5909, 5963, 95, 211, 5962, 5681, 2221, 2317, 2438}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = -\frac{a}{d^2x} - \frac{b\operatorname{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} - \frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} - \frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} - \frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}} - \frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}} + \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}} + \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}} + \frac{bc\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}}{d^2}$$

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/d^2 - a/(d^2*x) - (b*ArcSech[c*x
])/(d^2*x) + (e*(a + b*ArcSech[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e
*(a + b*ArcSech[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTan[(Sqrt
[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*S
qrt[-1 + 1/(c*x)])])/(2*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d
]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/
```

```
(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d^2*Sqrt[c*d - Sqrt[
-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (3*Sqrt[e]*(a + b*ArcSech[c*x]
)*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)
^(5/2)) + (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x
])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcSech
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4
*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSec
h[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog
[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*(-d)^(5
/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[
c^2*d + e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcS
ech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*Poly
Log[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5
/2))
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
```

```
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^4\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{d^2}+\frac{e^2\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2}-\frac{2e\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)\,dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad+\frac{(2e)\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2}-\frac{e^2\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2}\\
&= -\frac{a}{d^2x}-\frac{b\text{Subst}\left(\int \operatorname{arccosh}\left(\frac{x}{c}\right)\,dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad+\frac{(2e)\text{Subst}\left(\int \left(\frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad-\frac{e^2\text{Subst}\left(\int \left(-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right)}{d^2}\\
&= -\frac{a}{d^2x}-\frac{b\operatorname{sech}^{-1}(cx)}{d^2x}+\frac{b\text{Subst}\left(\int \frac{x}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}}\,dx, x, \frac{1}{x}\right)}{cd^2}\\
&\quad+\frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad+\frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{d^2}+\frac{e\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4d}\\
&\quad+\frac{e\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4d}+\frac{e\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2d}
\end{aligned}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\big(a+b\text{sech}^{-1}(cx)\big)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e\big(a+b\text{sech}^{-1}(cx)\big)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{d^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{d^2}$$
$$- \frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(\sqrt{-d}\sqrt{e}-dx)}\,dx, x, \frac{1}{x}\right)}{4cd^2}$$
$$+ \frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(\sqrt{-d}\sqrt{e}+dx)}\,dx, x, \frac{1}{x}\right)}{4cd^2}$$
$$+ \frac{e\text{Subst}\left(\int \left(-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}-\sqrt{-d}x)} - \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}+\sqrt{-d}x)}\right)dx, x, \frac{1}{x}\right)}{2d}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\operatorname{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$- \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d^2}$$

$$- \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d^2}$$

$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{d^2}$$

$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{d^2}$$

$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{d^2}$$

$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{d^2}$$

$$- \frac{(be)\text{Subst}\left(\int \frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2cd^2}$$

$$+ \frac{(be)\text{Subst}\left(\int \frac{1}{-d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2cd^2}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\operatorname{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$- \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$- \frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$- \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{(-d)^{5/2}}$$

$$- \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{(-d)^{5/2}}$$

$$+ \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{(-d)^{5/2}}$$

$$- \frac{\left(b\sqrt{e}\right)\operatorname{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{(-d)^{5/2}}$$

$$- \frac{\sqrt{e}\operatorname{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}dx, x, \operatorname{sech}^{-1}(cx)\right)}{4d^2}$$

$$- \frac{\sqrt{e}\operatorname{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}dx, x, \operatorname{sech}^{-1}(cx)\right)}{4d^2}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$- \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$- \frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$- \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\text{sech}^{-1}(cx)}\right)}{(-d)^{5/2}}$$

$$- \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\text{sech}^{-1}(cx)}\right)}{(-d)^{5/2}}$$

$$+ \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\text{sech}^{-1}(cx)}\right)}{(-d)^{5/2}}$$

$$- \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{\text{sech}^{-1}(cx)}\right)}{(-d)^{5/2}}$$

$$- \frac{\sqrt{e}\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{4d^2}$$

$$- \frac{\sqrt{e}\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{4d^2}$$

$$\sqrt{e}\text{Subst}\left(\int\frac{e^x(a+bx)}{\cdots}\,dx, x, \text{sech}^{-1}(cx)\right)$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$- \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$- \frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$- \frac{3\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$+ \frac{3\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$- \frac{3\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$+ \frac{3\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$+ \frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} - \frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} - \frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\text{sech}^{-1}(cx)\right)}{4(-d)^{5/2}}$$

$$+ \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\text{sech}^{-1}(cx)\right)}{4(-d)^{5/2}}$$

$$- \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\text{sech}^{-1}(cx)\right)}{4(-d)^{5/2}}$$

$$+ \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\text{sech}^{-1}(cx)\right)}{4(-d)^{5/2}}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\operatorname{sech}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$- \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$- \frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$

$$- \frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$+ \frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$- \frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$+ \frac{3\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$+ \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} - \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} - \frac{b\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{\left(b\sqrt{e}\right)\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{4(-d)^{5/2}}$$

$$+ \frac{\left(b\sqrt{e}\right)\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{4(-d)^{5/2}}$$

$$- \frac{\left(b\sqrt{e}\right)\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{4(-d)^{5/2}}$$

$$+ \frac{\left(b\sqrt{e}\right)\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{4(-d)^{5/2}}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{sech}^{-1}(cx)}{d^2x} + \frac{e(a+b\text{sech}^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e(a+b\text{sech}^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$
$$- \frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{2d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}}$$
$$- \frac{3\sqrt{e}(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3\sqrt{e}(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$- \frac{3\sqrt{e}(a+b\text{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3\sqrt{e}(a+b\text{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 1305, normalized size of antiderivative = 1.55

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$$

$$= \frac{-\frac{4a\sqrt{d}}{x} + 4bc\sqrt{d}\sqrt{\frac{1-cx}{1+cx}} + \frac{4b\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}}{x} - \frac{2a\sqrt{d}ex}{d+ex^2} - \frac{4b\sqrt{d}\text{sech}^{-1}(cx)}{x} - \frac{b\sqrt{d}e\text{sech}^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex} - \frac{b\sqrt{d}e\text{sech}^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex} - 6a\sqrt{e}\,\text{arc}}{}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-4*a*Sqrt[d])/x + 4*b*c*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)] + (4*b*Sqrt[d]*
Sqrt[(1 - c*x)/(1 + c*x)])/x - (2*a*Sqrt[d]*e*x)/(d + e*x^2) - (4*b*Sqrt[d]
*ArcSech[c*x])/x - (b*Sqrt[d]*e*ArcSech[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x)
- (b*Sqrt[d]*e*ArcSech[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - 6*a*Sqrt[e]*ArcTan
[(Sqrt[e]*x)/Sqrt[d]] + 12*b*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[
c^2*d + e]] - 12*b*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]
]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] +
 (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sq
rt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*
x])] - (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])
)/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Ar
cSech[c*x])] - (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*
d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 6*b*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d
]*E^ArcSech[c*x])] + (3*I)*b*Sqrt[e]*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 6*b*Sqrt[e]*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*
Sqrt[d]*E^ArcSech[c*x])] + (I*b*e*Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x
)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I
*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] - (I*b*e*Log[(2*Sqrt[e]*(I*Sqrt[d]*
Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^
2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)])/Sqrt[c^2*d + e] + (3*I)*b*Sqrt[e]*P
olyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] -
 (3*I)*b*Sqrt[e]*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^A
rcSech[c*x])] - (3*I)*b*Sqrt[e]*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])] + (3*I)*b*Sqrt[e]*PolyLog[2, (I*(Sqrt[e] + S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/(4*d^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 109.90 (sec) , antiderivative size = 1007, normalized size of antiderivative = 1.19

method	result	size
parts	Expression too large to display	1007
derivativedivides	Expression too large to display	1034
default	Expression too large to display	1034

```
[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/d^2*e*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/d^2
```

```
/x)+b*c*(-1/2*(-1+arcsech(c*x))/d^2*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)
^(1/2)+1)/x/c+1/2*((-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)-1)*(1+arcse
ch(c*x))/d^2/x/c-1/2*arcsech(c*x)/d^2*e*x*c/(c^2*e*x^2+c^2*d)+1/2*(-(c^2*d-
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*e*arc
tanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^
(1/2)-2*e)*d)^(1/2))/d^5/c^5-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2)*e)*e*ar
ctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))
^(1/2)-2*e)*d)^(1/2))/d^5/c^5/(c^2*d+e)+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*e*arctan(c*d*(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/
c^5-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(
1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e*arctan(c*d*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/c
^5/(c^2*d+e)-3/4/d^2*e*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((
_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2
*d))+3/4/d^2*e*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^2x^2}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{x^2(d+ex^2)^2}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**2,x)
[Out] Integral((a + b*asech(c*x))/(x**2*(d + e*x**2)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2 (d+ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2 (d+ex^2)^2} \, dx = \int \frac{b \,\text{arsech}\,(cx)+a}{(ex^2+d)^2 x^2} \, dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2 (d+ex^2)^2} \, dx = \int \frac{a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^2\,(e\,x^2+d)^2} \, dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

3.123 $\int \frac{x^5\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size $= 21$, antiderivative size $= 778$

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^3} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{b(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^3} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$

```
[Out] 1/4*(-a-b*arcsech(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arcsech(c*x))/e^2/(e+d/x^2)
-(a+b*arcsech(c*x))^2/b/e^3-(a+b*arcsech(c*x))*ln(1+1/(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))^2)/e^3+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*a
rcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*
arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,-1/(1/c/x+(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))^2)/e^3+1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1
/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/8*b*d*(c^2-1/x^2)/c/e
^2/(c^2*d+e)/(e+d/x^2)/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/8*b*(c^2*d+2*e)
*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*(-1+1/c^2/x^2)^(
1/2)/e^(5/2)/(c^2*d+e)^(3/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/2*b*arctanh
((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*(-1+1/c^2/x^2)^(1/2)/e^(
5/2)/(c^2*d+e)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {6438, 5959, 5882, 3799, 2221, 2317, 2438, 5957, 533, 390, 385, 214, 5962, 5681}

$$\int \frac{x^5(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2e^3} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2e^3} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e^2\left(\frac{d}{x^2}+e\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(\frac{d}{x^2}+e\right)^2} - \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{be^3} - \frac{\log\left(e^{-2\operatorname{sech}^{-1}(cx)}+1\right)(a+b\operatorname{sech}^{-1}(cx))}{e^3} + \frac{b\sqrt{\frac{1}{c^2x^2}-1}(c^2d+2e)\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{8e^{5/2}\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\,(c^2d+e)^{3/2}} + \frac{b\sqrt{\frac{1}{c^2x^2}-1}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{2e^{5/2}\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\sqrt{c^2d+e}} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3} + \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\,(c^2d+e)\left(\frac{d}{x^2}+e\right)} + \frac{b\operatorname{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e^3}$$

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*d*(c^2 - x^(-2)))/(8*c*e^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)]*Sq
rt[1 + 1/(c*x)]*x) - (a + b*ArcSech[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcS
ech[c*x])/(2*e^2*(e + d/x^2)) - (a + b*ArcSech[c*x])^2/(b*e^3) + (b*Sqrt[-1
 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x
)])/(2*e^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*(
c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt
[-1 + 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)*Sqrt[-1 + 1/(c*x)]*Sqr
t[1 + 1/(c*x)]) - ((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e^3 +
 ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[
c^2*d + e])])/(2*e^3) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcSech[c*x])*Log[1
- (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) + ((a +
 b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d
+ e])])/(2*e^3) + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/(2*e^3) + (b*PolyLog
[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e^3) +
(b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*
e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d +
e]))])/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[
c^2*d + e])])/(2*e^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
```

```
[n, 2] && IGtQ[q, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))),
```

```
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a+b\,\text{arccosh}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\,\text{arccosh}\left(\frac{x}{c}\right)}{e^3 x} - \frac{dx\left(a+b\,\text{arccosh}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^3} - \frac{dx\left(a+b\,\text{arccosh}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} - \frac{dx\left(a+b\,\text{arccosh}\left(\frac{x}{c}\right)\right)}{e^3\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\,\text{arccosh}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\,\text{arccosh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\,\text{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\,\text{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{\operatorname{Subst}\left(\int x\tanh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\operatorname{sech}^{-1}(cx)\right)}{be^3} + \frac{d\operatorname{Subst}\left(\int\left(-\frac{\sqrt{-d}\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e^3} + \frac{b\operatorname{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(e+dx^2)}\,dx, x, \frac{1}{x}\right)}{2ce^2} + \frac{b\operatorname{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)}{4ce}$$

$$= -\frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2be^3} + \frac{2\operatorname{Subst}\left(\int\frac{e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}x}{1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}}\,dx, x, a+b\operatorname{sech}^{-1}(cx)\right)}{be^3} - \frac{\sqrt{-d}\operatorname{Subst}\left(\int\frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^3} + \frac{\sqrt{-d}\operatorname{Subst}\left(\int\frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^3} + \frac{\left(b\sqrt{-1+\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x^2}{c^2}}(e+dx^2)}\,dx, x, \frac{1}{x}\right)}{2ce^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} + \frac{\left(b\sqrt{-1+\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x^2}{c^2}}(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)}{4ce\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2be^3} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^3}$$

$$+ \frac{\operatorname{Subst}\left(\int\log\left(1+e^{2\left(\frac{a}{b}-\frac{x}{b}\right)}\right)\,dx, x, a+b\operatorname{sech}^{-1}(cx)\right)}{e^3}$$

$$- \frac{\sqrt{-d}\operatorname{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{2e^3}$$

$$+ \frac{\sqrt{-d}\operatorname{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{2e^3}$$

$$+ \frac{\left(b\sqrt{-1+\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{1}{e-\left(d+\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{2ce^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$+ \frac{\left(b(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x^2}{c^2}}(e+dx^2)}\,dx, x, \frac{1}{x}\right)}{8ce^2\left(c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^3} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^3}$$

$$- \frac{b\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{sech}^{-1}(cx)}{b}\right)}\right)}{2e^3}$$

$$- \frac{\sqrt{-d}\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^3}$$

$$- \frac{\sqrt{-d}\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^3}$$

$$+ \frac{\sqrt{-d}\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^3}$$

$$+ \frac{\sqrt{-d}\operatorname{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{2e^3}$$

$$+ \frac{\left(b(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\right)\operatorname{Subst}\left(\int\frac{1}{e-\left(d+\frac{e}{c^2}\right)x^2}\,dx,x,\frac{1}{\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8ce^2\left(c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{a+b\text{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$

$$- \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{be^3} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$+ \frac{b(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+e^{-2\text{sech}^{-1}(cx)}\right)}{e^3}$$

$$+ \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+ \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+ \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+ \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+ \frac{b\,\text{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\text{sech}^{-1}(cx)}{b}\right)}\right)}{2e^3}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\text{sech}^{-1}(cx)\right)}{2e^3}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\text{sech}^{-1}(cx)\right)}{2e^3}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\text{sech}^{-1}(cx)\right)}{2e^3}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\text{sech}^{-1}(cx)\right)}{2e^3}$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^3} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$+ \frac{b(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^3}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{sech}^{-1}(cx)}{b}\right)}\right)}{2e^3}$$
$$- \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^3}$$
$$- \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^3}$$
$$- \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^3}$$
$$- \frac{b\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{\operatorname{sech}^{-1}(cx)}\right)}{2e^3}$$

$$= \frac{bd\left(c^2-\frac{1}{x^2}\right)}{8ce^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{be^3} + \frac{b\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$+ \frac{b(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+e^{-2\operatorname{sech}^{-1}(cx)}\right)}{e^3}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-e^{2\left(\frac{a}{b}-\frac{a+b\operatorname{sech}^{-1}(cx)}{b}\right)}\right)}{2e^3}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.93 (sec) , antiderivative size = 2000, normalized size of antiderivative = 2.57

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=-\frac{ad^2}{4e^3\left(d+ex^2\right)^2}+\frac{ad}{e^3\left(d+ex^2\right)}+\frac{a\log\left(d+ex^2\right)}{2e^3}$$

$$+b\left(-\frac{d\left(-\frac{i\sqrt{e}\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{\sqrt{d}(c^2d+e)\left(-i\sqrt{d}+\sqrt{e}x\right)}-\frac{\operatorname{sech}^{-1}(cx)}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\log(x)}{d\sqrt{e}}-\frac{\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d\sqrt{e}}+\frac{(2c^2d+e)\log\left(-\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{16e^{5/2}}\right.$$

$$-\frac{d\left(\frac{i\sqrt{e}\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{\sqrt{d}(c^2d+e)\left(i\sqrt{d}+\sqrt{e}x\right)}-\frac{\operatorname{sech}^{-1}(cx)}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\log(x)}{d\sqrt{e}}-\frac{\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d\sqrt{e}}+\frac{(2c^2d+e)\log\left(-\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(\sqrt{e}+ic^2\sqrt{d}x+\cdots\right.}{\left(2c^2d\cdots\right.}\right.}{d(c^2d+e)\cdots}\right)}{16e^{5/2}}$$

$$-\frac{7i\sqrt{d}\left(-\frac{\operatorname{sech}^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex}+\frac{i\left(\frac{\log(x)}{\sqrt{e}}-\frac{\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{\sqrt{e}}+\frac{\log\left(\frac{2i\sqrt{e}\left(\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx)+\frac{\sqrt{d}\sqrt{e}+ic^2dx}{\sqrt{c^2d+e}}\right)}{i\sqrt{d}+\sqrt{e}x}\right)}{\sqrt{c^2d+e}}\right)}{\sqrt{d}}\right)}{16e^{5/2}}$$

$$\frac{7i\sqrt{d}\left(-\frac{\operatorname{sech}^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex}-\frac{i\left(\frac{\log(x)}{\sqrt{e}}-\frac{\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{\sqrt{e}}+\frac{\log\left(\frac{2\sqrt{e}\left(i\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx)+\frac{i\sqrt{d}\sqrt{e}+c^2dx}{\sqrt{c^2d+e}}\right)}{-i\sqrt{d}+\sqrt{e}x}\right)}{\sqrt{c^2d+e}}\right)}{\sqrt{d}}\right.\cdots}{\cdots}$$

[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]

[Out] -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x^2])/(2*e^3) + b*(-1/16*(d*(((-I)*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^(5/2) - (d*((I*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)) - (((7*I)/16)*Sqrt[d]*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/e^(5/2) + (((7*I)/16)*Sqrt[d]*(-(ArcSech[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/e^(5/2) + (PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]))/(4*e^3) - (-PolyLog[2, -E^(-2*ArcSech[c*x])] + 2*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]

```
 + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x]
)] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]
))/(4*e^3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.87 (sec) , antiderivative size = 1549, normalized size of antiderivative = 1.99

method	result	size
parts	Expression too large to display	1549
derivativedivides	Expression too large to display	1562
default	Expression too large to display	1562

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2/e^3*ln(e*x^2+d)+d/e^3/(e*x^2+d)-1/4*d^2/e^3/(e*x^2+d)^2)+b/c^6*(-1/8
*c^6*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^5*d^2*x+(-(c*x-1)/c/x)^(1/
2)*((c*x+1)/c/x)^(1/2)*c^5*d*e*x^3+4*arcsech(c*x)*c^6*d^2*x^2+6*c^6*d*e*arc
sech(c*x)*x^4+4*c^4*d*e*arcsech(c*x)*x^2+6*arcsech(c*x)*e^2*c^4*x^4-c^4*d^2
-2*c^4*d*e*x^2-c^4*e^2*x^4)/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-3/4*(e*(c^2*d
+e))^(1/2)/(c^2*d+e)^2/e^2*c^6*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-5/8*(e*(c^2*d+e))^(1/
2)/(c^2*d+e)^2/e^3*c^8*d*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/
c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-1/(c^2*d+e)/e^2*c^6*arcsech
(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/(c^2*d+e)/e^2*c^6*
arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/(c^2*d+e)/e
^2*c^6*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/(c^2*d+e)/e^2*
c^6*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4/(c^2*d+e)/e^2*c
^6*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R
1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^
(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d
))+1/4/(c^2*d+e)/e^2*c^8*d*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c
*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*
_Z^2+c^2*d))-1/(c^2*d+e)/e^3*c^8*d*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2)))-1/(c^2*d+e)/e^3*c^8*d*arcsech(c*x)*ln(1-I*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/(c^2*d+e)/e^3*c^8*d*dilog(1+I*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2)))-1/(c^2*d+e)/e^3*c^8*d*dilog(1-I*(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4/(c^2*d+e)/e^3*c^8*d*sum((_R1^2*c^2*d+c^2*d
+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R
1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4/(c^2*d+e)/e^3*c^10
*d^2*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+
```

```
1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^5 (a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^5}{(ex^2 + d)^3} \, dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*x^5*arcsech(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 (a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} \, dx = \text{Timed out}$$

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^5 (a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^5}{(ex^2 + d)^3} \, dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*
x))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Giac [F]

$$\int \frac{x^5(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^5}{(ex^2 + d)^3}\,dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3}\,dx = \int \frac{x^5\,\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^3}\,dx$$

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.124 $\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\left(c^2d+e\right)^{3/2}}$$

```
[Out] 1/4*x^4*(a+b*arcsech(c*x))/d/(e*x^2+d)^2-1/8*b*(c^2*d+2*e)*arctanh(e^(1/2)*
(-c^2*x^2+1)^(1/2)/(c^2*d+e)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d/e^(3/
2)/(c^2*d+e)^(3/2)+1/8*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)
/e/(c^2*d+e)/(e*x^2+d)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 6436, 12, 457, 79, 65, 214}

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+2e)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\left(c^2d+e\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}$$

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(8*e*(c^2*d + e)*(
d + e*x^2)) + (x^4*(a + b*ArcSech[c*x]))/(4*d*(d + e*x^2)^2) - (b*(c^2*d +
2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[(Sqrt[e]*Sqrt[1 - c^2*x^2])
/Sqrt[c^2*d + e]])/(8*d*e^(3/2)*(c^2*d + e)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3}{4d\sqrt{1-c^2x^2}\left(d+ex^2\right)^2}\,dx\\
&= \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{x^3}{\sqrt{1-c^2x^2}(d+ex^2)^2}\,dx}{4d}\\
&= \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{x}{\sqrt{1-c^2x}(d+ex)^2}\,dx,x,x^2\right)}{8d}\\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}\\
&\quad+\frac{\left(b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-c^2x}(d+ex)}\,dx,x,x^2\right)}{16de\left(c^2d+e\right)}\\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}\\
&\quad-\frac{\left(b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{8c^2de\left(c^2d+e\right)}\\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8e\left(c^2d+e\right)\left(d+ex^2\right)}+\frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}\\
&\quad-\frac{b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\left(c^2d+e\right)^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.81

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx =$$

$$-\frac{-\frac{4ad}{(d+ex^2)^2}+\frac{8a}{d+ex^2}-\frac{2e\sqrt{\frac{1-cx}{1+cx}}(b+bcx)}{(c^2d+e)(d+ex^2)}+\frac{4b(d+2ex^2)\text{sech}^{-1}(cx)}{(d+ex^2)^2}+\frac{4b\log(x)}{d}-\frac{4b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d}+\frac{b\sqrt{e}(c^2d+2e)\cdots}{\cdots}}{\cdots}$$

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*e*Sqrt[(1 - c*x)/(1
+ c*x)]*(b + b*c*x))/((c^2*d + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcSech
[c*x])/(d + e*x^2)^2 + (4*b*Log[x])/d - (4*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*
x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(16*
d*e^(3/2)*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt
[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/(b*
(c^2*d + 2*e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2)) + (b*Sqrt
[e]*(c^2*d + 2*e)*Log[(16*d*e^(3/2)*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d
]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[
(1 - c*x)/(1 + c*x)]))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*
d + e)^(3/2)))/e^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1361 vs. 2(147) = 294.

Time = 5.09 (sec) , antiderivative size = 1362, normalized size of antiderivative = 7.87

method	result	size
parts	Expression too large to display	1362
derivativedivides	Expression too large to display	1395
default	Expression too large to display	1395

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/e^2/(e*x^2+d)+1/4*d/e^2/(e*x^2+d)^2)+b/c^4*(-1/2*c^6*arcsech(c*x)/e
^2/(c^2*e*x^2+c^2*d)+1/4*c^8*arcsech(c*x)*d/e^2/(c^2*e*x^2+c^2*d)^2-1/16*c^
5*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e*(4*((c^2*d+e)/e)^(1/2)*arcta
nh(1/(-c^2*x^2+1)^(1/2))*c^6*d^2*e*x^2+4*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^
2*x^2+1)^(1/2))*c^6*d^3-ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c
^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^6*d^2*e-ln(-2*(((c^2*
```

```
d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*
e)^(1/2)))*c^6*d^3-ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e
)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^6*d^2*e*x^2-ln(2*(((c^2*d+e)/e)^
(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2))
)*c^6*d^3+8*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*d*e^2*x^2
+8*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*d^2*e+2*(-c^2*x^2+
1)^(1/2)*((c^2*d+e)/e)^(1/2)*c^4*d^2*e-3*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x
^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^4*d*
e^2-3*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+
e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2*e-3*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x
^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e^2*x
^2-3*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)
/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2*e+4*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*
x^2+1)^(1/2))*e^3*c^2*x^2+4*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2
))*c^2*d*e^2+2*(-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*c^2*d*e^2-2*ln(-2*(((
c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^
2*d*e)^(1/2)))*x^2*c^2*e^3-2*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*
e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e^2-2*ln(2*(((c^
2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d
*e)^(1/2)))*e^3*c^2*x^2-2*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-
c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e^2)/(c*e*x+(-c^2*d*e
)^(1/2))/(-c*e*x+(-c^2*d*e)^(1/2))/((c^2*d+e)/e)^(1/2)/(-e+(-c^2*d*e)^(1/2)
)^2/(e+(-c^2*d*e)^(1/2))^2/d/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(115) = 230$.

Time = 0.38 (sec) , antiderivative size = 1346, normalized size of antiderivative = 7.78

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 2*(4*a - b)*c^2*d^3*e + 2*(2*a - b)*d^2*e^2 - 2*(b*c^
2*d*e^3 + b*e^4)*x^4 + 4*(2*a*c^4*d^3*e + (4*a - b)*c^2*d^2*e^2 + (2*a - b)
*d*e^3)*x^2 - (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c
^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*log((c^4*d^2 + 4*c^2*d*e - (
c^4*d*e + 2*c^2*e^2)*x^2 + 4*(c^3*d*e + c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x
^2)) + 4*e^2 + 2*(c^2*e*x^2 - c^2*d - (c^3*d + 2*c*e)*x*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) - 2*e)*sqrt(c^2*d*e + e^2))/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c
^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c
^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) - 1)/x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e
```

```
 + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
+ 1)/(c*x)) - 2*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e
^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e
^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3
*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + (4*a - b)*c^2*d^3*e + (2*a - b)*d
^2*e^2 - (b*c^2*d*e^3 + b*e^4)*x^4 + 2*(2*a*c^4*d^3*e + (4*a - b)*c^2*d^2*e
^2 + (2*a - b)*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*
d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*arctan((sqrt(
-c^2*d*e - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)
*(e*x^2 + d))/((c^2*d*e + e^2)*x^2)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2
*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c
^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x)
+ 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e
^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - ((
b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(-(c^2*
x^2 - 1)/(c^2*x^2)))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4
+ 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^
2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^3}{(ex^2 + d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3}\,dx = \int \frac{x^3\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^3}\,dx$$

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.125 $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 217

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx = -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d\,(c^2d+e)\,(d+ex^2)}-\frac{a+b\text{sech}^{-1}(cx)}{4e\,(d+ex^2)^2}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4d^2e}-\frac{b(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\,(c^2d+e)^{3/2}}$$

```
[Out] 1/4*(-a-b*arcsech(c*x))/e/(e*x^2+d)^2+1/4*b*arctanh((-c^2*x^2+1)^(1/2))*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^2/e-1/8*b*(3*c^2*d+2*e)*arctanh(e^(1/2)*(-c^
2*x^2+1)^(1/2)/(c^2*d+e)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*d+
e)^(3/2)/e^(1/2)-1/8*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d
/(c^2*d+e)/(e*x^2+d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

= {6434, 531, 457, 105, 162, 65, 214}

$$\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4d^2e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(3c^2d+2e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\left(c^2d+e\right)^{3/2}} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{8d\left(c^2d+e\right)\left(d+ex^2\right)}$$

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(d*(c^2*d + e
)*(d + e*x^2)) - (a + b*ArcSech[c*x])/(4*e*(d + e*x^2)^2) + (b*Sqrt[(1 + c*
x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(4*d^2*e) - (b*(3*c^2*d
+ 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[(Sqrt[e]*Sqrt[1 - c^2*x^2
])/Sqrt[c^2*d + e]])/(8*d^2*Sqrt[e]*(c^2*d + e)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6434

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Dist[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{4e\,(d+ex^2)^2}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}(d+ex^2)^2}\,dx}{4e}\\
&= -\frac{a+b\operatorname{sech}^{-1}(cx)}{4e\,(d+ex^2)^2}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{x\sqrt{1-c^2x^2}(d+ex^2)^2}\,dx}{4e}\\
&= -\frac{a+b\operatorname{sech}^{-1}(cx)}{4e\,(d+ex^2)^2}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}(d+ex)^2}\,dx,x,x^2\right)}{8e}\\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d\,(c^2d+e)\,(d+ex^2)}-\frac{a+b\operatorname{sech}^{-1}(cx)}{4e\,(d+ex^2)^2}\\
&\quad-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{c^2d+e-\frac{1}{2}c^2ex}{x\sqrt{1-c^2x}(d+ex)}\,dx,x,x^2\right)}{8de\,(c^2d+e)}
\end{aligned}
$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d\left(c^2d+e\right)\left(d+ex^2\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{8d^2e} + \frac{\left(b\left(\frac{1}{2}c^2de+e(c^2d+e)\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}(d+ex)}\,dx, x, x^2\right)}{8d^2e\left(c^2d+e\right)}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d\left(c^2d+e\right)\left(d+ex^2\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{4c^2d^2e} - \frac{\left(b\left(\frac{1}{2}c^2de+e(c^2d+e)\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{4c^2d^2e\left(c^2d+e\right)}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d\left(c^2d+e\right)\left(d+ex^2\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{4d^2e} - \frac{b(3c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\left(c^2d+e\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.24

$$\int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{1}{16}\left(-\frac{4a}{e(d+ex^2)^2}-\frac{2\sqrt{\frac{1-cx}{1+cx}}(b+bcx)}{d(c^2d+e)(d+ex^2)}-\frac{4b\operatorname{sech}^{-1}(cx)}{e(d+ex^2)^2}-\frac{4b\log(x)}{d^2e}+\frac{4b\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{d^2e}-\frac{b(3c^2d+2e)\log\left(\frac{16d^2\sqrt{e}\sqrt{c^2d+e}\left(\sqrt{e}-ic^2\sqrt{d}x+\sqrt{c^2d+e}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{c^2d+e}x\sqrt{\frac{1-cx}{1+cx}}\right)}{b(3c^2d+2e)\left(-i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2\sqrt{e}(c^2d+e)^{3/2}}-\frac{b(3c^2d+2e)\log\left(\frac{16d^2\sqrt{e}\sqrt{c^2d+e}\left(\sqrt{e}+ic^2\sqrt{d}x+\sqrt{c^2d+e}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{c^2d+e}x\sqrt{\frac{1-cx}{1+cx}}\right)}{b(3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2\sqrt{e}(c^2d+e)^{3/2}}\right)$$

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a)/(e*(d + e*x^2)^2) - (2*Sqrt[(1 - c*x)/(1 + c*x)]*(b + b*c*x))/(d*(c
^2*d + e)*(d + e*x^2)) - (4*b*ArcSech[c*x])/(e*(d + e*x^2)^2) - (4*b*Log[x]
)/(d^2*e) + (4*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1
+ c*x)]])/(d^2*e) - (b*(3*c^2*d + 2*e)*Log[(16*d^2*Sqrt[e]*Sqrt[c^2*d + e]*
(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*
Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/(b*(3*c^2*d + 2*e)*((-I)*Sqrt
[d] + Sqrt[e]*x))])/(d^2*Sqrt[e]*(c^2*d + e)^(3/2)) - (b*(3*c^2*d + 2*e)*Lo
g[(16*d^2*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d +
 e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x
)]))/(b*(3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*Sqrt[e]*(c^2*d + e)
^(3/2)))/16
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. 2(186) = 372.

Time = 5.10 (sec) , antiderivative size = 1318, normalized size of antiderivative = 6.07

method	result	size
parts	Expression too large to display	1318
derivativedivides	Expression too large to display	1329
default	Expression too large to display	1329

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arcsech(c*x)-1/1
6*c^3*e^2*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(4*((c^2*d+e)/e)^(1/2)
*arctanh(1/(-c^2*x^2+1)^(1/2))*c^6*d^2*e*x^2+4*((c^2*d+e)/e)^(1/2)*arctanh(
1/(-c^2*x^2+1)^(1/2))*c^6*d^3-3*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/
2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^6*d^2*e-3*ln(
-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*
x+(-c^2*d*e)^(1/2)))*c^6*d^3-3*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)
*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^6*d^2*e*x^2-3*ln(2*(
((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c
^2*d*e)^(1/2)))*c^6*d^3+8*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))
*c^4*d*e^2*x^2+8*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*d^2*
e-2*(-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*c^4*d^2*e-5*ln(-2*(((c^2*d+e)/e)
^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2
)))*x^2*c^4*d*e^2-5*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d
*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2*e-5*ln(2*(((c^2*d+e)/e)
^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)
))*c^4*d*e^2*x^2-5*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e
)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2*e+4*((c^2*d+e)/e)^(1/2)*ar
ctanh(1/(-c^2*x^2+1)^(1/2))*e^3*c^2*x^2+4*((c^2*d+e)/e)^(1/2)*arctanh(1/(-c
^2*x^2+1)^(1/2))*c^2*d*e^2-2*(-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*c^2*d*e
^2-2*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e
)/(-c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^2*e^3-2*ln(-2*(((c^2*d+e)/e)^(1/2)*(-c^2
*x^2+1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e^
2-2*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/
(c*e*x+(-c^2*d*e)^(1/2)))*e^3*c^2*x^2-2*ln(2*(((c^2*d+e)/e)^(1/2)*(-c^2*x^2
+1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e^2)/(-
c*e*x+(-c^2*d*e)^(1/2))/(c*e*x+(-c^2*d*e)^(1/2))/((c^2*d+e)/e)^(1/2)/(-e+(-
c^2*d*e)^(1/2))^2/(e+(-c^2*d*e)^(1/2))^2/d^2/(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(135) = 270.

Time = 0.37 (sec) , antiderivative size = 1232, normalized size of antiderivative = 5.68

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 2*(4*a + b)*c^2*d^3*e + 2*(2*a + b)*d^2*e^2 + 2*(b*c^
2*d*e^3 + b*e^4)*x^4 + 4*(b*c^2*d^2*e^2 + b*d*e^3)*x^2 - (3*b*c^2*d^3 + (3*
b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)
*sqrt(c^2*d*e + e^2)*log((c^4*d^2 + 4*c^2*d*e - (c^4*d*e + 2*c^2*e^2)*x^2 +
 4*(c^3*d*e + c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 4*e^2 + 2*(c^2*e*x^
2 - c^2*d - (c^3*d + 2*c*e)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*e)*sqrt(c^
2*d*e + e^2))/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*
c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2
 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 4*(b*c^4
*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
 1)/(c*x)) + 2*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^
2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3
+ (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*
e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + (4*a + b)*c^2*d^3*e + (2*a + b)*d^
2*e^2 + (b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^2*d^2*e^2 + b*d*e^3)*x^2 + (3*b*
c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*
b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*arctan((sqrt(-c^2*d*e - e^2)*c*d*x*sqrt(
-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)*(e*x^2 + d))/((c^2*d*e + e
^2)*x^2)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b
*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*
log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 2*(b*c^4*d^4 + 2*b*c^2*d^
3*e + b*d^2*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + ((b*
c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*
c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3} dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3} dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x}{(ex^2 + d)^3} dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^3} dx = \int \frac{x\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^3} dx$$

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.126 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 741

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^3}\,dx = -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2(c^2d+e)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd^3} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{b\sqrt{e}(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8d^3(c^2d+e)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

```
[Out] 1/4*e^2*(a+b*arcsech(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arcsech(c*x))/d^3/(e+d/x^
2)+1/2*(a+b*arcsech(c*x))^2/b/d^3-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2
*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1
/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/
2*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1
/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*b*polylo
g(2,c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(
```

```
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,c*(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/8*b*e*
(c^2-1/x^2)/c/d^2/(c^2*d+e)/(e+d/x^2)/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/
8*b*(c^2*d+2*e)*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*e
^(1/2)*(-1+1/c^2/x^2)^(1/2)/d^3/(c^2*d+e)^(3/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^
(1/2)+b*arctanh((c^2*d+e)^(1/2)/c/x/e^(1/2)/(-1+1/c^2/x^2)^(1/2))*e^(1/2)*(
-1+1/c^2/x^2)^(1/2)/d^3/(c^2*d+e)^(1/2)/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6438, 5959, 5957, 533, 390, 385, 214, 5962, 5681, 2221, 2317, 2438}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = -\frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}+1\right)}{2d^3} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3} - \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}+1\right)}{2d^3} + \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{4d^3\left(\frac{d}{x^2}+e\right)^2} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{d^3\left(\frac{d}{x^2}+e\right)} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd^3} - \frac{b\sqrt{e}\sqrt{\frac{1}{c^2x^2}-1}(c^2d+2e)\,\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{8d^3\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\left(c^2d+e\right)^{3/2}} + \frac{b\sqrt{e}\sqrt{\frac{1}{c^2x^2}-1}\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}-1}}\right)}{d^3\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\sqrt{c^2d+e}} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3} - \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3} - \frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}\left(c^2d+e\right)\left(\frac{d}{x^2}+e\right)}$$

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -1/8*(b*e*(c^2 - x^(-2)))/(c*d^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)]
*Sqrt[1 + 1/(c*x)]*x) + (e^2*(a + b*ArcSech[c*x]))/(4*d^3*(e + d/x^2)^2) -
(e*(a + b*ArcSech[c*x]))/(d^3*(e + d/x^2)) + (a + b*ArcSech[c*x])^2/(2*b*d^
3) + (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*S
qrt[-1 + 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 +
 1/(c*x)]) - (b*Sqrt[e]*(c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c
^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d + e)^(3/2)*S
qrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqr
t[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSe
ch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/
(2*d^3) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e
] + Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*
E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - (b*PolyLog[2, -((c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d^3) - (b*PolyLo
g[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - (b
*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2
*d^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e
])])/(2*d^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
```

b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ[n, 2] && IGtQ[q, 0])

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))), x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5957

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCosh[c*x])/(2*e*(p + 1))), x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^5\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{e^2x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3}-\frac{2ex\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2}+\frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2}+\frac{(2e)\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad -\frac{e^2\text{Subst}\left(\int \frac{x\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d^2}\\
&= \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2}-\frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)}\\
&\quad -\frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad +\frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{cd^3}\\
&\quad -\frac{\left(be^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{4cd^3}
\end{aligned}$$

$$= \frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{5/2}} + \frac{\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{5/2}} + \frac{\left(be\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x^2}{c^2}}(e+dx^2)}\,dx, x, \frac{1}{x}\right)}{cd^3\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{\left(be^2\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x^2}{c^2}}(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)}{4cd^3\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{2(-d)^{5/2}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{2(-d)^{5/2}} + \frac{\left(be\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int \frac{1}{e-\left(d+\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{cd^3\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{\left(be(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x^2}{c^2}}(e+dx^2)}\,dx, x, \frac{1}{x}\right)}{8cd^3\left(c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd^3} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}} - \frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\text{sech}^{-1}(cx)\right)}{2(-d)^{5/2}} - \frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\text{sech}^{-1}(cx)\right)}{2(-d)^{5/2}} + \frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\text{sech}^{-1}(cx)\right)}{2(-d)^{5/2}} + \frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\text{sech}^{-1}(cx)\right)}{2(-d)^{5/2}} - \frac{\left(be(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\right)\text{Subst}\left(\int\frac{1}{e-\left(d+\frac{e}{c^2}\right)x^2}\,dx,x,\frac{1}{\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8cd^3\left(c^2d+e\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd^3} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{b\sqrt{e}(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d+e\right)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\operatorname{sech}^{-1}(cx)\right)}{2d^3}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{e^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)^2}{2bd^3} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{b\sqrt{e}(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d+e\right)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{\operatorname{sech}^{-1}(cx)}\right)}{2d^3}$$

$$= -\frac{be\left(c^2-\frac{1}{x^2}\right)}{8cd^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{e^2\left(a+b\text{sech}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{sech}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)}$$
$$+ \frac{\left(a+b\text{sech}^{-1}(cx)\right)^2}{2bd^3} + \frac{b\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{d^3\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$- \frac{b\sqrt{e}(c^2d+2e)\sqrt{-1+\frac{1}{c^2x^2}}\,\text{arctanh}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}\,x}\right)}{8d^3\left(c^2d+e\right)^{3/2}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$
$$- \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{\left(a+b\text{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.09 (sec) , antiderivative size = 2054, normalized size of antiderivative = 2.77

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] a/(4*d*(d + e*x^2)^2) + a/(2*d^2*(d + e*x^2)) + (a*Log[x])/d^3 - (a*Log[d +
 e*x^2])/(2*d^3) + b*((Sqrt[e]*(((-I)*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1
+ c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sq
rt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1
```

- c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d
+ e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^
2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1
+ c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2)
)))/(16*d^2) + (Sqrt[e]*((I*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(S
qrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*(I*Sqrt
[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)
] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*S
qrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1
 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^
2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*d^2) - (((5
*I)/16)*Sqrt[e]*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(Log[x]/Sqr
t[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/S
qrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) +
(Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)]/Sq
rt[c^2*d + e]))/Sqrt[d]))/d^(5/2) + (((5*I)/16)*Sqrt[e]*(-(ArcSech[c*x]/((-
I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1
 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[
d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt
[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/d^(5/
2) + (-(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])])) + Pol
yLog[2, -E^(-2*ArcSech[c*x])])/(2*d^3) - (PolyLog[2, -E^(-2*ArcSech[c*x])]
- 2*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c
*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*L
og[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d
 + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Arc
Sech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[
d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2
]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + Po
lyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + Pol
yLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]))/(4
*d^3) + (-PolyLog[2, -E^(-2*ArcSech[c*x])] + 2*((-4*I)*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcS
ech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] -
 ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech
[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (
I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*
Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)
*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sq
rt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(-Sqrt[e] +
Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sq
rt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])]))/(4*d^3))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.33 (sec) , antiderivative size = 3727, normalized size of antiderivative = 5.03

method	result	size
parts	Expression too large to display	3727
derivativedivides	Expression too large to display	3801
default	Expression too large to display	3801

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d^3*ln(e*x^2+d)+1/4*a/d/(e*x^2+d)^2+1/2*a/d^2/(e*x^2+d)+a/d^3*ln(x)+
b*(-2*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*
e/(c^4*d^2+2*c^2*d*e+e^2)/d^4/c^2*arcsech(c*x)^2+(-c^2*d*(e*(c^2*d+e))^(1/2
)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e/(c^4*d^2+2*c^2*d*e+e^2)/d^4/c^
2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(
c^2*d+e))^(1/2)-2*e))-3/4*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^4/(c^2*d+e)/c
^2*e*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(
e*(c^2*d+e))^(1/2)-2*e))-(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(
c^2*d+e))^(1/2)*e)/(c^4*d^2+2*c^2*d*e+e^2)*e^2/d^5/c^4*arcsech(c*x)^2-1/4*(
e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d/e*c^4*arcsech(c*x)^2+1/8*(e*(c^2*d+e))^(1/
2)/(c^2*d+e)^2/d/e*c^4*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1
/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/2*(-c^2*d*(e*(c^2*d+e))^(1/2)+
2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^4*d^2+2*c^2*d*e+e^2)*e^2/d^5/c^
4*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(
c^2*d+e))^(1/2)-2*e))+1/2*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*arcsech(c*x
)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d
+e))^(1/2)-2*e))+(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^5/(c^2*d+e)/c^4*e^2*ar
csech(c*x)^2+3/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^4/(c^2*d+e)/c^2*e*arcs
ech(c*x)^2+3/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*c^2*arcsech(c*x)*ln(1-d*
c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2
)-2*e))+1/8*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/
2)*e)/e/(c^4*d^2+2*c^2*d*e+e^2)*c^2/d^2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/2*(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)/d^5/(c^2*d+e)/c^4*e^2*polylog(2,d*c^2*(1/c/x+(-1+1/c
/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-1/4*(-c^2*
d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/e/(c^4*d^2+2
*c^2*d*e+e^2)*c^2/d^2*arcsech(c*x)^2-3/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/
d^4/(c^2*d+e)/c^2*e*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(
-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+1/4*(e*(c^2*d+e))^(1/2)/(c^
2*d+e)^2/d/e*c^4*arcsech(c*x)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+(-c^2*d*(e*(c^2*d+e))^(1/2)+2*
c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/(c^4*d^2+2*c^2*d*e+e^2)*e^2/d^5/c^4*
ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e
```

))^(1/2)-2*e))*arcsech(c*x)+2*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e/(c^4*d^2+2*c^2*d*e+e^2)/d^4/c^2*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+1/4*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/e/(c^4*d^2+2*c^2*d*e+e^2)*c^2/d^2*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^5/(c^2*d+e)/c^4*e^2*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)-7/8*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*c^2*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-1/8*e*(8*arcsech(c*x)*c^6*d^2*x^2+6*c^6*d*e*arcsech(c*x)*x^4-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^5*d^2*x-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^5*d*e*x^3+8*c^4*d*e*arcsech(c*x)*x^2+6*arcsech(c*x)*e^2*c^4*x^4+c^4*d^2+2*c^4*d*e*x^2+c^4*e^2*x^4)/d^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)+5/4*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/d^3/(c^4*d^2+2*c^2*d*e+e^2)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)-3/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*c^2*arcsech(c*x)^2-3/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-1/2*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*arcsech(c*x)^2-1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^3/(c^2*d+e)*ln(1-d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsech(c*x)+1/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+3/8*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*c^2*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/2/(c^2*d+e)/d^2*c^2*sum((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+5/8*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/d^3/(c^4*d^2+2*c^2*d*e+e^2)*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))-5/4*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)/d^3/(c^4*d^2+2*c^2*d*e+e^2)*arcsech(c*x)^2+1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^3/(c^2*d+e)*arcsech(c*x)^2-1/4*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^3/(c^2*d+e)*polylog(2,d*c^2*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))+1/(c^2*d+e)/d^3*e*arcsech(c*x)^2+1/(c^2*d+e)/d^2*c^2*arcsech(c*x)^2-1/2/(c^2*d+e)/d^3*e*sum((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*arcsech(c*x)^2/d^3)

Fricas [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\left(d + ex^2\right)^3}\, dx = \int \frac{b\,\text{arsech}\left(cx\right) + a}{\left(ex^2 + d\right)^3 x}\, dx$$

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\left(d + ex^2\right)^3}\, dx = \text{Timed out}$$

[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\left(d + ex^2\right)^3}\, dx = \int \frac{b\,\text{arsech}\left(cx\right) + a}{\left(ex^2 + d\right)^3 x}\, dx$$

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")

[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)/d^3 + 4*log(x)/d^3) + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\left(d + ex^2\right)^3}\, dx = \int \frac{b\,\text{arsech}\left(cx\right) + a}{\left(ex^2 + d\right)^3 x}\, dx$$

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^3*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x(d + ex^2)^3} \, dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2 + d)^3} \, dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^3),x)
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

3.127 $\int \frac{x^4\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size $=21$, antiderivative size $=1272$

$$
\begin{aligned}
\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx &= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&+\frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} \\
&+\frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&-\frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{3b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} \\
&-\frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}e} \\
&-\frac{3b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} \\
&-\frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}e} \\
&+\frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&-\frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+\frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&-\frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&-\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&-\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&\quad 3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\cdots}\right.
\end{aligned}
$$

```
[Out] 3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arcsech(c*x
))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)
/(-d)^(1/2)-3/16*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*b*po
lylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^
2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)
-3/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/
(-d)^(1/2)-1/8*b*d*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-
1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/e/(c*d-(-d)^(1/2)*e^(1/2))^(
3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)-1/8*b*d*arctan((1+1/c/x)^(1/2)*(c*d+(-d
)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/e/(
c*d-(-d)^(1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/16*(a+b*arcs
ech(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2+3/16*(a+b*arcsech(
c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arcsech(c*x))*(-d)^(1/2)/e^(3
/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/16*(a+b*arcsech(c*x))/e^2/(d/x+(-d)^(1/2)*
e^(1/2))+1/16*b*c*(-d)^(1/2)*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/e^(3/2)/(c^2*
d+e)/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*b*c*(-d)^(1/2)*(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2)/e^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))-3/8*b*arctan((1+1/c/x)^
(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/
2))^(1/2))/e^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2
)-3/8*b*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1
/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/e^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d
+(-d)^(1/2)*e^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 1272, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules

used = {6438, 5909, 5963, 98, 95, 211, 5962, 5681, 2221, 2317, 2438}

$$\int \frac{x^4 (a + b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16e^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16e^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{3(a+b\operatorname{sech}^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{3(a+b\operatorname{sech}^{-1}(cx))}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))}{16e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2} - \frac{bd\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}e} - \frac{3b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} - \frac{bd\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}e} - \frac{3b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} + \frac{3(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\operatorname{sech}^{-1}(cx))\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}} - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*e^(3/2)*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c
*x)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*
ArcSech[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcSech[
c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcSech[c*x]))/
(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcSech[c*x]))/(16*e^2*
(Sqrt[-d]*Sqrt[e] + d/x)) - (3*b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[
1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*Sqrt[c
*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*d*ArcTan[(Sqr
t[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*
Sqrt[-1 + 1/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqr
t[e])^(3/2)*e) - (3*b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)
])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*Sqrt[c*d - Sqrt[-
d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*d*ArcTan[(Sqrt[c*d + Sqr
t[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1
/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)
*e) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e]
- Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSech[c*x])*Log[1
+ (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^
(5/2)) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[
e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSech[c*x])*Log
[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]
*e^(5/2)) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c
^2*d + e]))])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2
, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(16*Sqrt[-d]
*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2
*d + e])])/(16*Sqrt[-d]*e^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

, 1])

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3}-\frac{3d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\right.\right.\\
&\qquad -\frac{d^3\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3}-\frac{3d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\\
&\qquad \left.\left.-\frac{3d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{8e^2\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{(3d)\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2}+\frac{(3d)\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2}\\
&\quad +\frac{(3d)\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8e^2}\\
&\quad -\frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8e^{3/2}}\\
&\quad -\frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8e^{3/2}}
\end{aligned}$$

$$= \frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}+\frac{3\left(a+b\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$-\frac{3\left(a+b\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{(3b)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)}\,dx,x,\frac{1}{x}\right)}{16ce^2}$$

$$+\frac{(3b)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)}\,dx,x,\frac{1}{x}\right)}{16ce^2}$$

$$+\frac{(3d)\text{Subst}\left(\int\left(-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{8e^2}$$

$$-\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx,x,\frac{1}{x}\right)}{16ce^{3/2}}$$

$$+\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx,x,\frac{1}{x}\right)}{16ce^{3/2}}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+\frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}+\frac{3\left(a+b\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$-\frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}-\frac{3\left(a+b\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$-\frac{3\text{Subst}\left(\int\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{16e^{5/2}}-\frac{3\text{Subst}\left(\int\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{16e^{5/2}}$$

$$-\frac{(3b)\text{Subst}\left(\int\frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx,x,\frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{8ce^2}$$

$$+\frac{(3b)\text{Subst}\left(\int\frac{1}{-d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx,x,\frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{8ce^2}$$

$$-\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)}\,dx,x,\frac{1}{x}\right)}{16ce\left(c^2d+e\right)}$$

$$+\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)}\,dx,x,\frac{1}{x}\right)}{16ce\left(c^2d+e\right)}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$- \frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3\left(a+b\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$- \frac{3b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2} - \frac{3b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$

$$- \frac{3\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{16e^{5/2}}$$

$$- \frac{3\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \text{sech}^{-1}(cx)\right)}{16e^{5/2}}$$

$$- \frac{b\text{Subst}\left(\int \frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{8ce\left(c^2d+e\right)}$$

$$+ \frac{b\text{Subst}\left(\int \frac{1}{-d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{8ce\left(c^2d+e\right)}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\text{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$- \frac{3\left(a+b\text{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$

$$- \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e\left(c^2d+e\right)}$$

$$- \frac{3b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$

$$- \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e\left(c^2d+e\right)}$$

$$- \frac{3\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{16e^{5/2}}$$

$$- \frac{3\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{16e^{5/2}}$$

$$- \frac{3\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{16e^{5/2}}$$

$$- \frac{3\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx, x, \text{sech}^{-1}(cx)\right)}{16e^{5/2}}$$

$$= \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$- \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$

$$- \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e\left(c^2d+e\right)}$$

$$- \frac{3b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e^2}$$

$$- \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e\left(c^2d+e\right)}$$

$$+ \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{(3b)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{(3b)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{(3b)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{(3b)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \operatorname{sech}^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

= Too large to display

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.11 (sec) , antiderivative size = 2022, normalized size of antiderivative = 1.59

$$\int \frac{x^4(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*(((I/16)*Sqrt[d]*(((-I)*Sqrt[e
]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] +
 Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]
/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 +
c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt
[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[
c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqr
t[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 - ((I/16)*Sqrt[d]*((I*Sqrt[e]*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)
) - ArcSech[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) -
 Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt
[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sq
rt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*S
qrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^
2*d + e)^(3/2))))/e^2 + (5*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*
(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1
 + c*x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqr
t[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/(16*e^2) + (5*(-(ArcSech[c*x]/((-I)*Sq
rt[d]*Sqrt[e] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*
x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[d]*Sq
rt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*
d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/(16*e^2) -
 (((3*I)/32)*(PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 +
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[Arc
Sech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])]
- ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech
[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (
I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*L
og[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*
ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqr
```

```
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt
[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])))/(Sqrt[d]*e^(5/2)) - (((3*I)/3
2)*(-PolyLog[2, -E^(-2*ArcSech[c*x])] + 2*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e
])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c
*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcS
ech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x]
)] + (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-S
qrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1
 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[
c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^
2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])))/(Sqrt[d]*e^(5/2)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 110.96 (sec) , antiderivative size = 1960, normalized size of antiderivative = 1.54

method	result	size
parts	Expression too large to display	1960
derivativedivides	Expression too large to display	1983
default	Expression too large to display	1983

```
[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*((-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*e
)^(1/2)))+b/c^5*(-1/8*x*c^7*(-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^3*
d*e*x-(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e^2*c^3*x^3+3*d^2*c^4*arcsec
h(c*x)+5*c^4*d*e*arcsech(c*x)*x^2+3*c^2*d*e*arcsech(c*x)+5*e^2*arcsech(c*x)
*c^2*x^2)/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-3/8*(-(c^2*d-2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e)
)^(1/2)*e)*c^3*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*
d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e^2/d^2-3/8*((c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2
-2*(e*(c^2*d+e))^(1/2)*e)*c^3*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e^2/d^2+1/2
*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+
2*e)*c*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(
c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d^3-1/2*(-(c^2*d-2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d
+e))^(1/2)*e)*c*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3+1/2*((c^2*d+2*(e*
```

```
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c
*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2))/(c^2*d+e)/e/d^3-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*c*a
rctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3+3/16/(c^2*d+e)/e*c^6*sum(1/_R1/(_R1^
2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2
)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c^3*arctanh(c*d*(1/c/x+(-
1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2
))/(c^2*d+e)/e^2/d^2+3/8*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d
-2*(e*(c^2*d+e))^(1/2)+2*e)*c^3*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/e^2/d^2-3/1
6/(c^2*d+e)/e*c^6*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c
/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)
*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/
16/(c^2*d+e)/e^2*c^8*d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((
_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2
*d))-3/16/(c^2*d+e)/e^2*c^8*d*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)
*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^
2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^4}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*x^4*arcsech(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\operatorname{arsech}(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

```
[In] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.128 $\int \frac{x^2\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 1276

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} - \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} + \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} - \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} - \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\operatorname{sech}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

$b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\ldots}\right)$

```
[Out] -1/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsech(c*
x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^
2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(
3/2)/e^(3/2)+1/16*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/
x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*b*p
olylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c
^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/
2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2
)+1/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(
e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3
/2)/e^(3/2)+1/8*b*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1
+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/(c*d-(-d)^(1/2)*e^(1/2))^(3/2
)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/8*b*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/
2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/(c*d-(-d
)^(1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/16*(a+b*arcsech(c*x
))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(a+b*arcsech(c*x))/d
/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*(-a-b*arcsech(c*x))/(-d)^(1/2)/e^(1/2)/(d
/x+(-d)^(1/2)*e^(1/2))^2+1/16*(-a-b*arcsech(c*x))/d/e/(d/x+(-d)^(1/2)*e^(1/
2))+1/16*b*c*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/(c^2*d+e)/(-d)^(1/2)/e^(1/2)/
(-d/x+(-d)^(1/2)*e^(1/2))+1/16*b*c*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/(c^2*d+
e)/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))-1/8*b*arctan((1+1/c/x)^(1/2)
*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(
1/2))/d/e/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2)-1/8
*b*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(
c*d-(-d)^(1/2)*e^(1/2))^(1/2))/d/e/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)
^(1/2)*e^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.71 (sec) , antiderivative size = 1276, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules

used = {6438, 5959, 5909, 5963, 98, 95, 211, 5962, 5681, 2221, 2317, 2438}

$$
\begin{aligned}
\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx =& \frac{b\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&+ \frac{b\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16\sqrt{-d}\sqrt{e}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} \\
&+ \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} \\
&+ \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2} \\
&- \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} \\
&+ \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} \\
&- \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} \\
&+ \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} \\
&- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}} \\
&- \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&- \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} \\
&+ \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}
\end{aligned}
$$

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16
*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcSech[c*
x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcSech[c*x])
/(16*d*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSech[c*x])/(16*Sqrt[-d]*Sqrt
[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (a + b*ArcSech[c*x])/(16*d*e*(Sqrt[-d]*Sq
rt[e] + d/x)) + (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/
(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt
[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)) - (b*ArcTan[(Sqrt[c*d - Sqrt[-d]
*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x
)])])/(8*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) + (
b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[
-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d +
 Sqrt[-d]*Sqrt[e])^(3/2)) - (b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1
+ 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d*Sqrt[c
*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) - ((a + b*ArcSech[c*
x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(
-d)^(3/2)*e^(3/2)) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcSech
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(1
6*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2
, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(16*(-d)^(3/
2)*e^(3/2)) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2
*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2
, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*
e^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
```

```
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6438

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^3}+\frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)^2}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{a+b\operatorname{arccosh}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{d}\\
&\quad+\frac{e\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3}-\frac{3d\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d^3\left(a+b\operatorname{arccosh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3}-\frac{3d\left(a+b\operatorname{arcc\right.}}{16e^2\left(\sqrt{-d}\sqrt{\right.}}\right.}{d}
\end{aligned}
$$

$$= -\frac{3\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}dx, x, \frac{1}{x}\right)}{16e} - \frac{3\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}dx, x, \frac{1}{x}\right)}{16e}$$
$$+ \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}dx, x, \frac{1}{x}\right)}{4e} + \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}dx, x, \frac{1}{x}\right)}{4e}$$
$$- \frac{3\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{-de-d^2x^2}dx, x, \frac{1}{x}\right)}{8e} + \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{-de-d^2x^2}dx, x, \frac{1}{x}\right)}{2e}$$
$$- \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}dx, x, \frac{1}{x}\right)}{8\sqrt{e}} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}dx, x, \frac{1}{x}\right)}{8\sqrt{e}}$$

$$= \frac{a+b\text{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\text{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$
$$- \frac{a+b\text{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3\text{Subst}\left(\int \left(-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{8e}$$
$$+ \frac{\text{Subst}\left(\int \left(-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{2e}$$
$$+ \frac{(3b)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)}dx, x, \frac{1}{x}\right)}{16cde}$$
$$- \frac{(3b)\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)}dx, x, \frac{1}{x}\right)}{16cde}$$
$$- \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)}dx, x, \frac{1}{x}\right)}{4cde}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)}dx, x, \frac{1}{x}\right)}{4cde}$$
$$- \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)^2}dx, x, \frac{1}{x}\right)}{16c\sqrt{-d}\sqrt{e}}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)^2}dx, x, \frac{1}{x}\right)}{16c\sqrt{-d}\sqrt{e}}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+ \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+ \frac{3\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16de^{3/2}} + \frac{3\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16de^{3/2}}$$
$$- \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4de^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4de^{3/2}}$$
$$+ \frac{(3b)\text{Subst}\left(\int \frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{8cde}$$
$$- \frac{(3b)\text{Subst}\left(\int \frac{1}{-d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{8cde}$$
$$- \frac{b\text{Subst}\left(\int \frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2cde}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{-d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{2cde}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)}\,dx, x, \frac{1}{x}\right)}{16cd\left(c^2d+e\right)}$$
$$- \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)}\,dx, x, \frac{1}{x}\right)}{16cd\left(c^2d+e\right)}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+ \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$- \frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e} - \frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}$$
$$+ \frac{3\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{16de^{3/2}}$$
$$+ \frac{3\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{16de^{3/2}}$$
$$- \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4de^{3/2}}$$
$$- \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(cx)\right)}{4de^{3/2}}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(-d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{8cd\left(c^2d+e\right)}$$
$$- \frac{b\text{Subst}\left(\int \frac{1}{-d+\frac{\sqrt{-d}\sqrt{e}}{c}-\left(d+\frac{\sqrt{-d}\sqrt{e}}{c}\right)x^2}\,dx, x, \frac{\sqrt{1+\frac{1}{cx}}}{\sqrt{-1+\frac{1}{cx}}}\right)}{8cd\left(c^2d+e\right)}$$

$$= \frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{bc\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+\frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}+\frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}$$

$$+\frac{b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}\left(c^2d+e\right)}$$

$$-\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}e}$$

$$+\frac{b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}\left(c^2d+e\right)}$$

$$+\frac{3\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{16de^{3/2}}$$

$$+\frac{3\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{16de^{3/2}}$$

$$+\frac{3\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{16de^{3/2}}$$

$$+\frac{3\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{16de^{3/2}}$$

$$-\frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{4de^{3/2}}$$

$$-\frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{4de^{3/2}}$$

$$-\frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{4de^{3/2}}$$

$$-\frac{\text{Subst}\left(\int\frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^x}\,dx,x,\operatorname{sech}^{-1}(cx)\right)}{4de^{3/2}}$$

= Too large to display

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.11 (sec) , antiderivative size = 2030, normalized size of antiderivative = 1.59

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(((-1/16*I)*(((-I)*Sqrt[e]*Sqrt[(1
- c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x
)) - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e
]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*
Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^
2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]
*x*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])
/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*((I*Sqrt[e]*Sqrt[(1 - c*x)/(
1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSec
h[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + S
qrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2
*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x +
Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c
*x)/(1 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(
3/2))))/(Sqrt[d]*e) - (-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(Log[
x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*
x)]]/Sqrt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c
*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*
x)]/Sqrt[c^2*d + e]))/Sqrt[d])/(16*d*e) - (-(ArcSech[c*x]/((-I)*Sqrt[d]*Sqr
t[e] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x
*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c
*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/
((-I)*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d])/(16*d*e) - ((I/32)*(
PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/
Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]
*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I
)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqr
t[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1
+ (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))
```

```
/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])]))/(d^(3/2)*e^(3/2)) - ((I/32)*(-PolyLog[2, -E
^(-2*ArcSech[c*x])] + 2*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d
 + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (
I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*
d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + S
qrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt
[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sq
rt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt
[d]*E^ArcSech[c*x])])))/(d^(3/2)*e^(3/2)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 150.99 (sec) , antiderivative size = 1398, normalized size of antiderivative = 1.10

method	result	size
parts	Expression too large to display	1398
derivativedivides	Expression too large to display	1417
default	Expression too large to display	1417

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/
2)))+b/c^3*(1/8*x*c^5*(c^4*d*e*arcsech(c*x)*x^2-d^2*c^4*arcsech(c*x)-(-(c*x
-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e^2*c^3*x^3-(-(c*x-1)/c/x)^(1/2)*((c*x+1
)/c/x)^(1/2)*c^3*d*e*x+e^2*arcsech(c*x)*c^2*x^2-c^2*d*e*arcsech(c*x))/d/e/(
c^2*d+e)/(c^2*e*x^2+c^2*d)^2-1/16/d/(c^2*d+e)*c^4*sum(_R1/(_R1^2*c^2*d+c^2*
d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+d
ilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z
^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/16/(c^2*d+e)/e*c^6*sum(_R1/(_R1^2*c^2*d+c^2
*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+
dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_
Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/16/d/(c^2*d+e)*c^4*sum(1/_R1/(_R1^2*c^2*d+
c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R
1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*
d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+
e)/e/d^3-1/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+
```

```
e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2)*e)*c*arctanh(c*d*(1/c/x+(-1
+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2)
)/(c^2*d+e)^2/e/d^3+1/8*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-
2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(
1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/e/d^3-1/8*((c^
2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d
*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*c*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1
/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3
+1/16/(c^2*d+e)/e*c^6*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_
R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)
^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*
d)))
```

Fricas [F]

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \frac{\left(b\operatorname{arsech}\left(cx\right)+a\right)x^2}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.129 $\int \frac{a+b\,\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1272

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} + \frac{5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{5b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} - \frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} + \frac{5b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} - \frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{-1+\frac{1}{cx}}}\right)}{8d\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} + \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

```
[Out] 3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arcsech(c*x
))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arcsech(c*x))*ln(1-c*(1/c/x+(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5
/2)/e^(1/2)-3/16*(a+b*arcsech(c*x))*ln(1+c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*po
lylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^
2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1/c/x)^(1/2
)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)
-3/16*b*polylog(2,-c*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,c*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/
2)/e^(1/2)-1/8*b*e*arctan((1+1/c/x)^(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-
1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2))/d/(c*d-(-d)^(1/2)*e^(1/2))^(
3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)-1/8*b*e*arctan((1+1/c/x)^(1/2)*(c*d+(-d
)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/d/(
c*d-(-d)^(1/2)*e^(1/2))^(3/2)/(c*d+(-d)^(1/2)*e^(1/2))^(3/2)+1/16*(a+b*arcs
ech(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2-5/16*(a+b*arcsech(
c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arcsech(c*x))*e^(1/2)/(-d)^(3
/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/16*(a+b*arcsech(c*x))/d^2/(d/x+(-d)^(1/2)*
e^(1/2))+1/16*b*c*e^(1/2)*(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)/(-d)^(3/2)/(c^2*
d+e)/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*b*c*e^(1/2)*(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2)/(-d)^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))+5/8*b*arctan((1+1/c/x)^
(1/2)*(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1/2)/(c*d+(-d)^(1/2)*e^(1/
2))^(1/2))/d^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d+(-d)^(1/2)*e^(1/2))^(1/2
)+5/8*b*arctan((1+1/c/x)^(1/2)*(c*d+(-d)^(1/2)*e^(1/2))^(1/2)/(-1+1/c/x)^(1
/2)/(c*d-(-d)^(1/2)*e^(1/2))^(1/2))/d^2/(c*d-(-d)^(1/2)*e^(1/2))^(1/2)/(c*d
+(-d)^(1/2)*e^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 3.40 (sec) , antiderivative size = 1272, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {6428, 5959, 5909, 5963, 98, 95, 211, 5962, 5681, 2221, 2317, 2438}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16(-d)^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}}c}{16(-d)^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} - \frac{5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{\sqrt{e}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2} - \frac{be\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} + \frac{5b\arctan\left(\frac{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} - \frac{be\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d\left(cd-\sqrt{-d}\sqrt{e}\right)^{3/2}\left(cd+\sqrt{-d}\sqrt{e}\right)^{3/2}} + \frac{5b\arctan\left(\frac{\sqrt{cd+\sqrt{-d}\sqrt{e}}\sqrt{1+\frac{1}{cx}}}{\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{\frac{1}{cx}-1}}\right)}{8d^2\sqrt{cd-\sqrt{-d}\sqrt{e}}\sqrt{cd+\sqrt{-d}\sqrt{e}}} + \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3\left(a+b\operatorname{sech}^{-1}(cx)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*(-d)^(3/2)*(c^2*d +
e)*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(
c*x)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a +
 b*ArcSech[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*Ar
cSech[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcSech[c*
x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcSech[c*x]))/
(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (5*b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e
]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(
8*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*Arc
Tan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*S
qrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sq
rt[-d]*Sqrt[e])^(3/2)) + (5*b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 +
 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d^2*Sqrt[
c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c
*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqr
t[-1 + 1/(c*x)])])/(8*d*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt
[e])^(3/2)) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSech[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(
-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*Arc
Sech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] - Sqrt[c^2*d + e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[
e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d +
 e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x
])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e])
```

<u>Rule 95</u>

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

<u>Rule 98</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5959

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
```

```
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 6428

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCosh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^4\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{(e+dx^2)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{e^2\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{d^2(e+dx^2)^3} - \frac{2e\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{d^2(e+dx^2)^2} + \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{d^2(e+dx^2)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{(e+dx^2)^3}\,dx, x, \frac{1}{x}\right)}{d^2}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \left(-\frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{2e(-de-d^2x^2)}\right)dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d^3\left(a+b\text{arccosh}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3} - \frac{3d\left(a+b\text{arcc}\right.}{16e^2\left(\sqrt{-d}\sqrt{e}\right.}\right.\right.}{d^2}$$

$$= \frac{3\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx,x,\frac{1}{x}\right)}{16d}+\frac{3\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx,x,\frac{1}{x}\right)}{16d}$$

$$+\frac{3\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx,x,\frac{1}{x}\right)}{8d}-\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx,x,\frac{1}{x}\right)}{2d}$$

$$-\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx,x,\frac{1}{x}\right)}{2d}-\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx,x,\frac{1}{x}\right)}{d}$$

$$-\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{2d^2\sqrt{e}}-\frac{\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{2d^2\sqrt{e}}$$

$$-\frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx,x,\frac{1}{x}\right)}{8\sqrt{-d}}-\frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx,x,\frac{1}{x}\right)}{8\sqrt{-d}}$$

$$= \frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{5\left(a+b\text{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{\sqrt{e}\left(a+b\text{sech}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$
$$+\frac{5\left(a+b\text{sech}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{(3b)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(\sqrt{-d}\sqrt{e}-dx)}\,dx,x,\frac{1}{x}\right)}{16cd^2}$$
$$+\frac{(3b)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(\sqrt{-d}\sqrt{e}+dx)}\,dx,x,\frac{1}{x}\right)}{16cd^2}$$
$$+\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(\sqrt{-d}\sqrt{e}-dx)}\,dx,x,\frac{1}{x}\right)}{2cd^2}$$
$$-\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}(\sqrt{-d}\sqrt{e}+dx)}\,dx,x,\frac{1}{x}\right)}{2cd^2}$$
$$+\frac{3\text{Subst}\left(\int\left(-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{8d}$$
$$-\frac{\text{Subst}\left(\int\left(-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\text{arccosh}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{d}$$
$$-\frac{\text{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cosh(x)}\,dx,x,\text{sech}^{-1}(cx)\right)}{2d^2\sqrt{e}}$$
$$-\frac{\text{Subst}\left(\int\frac{(a+bx)\sinh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cosh(x)}\,dx,x,\text{sech}^{-1}(cx)\right)}{2d^2\sqrt{e}}$$
$$-\frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx,x,\frac{1}{x}\right)}{16c(-d)^{3/2}}$$
$$+\frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+\frac{x}{c}}\sqrt{1+\frac{x}{c}}\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx,x,\frac{1}{x}\right)}{16c(-d)^{3/2}}$$

= Too large to display

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.07 (sec) , antiderivative size = 2015, normalized size of antiderivative = 1.58

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*(((I/16)*(((-I)*Sqrt[e]*Sqrt[(1 -
c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))
 - ArcSech[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e])
 - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sq
rt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*
Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x
*Sqrt[(1 - c*x)/(1 + c*x)]))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(
d*(c^2*d + e)^(3/2))))/d^(3/2) - ((I/16)*((I*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*
x)]*(1 + c*x))/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]
/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1
 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d
 + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c
^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1
 + c*x)]))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2)))
)/d^(3/2) - (3*(-(ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(Log[x]/Sqrt
[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sq
rt[e] + Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (
Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x)]/Sqr
t[c^2*d + e]))/Sqrt[d]))/(16*d^2) - (3*(-(ArcSech[c*x]/((-I)*Sqrt[d]*Sqrt[e
] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sq
rt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c*x)
/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/((-
I)*Sqrt[d] + Sqrt[e]*x)]/Sqrt[c^2*d + e]))/Sqrt[d]))/(16*d^2) - (((3*I)/32)
*(PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/
(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2]
)/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*
x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2
*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] -
 Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(S
qrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[
1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])
)/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]
))/(c*Sqrt[d]*E^ArcSech[c*x])])))/(d^(5/2)*Sqrt[e]) - (((3*I)/32)*(-PolyLog
```

```
[2, -E^(-2*ArcSech[c*x])] + 2*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt
[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log
[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*A
rcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[
e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 -
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(
c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e]))
/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(
c*Sqrt[d]*E^ArcSech[c*x])])))/(d^(5/2)*Sqrt[e]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 181.43 (sec) , antiderivative size = 1950, normalized size of antiderivative = 1.53

method	result	size
parts	Expression too large to display	1950
derivativedivides	Expression too large to display	1975
default	Expression too large to display	1975

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e*
x/(d*e)^(1/2))+b/c*(1/8*x*c^3*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c^3
*d*e*x+(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*e^2*c^3*x^3+5*d^2*c^4*arcse
ch(c*x)+3*c^4*d*e*arcsech(c*x)*x^2+5*c^2*d*e*arcsech(c*x)+3*e^2*arcsech(c*x
)*c^2*x^2)/d^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)+5/8*(-(c^2*d-2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e
))^(1/2)*e)*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2
*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)^2/c+5/8*((c^2*d+2*(e*(c^2
*d+e))^(1/2)+2*e)*d)^(1/2)*(-c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e
*(c^2*d+e))^(1/2)*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((
c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)^2/c-1/2*(-(c^2*d-2
*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*e*arct
anh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(
1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)/c^3+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2)*(c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2+2*(e*(c^2*d+e))^(1/2
)*e)*e*arctanh(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(
c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)^2/c^3-1/2*((c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*e*arctan(c*d*(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2))/d^5/(c^2*d+e)/c^3+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-
```

```
c^2*d*(e*(c^2*d+e))^(1/2)+2*c^2*d*e+2*e^2-2*(e*(c^2*d+e))^(1/2)*e)*e*arctan
(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2))/d^5/(c^2*d+e)^2/c^3+3/16/d^2/(c^2*d+e)*c^2*e*sum(1/_R1/(_R1
^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^
(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=Ro
otOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-5/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/
2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/
d^4/(c^2*d+e)/c-5/8*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/
((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)/c-3/16/d^2/(c^2*
d+e)*c^2*e*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+
1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c
/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/16/d/(c
^2*d+e)*c^4*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(
-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+
1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-3/16/d
/(c^2*d+e)*c^4*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1
+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

Sympy [F]

$$\int \frac{a+b\,\text{sech}^{-1}(cx)}{(d+ex^2)^3}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{(d+ex^2)^3}\,dx$$

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**3,x)
[Out] Integral((a + b*asech(c*x))/(d + e*x**2)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex^2)^3}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex^2)^3}\, dx = \int \frac{b\,\operatorname{arsech}\,(cx) + a}{(ex^2 + d)^3}\, dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{(d + ex^2)^3}\, dx = \int \frac{a + b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^3}\, dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^3, x)
```

3.130 $\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 447

$$\begin{aligned}
&\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx \\
&= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} \\
&+ \frac{b(29c^2d-25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^4e^2} \\
&- \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42c^2e^2} + \frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} \\
&- \frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3} \\
&- \frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^7e^{5/2}} \\
&- \frac{8bd^{7/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{105e^3}
\end{aligned}$$

```
[Out] 1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*a
rcsech(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arcsech(c*x))/e^3-1/1680*b*(105*c
^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2+75*e^3)*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/
c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^7/e^(5/2)-8/105*b*d^(7
/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(
c*x+1)^(1/2)/e^3+1/840*b*(29*c^2*d-25*e)*(e*x^2+d)^(3/2)*(1/(c*x+1))^(1/2)*
(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2-1/42*b*(e*x^2+d)^(5/2)*(1/(c*x+1))
^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2+1/1680*b*(23*c^4*d^2+12*c^2
*d*e-75*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(
1/2)/c^6/e^2
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6436, 12, 1629, 159, 163, 65, 223, 209, 95, 213}

$$\begin{aligned}
&\int x^5\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx\\
&=\frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^3}\\
&+\frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^3}\\
&-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^7e^{5/2}}\\
&-\frac{8bd^{7/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{105e^3}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2}\\
&+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(29c^2d-25e)\left(d+ex^2\right)^{3/2}}{840c^4e^2}\\
&+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(23c^4d^2+12c^2de-75e^2)\sqrt{d+ex^2}}{1680c^6e^2}
\end{aligned}$$

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sq
rt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(1680*c^6*e^2) + (b*(29*c^2*d - 25*e)*Sqrt
[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(840*c^
4*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2
)^(5/2))/(42*c^2*e^2) + (d^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e^3
) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^3) + ((d + e*x^2)^(7/
2)*(a + b*ArcSech[c*x]))/(7*e^3) - (b*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*
d*e^2 + 75*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 -
 c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^7*e^(5/2)) - (8*b*d^(7/2)*Sqrt[(1
+ c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^
2])])/(105*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d^2(d+ex^2)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{3e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{5e^3}\\
&\quad+\frac{(d+ex^2)^{7/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{7e^3}\\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{3/2}\left(8d^2-12dex^2+15e^2x^4\right)}{105e^3x\sqrt{1-c^2x^2}}\,dx
\end{aligned}
$$

$$= \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(d+ex^2)^{3/2}(8d^2-12dex^2+15e^2x^4)}{x\sqrt{1-c^2x^2}}\,dx}{105e^3}$$

$$= \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{(d+ex)^{3/2}(8d^2-12dex+15e^2x^2)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{210e^3}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{(d+ex)^{3/2}\left(-24c^2d^2e+\frac{3}{2}(29c^2d-25e)e^2x\right)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{630c^2e^4}$$

$$= \frac{b(29c^2d-25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(48c^4d^3e-\frac{3}{4}e^2(23c^4d^2+12c^2de-75e^2)x\right)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{1260c^4e^4}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{-48c^6d^4e-\frac{3}{8}e^2(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)x}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{1260c^6e^4}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3} + \frac{\left(4bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{105e^3} + \frac{\left(b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3360c^6e^2}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^3} + \frac{\left(8bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{-d+x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{105e^3} - \frac{\left(b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{1-c^2x^2}\right)}{1680c^8e^2}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^3} - \frac{8bd^{7/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{105e^3} - \frac{\left(b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{1680c^8e^2}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^3} - \frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^7e^{5/2}} - \frac{8bd^{7/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{105e^3}$$

Mathematica [A] (verified)

Time = 37.75 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int x^5\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{\sqrt{d+ex^2}\left(16ac^6(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6)-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(75e^2+2c^2e(19d+25ex^2)+c^4(-\right.\right.}{1680c^6e^3} - \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{-1+c^2x^2}\left(128c^7d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\operatorname{arctan}\right.}{1680c^7e^3(-1+cx)}$$

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*a*c^6*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)
 - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(19*d + 25*e*x
^2) + c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^6*(8*d^3 - 4*d^2*e*
x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)*ArcSech[c*x]))/(1680*c^6*e^3) - (b*Sqrt[(1
- c*x)/(1 + c*x)]*Sqrt[-1 + c^2*x^2]*(128*c^7*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[
-1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(105*c^6*d^3 - 35*c^4*d^2*e + 63*
c^2*d*e^2 + 75*e^3)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2]
)]))/(1680*c^7*e^3*(-1 + c*x))
```

Maple [F]

$$\int x^5(a+b\ \mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.45 (sec) , antiderivative size = 1995, normalized size of antiderivative = 4.46

$$\int x^5\sqrt{d+ex^2}\big(a+b\mathrm{sech}^{-1}(cx)\big)\;dx=\text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6720*(128*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 - 35*b*c^4*d^2*e
+ 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d
*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e
*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 64*(15*b*c^7*e^3
*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)
*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6
 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 - (40*b*c^6*e^3*
x^5 + 2*(11*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e - 38*b*c^4*d*
e^2 - 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^
7*e^3), 1/3360*(64*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d
)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 - 35*b*c^4*
d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d
 - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^
```

```
4 + (c^2*d*e - e^2)*x^2 - d*e)) + 32*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4
- 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 6
4*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 +
 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e - 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sq
rt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3), -1/6720*(256*b*c^
7*sqrt(-d)*d^3*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sq
rt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 -
d^2)) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(-
e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*
(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + e^2) - 64*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*
d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*
x^2)) + 1)/(c*x)) - 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^
2*e*x^2 + 128*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 + 25*b*c^4*
e^3)*x^3 - (41*b*c^6*d^2*e - 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3), -1/3360*(128*b*c^7*sqrt(-d)
*d^3*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (1
05*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*arctan(1
/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) - 32*(15*b*c^7*e^3*
x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*
log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(240*a*c^7*e^3*x^6
+ 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 - (40*b*c^6*e^3*x
^5 + 2*(11*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e - 38*b*c^4*d*e
^2 - 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7
*e^3)]
```

Sympy [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int x^5(a+b\operatorname{asech}(cx))\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**5*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**5*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x^5\,dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^5*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^5*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.131 $\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 329

$$\begin{aligned}\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx &= -\frac{b(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e} \\ &\quad -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c^2e} \\ &\quad -\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2} \\ &\quad +\frac{b(15c^4d^2-10c^2de-9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^5e^{3/2}} \\ &\quad +\frac{2bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^2}\end{aligned}$$

```
[Out] -1/3*d*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arcs
ech(c*x))/e^2+1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*arctan(e^(1/2)*(-c^2*x^
2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5/e^(3/2)+2
/15*b*d^(5/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1
))^(1/2)*(c*x+1)^(1/2)/e^2-1/20*b*(e*x^2+d)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e-1/120*b*(c^2*d+9*e)*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^4/e
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 6436, 12, 587, 159, 163, 65, 223, 209, 95, 213}

$$\begin{aligned}
&\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx\\
&=-\frac{d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}\\
&+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(15c^4d^2-10c^2de-9e^2)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^5e^{3/2}}\\
&+\frac{2bd^{5/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^2}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e}\\
&-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(c^2d+9e)\sqrt{d+ex^2}}{120c^4e}
\end{aligned}$$

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/120*(b*(c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
]*Sqrt[d + e*x^2])/(c^4*e) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 -
 c^2*x^2]*(d + e*x^2)^(3/2))/(20*c^2*e) - (d*(d + e*x^2)^(3/2)*(a + b*ArcSe
ch[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^2) + (b*(
15*c^4*d^2 - 10*c^2*d*e - 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[
(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^5*e^(3/2)) + (2*b*
d^(5/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]
*Sqrt[1 - c^2*x^2])])/(15*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2}\\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{3/2}\left(-2d+3ex^2\right)}{15e^2x\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2}\\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{3/2}\left(-2d+3ex^2\right)}{x\sqrt{1-c^2x^2}}\,dx}{15e^2}\\
&= -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2}\\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{(d+ex)^{3/2}(-2d+3ex)}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{30e^2}
\end{aligned}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e} - \frac{d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(4c^2d^2-\frac{1}{2}e(c^2d+9e)x\right)}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{60c^2e^2}$$

$$= -\frac{b(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e} - \frac{d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{-4c^4d^3-\frac{1}{4}e(15c^4d^2-10c^2de-9e^2)x}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{60c^4e^2}$$

$$= -\frac{b(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e} - \frac{d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2} - \frac{\left(bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{15e^2} - \frac{\left(b(15c^4d^2-10c^2de-9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{240c^4e}$$

$$= -\frac{b(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e} - \frac{d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2} - \frac{\left(2bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{-d+x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{15e^2} + \frac{\left(b(15c^4d^2-10c^2de-9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1-c^2x^2}\right)}{120c^6e}$$

$$= -\frac{b(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e} - \frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2} + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2} + \frac{2bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^2} + \frac{\left(b(15c^4d^2-10c^2de-9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{120c^6e}$$

$$= -\frac{b(c^2d+9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e} - \frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2} + \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2} + \frac{b(15c^4d^2-10c^2de-9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^5e^{3/2}} + \frac{2bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^2}$$

Mathematica [A] (verified)

Time = 23.45 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.11

$$\int x^3\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = -\frac{\sqrt{d+ex^2}\left(8ac^4(2d^2-dex^2-3e^2x^4)+be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(9e+c^2(7d+6ex^2)\right)+8bc^4(2d^2-dex^2-3e^2x\right.}{120c^4e^2} - \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}(15c^4d^2-10c^2de-9e^2)\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+16c^7\right.}{120c^7e^2(-1+cx)\sqrt{d+ex^2}}$$

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/120*(Sqrt[d + e*x^2]*(8*a*c^4*(2*d^2 - d*e*x^2 - 3*e^2*x^4) + b*e*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(7*d + 6*e*x^2)) + 8*b*c^4*(2*d^2
- d*e*x^2 - 3*e^2*x^4)*ArcSech[c*x]))/(c^4*e^2) - (b*Sqrt[(1 - c*x)/(1 + c*
x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(15*c^4*d^2 -
10*c^2*d*e - 9*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*S
```

```
qrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 16*c^7*d^(5/2)*Sqrt[-d
 - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(120*c^7*e
^2*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^3(a+b\ \mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 1669, normalized size of antiderivative = 5.07

$$\int x^3\sqrt{d+ex^2}\big(a+b\mathrm{sech}^{-1}(cx)\big)\ dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b
*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e
^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2
*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c
*x)) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*
x^3 + (7*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e
*x^2 + d))/(c^5*e^2), 1/240*(8*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^
2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2
 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2
- 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*
sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*
d*e - e^2)*x^2 - d*e)) + 16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)
*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2
4*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7*b*
c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/
(c^5*e^2), 1/480*(32*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*
c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4
 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sq
rt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2
```

```
- 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d
^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4
*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7
*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d
))/(c^5*e^2), 1/240*(16*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 -
 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*
x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)
*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) +
16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5
*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e + 9*b*c^2*e^2)*x)
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2)]
```

Sympy [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^3(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**3*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**3*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.132 $\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 221

$$\begin{aligned}\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx =&-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2}\\&+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}\\&-\frac{b(3c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}}\\&-\frac{bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e}\end{aligned}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e-1/3*b*d^(3/2)*arctanh((e*x^2+d)^(1
/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/6*b*(3*
c^2*d+e)*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(
1/2)*(c*x+1)^(1/2)/c^3/e^(1/2)-1/6*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*
x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules

used = {6434, 531, 457, 104, 163, 65, 223, 209, 95, 213}

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(3c^2d+e)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}} - \frac{bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2}$$

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/6*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2
])/c^2 + ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e) - (b*(3*c^2*d + e)*
Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sq
rt[d + e*x^2])])/(6*c^3*Sqrt[e]) - (b*d^(3/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
 c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(3*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6434

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Dist[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^{3/2}}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{3e}$$

$$=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(d+ex^2\right)^{3/2}}{x\sqrt{1-c^2x^2}}\,dx}{3e}$$

$$=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{6e}$$

$$=-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{-c^2d^2-\frac{1}{2}e(3c^2d+e)x}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{6c^2e}$$

$$=-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{6e}+\frac{\left(b(3c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{12c^2}$$

$$=-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}+\frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{-d+x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{3e}-\frac{\left(b(3c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1-c^2x^2}\right)}{6c^4}$$

$$=-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e}-\frac{bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e}-\frac{\left(b(3c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^4}$$

$$
= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e} - \frac{b(3c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}} - \frac{bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e}
$$

Mathematica [A] (verified)

Time = 22.76 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.39

$$
\int x\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \frac{\sqrt{d+ex^2}\left(-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)+2ac^2(d+ex^2)+2bc^2(d+ex^2)\operatorname{sech}^{-1}(cx)\right)}{6c^2e} + \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}(3c^2d+e)\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+2c^5d^{3/2}\sqrt{-d-ex^2}\right.}{6c^5e(-1+cx)\sqrt{d+ex^2}}
$$

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + 2*a*c^2*(d +
 e*x^2) + 2*b*c^2*(d + e*x^2)*ArcSech[c*x]))/(6*c^2*e) + (b*Sqrt[(1 - c*x)/
(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(3*c^2*
d + e)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x
^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 2*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcT
an[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(6*c^5*e*(-1 + c*x)*Sqrt
[d + e*x^2])
```

Maple [F]

$$
\int x(a+b\operatorname{arcsech}(cx))\sqrt{e\,x^2+d}dx
$$

```
[In] int(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(131) = 262$.

Time = 0.45 (sec) , antiderivative size = 1382, normalized size of antiderivative = 6.25

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(2*b*c^3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (3*b*c^2*d + b*e)*sqrt(-e)*log(8*c^4
*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) + e^2) + 8*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), 1/12*(b*c^3*d^(3/2
)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d
- c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) + 8*d^2)/x^4) - (3*b*c^2*d + b*e)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*
d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x
^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 4*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d
)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 -
b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3
*e), -1/24*(4*b*c^3*sqrt(-d)*d*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sq
rt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d
^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d + b*e)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*
d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2
*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 8*(
b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^
2)) + 1)/(c*x)) - 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), -1/12*(2*b*c^3*sqrt(-d)*d*arctan(
-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2
- 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d + b
*e)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e
)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e))
 - 4*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 1)/(c*x)) - 2*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e)]
```

Sympy [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/3*((e*x^2 + d)^(3/2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/e - 3*integrat
e(1/3*sqrt(e*x^2 + d)*(6*(c^2*e*x^2 - e)*x*log(sqrt(x)) + 3*(c^2*e*x^2*log(
c) - e*log(c))*x + (6*(c^2*e*x^2 - e)*x*log(sqrt(x)) + ((3*e*log(c) + e)*c^
2*x^2 + c^2*d - 3*e*log(c))*x)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/(c
^2*e*x^2 + (c^2*e*x^2 - e)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)) - e), x
))*b + 1/3*(e*x^2 + d)^(3/2)*a/e
```

Giac [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.133 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx = \mathrm{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\text{arcsech}\,(cx))\,\sqrt{e\,x^2+d}}{x}dx$$

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}}{x}\,dx$$

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x,x)
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\,(a+b\,\mathrm{sech}^{-1}(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\,(a+b\,\mathrm{sech}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arsech}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}\,(a+b\,\mathrm{sech}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x, x)
```

3.134 $\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \operatorname{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx=\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\ \text{arcsech}\,(cx))\sqrt{e\,x^2+d}}{x^3}dx$$

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x)
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^3}\,dx=\int \frac{(a+b\,\text{asech}\,(cx))\sqrt{d+ex^2}}{x^3}\,dx$$

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**3,x)
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}\,(b\operatorname{arsech}(cx)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^3, x)
```

3.135 $\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Int}\left(x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\text{integral}=\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))\ dx=\int x^2\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))\ dx$$

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+b\ \text{arcsech}\,(cx))\sqrt{e\,x^2+d}dx$$

```
[In] int(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
[Out] int(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))\ dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 7.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))\ dx=\int x^2(a+b\,\text{asech}\,(cx))\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.136 $\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \operatorname{Int}\left(\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ \text{arcsech}\left(cx\right)\right)\sqrt{e\,x^{2}+d}dx$$

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (a+b\,\text{asech}\,(cx))\sqrt{d+ex^2}\,dx$$

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.137 $\int \frac{\sqrt{d+ex^2}\left(a+b\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\mathrm{sech}^{-1}(cx))}{x^2}\,dx = \mathrm{Int}\left(\frac{\sqrt{d+ex^2}(a+b\mathrm{sech}^{-1}(cx))}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b\mathrm{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\mathrm{sech}^{-1}(cx))}{x^2}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}(a+b\mathrm{sech}^{-1}(cx))}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^2}\,dx=\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^2}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\,\operatorname{arcsech}\left(cx\right)\right)\sqrt{e\,x^{2}+d}}{x^{2}}dx$$

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x)
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^2}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\left(cx\right)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^2}\,dx=\int \frac{\left(a+b\,\text{asech}\left(cx\right)\right)\sqrt{d+ex^2}}{x^2}\,dx$$

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**2,x)
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\operatorname{arsech}(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^2, x)
```

3.138 $\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 312

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^4}\,dx
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} \\
&+ \frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx} - \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3dx^3} \\
&+ \frac{2bc(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{b(c^2d+e)(2c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\mathrm{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9cd\sqrt{d+ex^2}}
\end{aligned}$$

```
[Out] -1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/d/x^3+1/9*b*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/x^3+2/9*b*(c^2*d+2*e)*(1/(c*x+1
))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x+2/9*b*c*(c^2*
d+2*e)*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x
^2+d)^(1/2)/d/(1+e*x^2/d)^(1/2)-1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*EllipticF(c*x
,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/(e
*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {270, 6436, 12, 485, 597, 538, 437, 435, 432, 430}

$$\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^4}\,dx$$
$$= -\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)\left(2c^2d+3e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9cd\sqrt{d+ex^2}} + \frac{2bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+2e)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d\sqrt{\frac{ex^2}{d}+1}} + \frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{d+ex^2}}{9dx} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*x^3) + (2*b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*
x^2]*Sqrt[d + e*x^2])/(9*d*x) - ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3
*d*x^3) + (2*b*c*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d +
e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*Sqrt[1 + (e*x^2)/d]) - (b
*(c^2*d + e)*(2*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e
*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*c*d*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{(d+ex^2)^{3/2}}{3dx^4\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(d+ex^2)^{3/2}}{x^4\sqrt{1-c^2x^2}}\,dx}{3d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} - \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{2d(c^2d+2e)+e(c^2d+3e)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx} - \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-de(c^2d+3e)+2c^2de(c^2d+2e)x^2}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx} - \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3dx^3} + \frac{\left(2bc^2(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{9d} - \frac{\left(b(c^2d+e)(2c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx} - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} + \frac{\left(2bc^2(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{9d\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(b(c^2d+e)\,(2c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{9d\sqrt{d+ex^2}}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx} - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} + \frac{2bc(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{9d\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)\,(2c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9cd\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.18 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.85

$$\int\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^4}\,dx$$

$$= \frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x^3} + \frac{bc\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x^2} + \frac{2b(c^2d+2e)\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{dx} - \frac{3a(d+ex^2)^2}{dx^3} - \frac{3b(d+ex^2)^2\operatorname{sech}^{-1}(cx)}{dx^3} - \frac{2ib\left(c\sqrt{d}-i\sqrt{e}\right)^2\sqrt{\frac{1-}{1-}}}{}}{}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^4,x]
```

[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^3 + (b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^2 + (2*b*(c^2*d + 2*e)*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/(d*x) - (3*a*(d + e*x^2)^2)/(d*x^3) - (3*b*(d + e*x^2)^2*ArcSech[c*x])/(d*x^3) - ((2*I)*b*(c*Sqrt[d] - I*Sqrt[e])^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*((c^2*d + 2*e)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + ((2*I)*c*Sqrt[d] - 3*Sqrt[e])*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(c*d*Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]))/(9*Sqrt[d + e*x^2])

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2+d}}{x^4}dx$$

[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x)

[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x)

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d+ex^2}\,(a+b\operatorname{sech}^{-1}(cx))}{x^4}\,dx =$$
$$-\frac{3\,(bcdex^2+bcd^2)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(3\,acdex^2+3\,acd^2-(bc^2d^2x+2\,(bc^4d^2+2\,bc^2de)x^3\right.}{}$$

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")

[Out] -1/9*(3*(b*c*d*e*x^2 + b*c*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (3*a*c*d*e*x^2 + 3*a*c*d^2 - (b*c^2*d^2*x + 2*(b*c^4*d^2 + 2*b*c^2*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) - (2*(b*c^6*d^2 + 2*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (2*b*c^6*d^2 + (4*b*c^4 + b*c^2)*d*e + 3*b*e^2)*x^3*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c*d^2*x^3)

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{asech}\,(cx))\,\sqrt{d+ex^2}}{x^4}\,dx$$

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^4, x)
```

3.139 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6} dx$

Optimal result

Integrand size = 23, antiderivative size = 446

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{x^6} dx &= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} \\
&+ \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225d^2x} \\
&+ \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} \\
&- \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3} \\
&+ \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{225d^2\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{b(c^2d+e)\left(24c^4d^2+7c^2de-30e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225cd^2\sqrt{d+ex^2}}
\end{aligned}$$

```
[Out] -1/5*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*a
rcsech(c*x))/d^2/x^3+1/25*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/x^5+1/45*b*e*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+
1)^(1/2)*(e*x^2+d)^(1/2)/d/x^3+1/75*b*(4*c^2*d+e)*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x^3-2/15*b*e^2*(1/(c*x+1))^(1/2
)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x+1/45*b*e*(2*c^2*d+
e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x
+1/75*b*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x
^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x-2/15*b*c*e^2*EllipticE(c*x,(-e/c^2/d)^(1/
```

```
2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)+1
/45*b*c*e*(2*c^2*d+e)*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*
x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)+1/75*b*c*(8*c^4*d^2+3*c^2*
d*e-2*e^2)*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)-1/75*b*c*(8*c^2*d-e)*(c^2*d+e)*Ellipt
icF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)
/d/(e*x^2+d)^(1/2)-2/45*b*c*e*(c^2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/
(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(e*x^2+d)^(1/2)+2/15*b*e^2
*(c^2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {277, 270, 6436, 12, 594, 597, 538, 437, 435, 432, 430}

$$\begin{aligned}
&\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx \\
&= \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{15d^2x^3} - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5} \\
&\quad - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)\left(24c^4d^2+7c^2de-30e^2\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225cd^2\sqrt{d+ex^2}} \\
&\quad + \frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{225d^2\sqrt{\frac{ex^2}{d}+1}} \\
&\quad + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} \\
&\quad + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(12c^2d-e)\sqrt{d+ex^2}}{225dx^3} \\
&\quad + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{225d^2x}
\end{aligned}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (b*(12*c^2*d - e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt
[d + e*x^2])/(225*d*x^3) + (b*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d^2*x) + (
b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(
25*d*x^5) - ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(5*d*x^5) + (2*e*(d +
e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(15*d^2*x^3) + (b*c*(24*c^4*d^2 + 19*c^2
```

```
*d*e - 31*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE
[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)
*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[
1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(225*c*d^2*Sqrt[d + e*
x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{15d^2x^6\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{x^6\sqrt{1-c^2x^2}}\,dx}{15d^2}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} - \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}\left(-d(12c^2d-e)-(3c^2d-10e)ex^2\right)}{x^4\sqrt{1-c^2x^2}}\,dx}{75d^2}$$

$$= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} - \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-d(24c^4d^2+19c^2de-31e^2)-2e(6c^4d^2+4c^2de-15e^2)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^2}$$

$$= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} + \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225d^2x} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} - \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{15d^2x^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{2de(6c^4d^2+4c^2de-15e^2)-c^2de(24c^4d^2+19c^2de-31e^2)x^2}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^3}$$

$$= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} + \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225d^2x} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{15d^2x^3} + \frac{\left(bc^2(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{225d^2} - \frac{\left(b(c^2d+e)\left(24c^4d^2+7c^2de-30e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^2}$$

$$= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} + \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225d^2x} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} - \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{15d^2x^3} + \frac{\left(bc^2(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{225d^2\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(b(c^2d+e)\left(24c^4d^2+7c^2de-30e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{225d^2\sqrt{d+ex^2}}$$

$$
\begin{aligned}
&= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} \\
&+ \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225d^2x} \\
&+ \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} \\
&- \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{15d^2x^3} \\
&+ \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{225d^2\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{b(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225cd^2\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.93 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.44

$$
\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx
$$

$$
= \frac{15a(d+ex^2)^2(-3d+2ex^2)}{x^5} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2)\left(-31e^2x^4+dex^2(8+19c^2x^2)+3d^2(3+4c^2x^2+8c^4x^4)\right)}{x^5} + \frac{15b(d+ex^2)^2(-3d+2ex^2)\operatorname{sec}}{x^5}
$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] ((15*a*(d + e*x^2)^2*(-3*d + 2*e*x^2))/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x)*(d + e*x^2)*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4
*c^2*x^2 + 8*c^4*x^4)))/x^5 + (15*b*(d + e*x^2)^2*(-3*d + 2*e*x^2)*ArcSech[
c*x])/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*(24*c^4*d^2 + 19*c^2*d*e -
31*e^2)*(d + e*x^2)) - (I*(c*Sqrt[d] - I*Sqrt[e])^2*(1 + c*x)*Sqrt[(c*(Sqrt
[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] +
 I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*((24*c^4*d^2 + 19*c^2*d
*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] +
 I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e
])^2] + 2*Sqrt[e]*((24*I)*c^3*d^(3/2) - 36*c^2*d*Sqrt[e] - (29*I)*c*Sqrt[d]
*e + 30*e^(3/2))*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[
d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sq
```

```
rt[e])^2]))/Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqr
t[e])*(1 + c*x)))]))/c)/(225*d^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{(a + b\,\mathrm{arcsech}\,(cx))\,\sqrt{e\,x^2 + d}}{x^6}dx$$

```
[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x)
[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{d + ex^2}(a + b\mathrm{sech}^{-1}(cx))}{x^6}\,dx = \frac{15\,(2\,bcde^2x^4 - bcd^2ex^2 - 3\,bcd^3)\sqrt{ex^2 + d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + \left(30\,acde^2x^4 - 15\,acd^2ex^2 - 45\,acd^3 + (9\right.}{}$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
[Out] 1/225*(15*(2*b*c*d*e^2*x^4 - b*c*d^2*e*x^2 - 3*b*c*d^3)*sqrt(e*x^2 + d)*log
((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (30*a*c*d*e^2*x^4 - 15*a
*c*d^2*e*x^2 - 45*a*c*d^3 + (9*b*c^2*d^3*x + (24*b*c^6*d^3 + 19*b*c^4*d^2*e
 - 31*b*c^2*d*e^2)*x^5 + 4*(3*b*c^4*d^3 + 2*b*c^2*d^2*e)*x^3)*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) + ((24*b*c^8*d^3 + 19*b*c^6*d^2*e - 31*b
*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3 + (19*b
*c^6 + 12*b*c^4)*d^2*e - (31*b*c^4 - 8*b*c^2)*d*e^2 - 30*b*e^3)*x^5*ellipti
c_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c*d^3*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{d + ex^2}(a + b\mathrm{sech}^{-1}(cx))}{x^6}\,dx = \int \frac{(a + b\,\mathrm{asech}\,(cx))\,\sqrt{d + ex^2}}{x^6}\,dx$$

```
[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**6,x)
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arsech}\,(cx)+a)}{x^6}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))))/x^6, x)
```

3.140 $\int x^3(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 418

$$\begin{aligned}\int x^3\left(d+ex^2\right)^{3/2}&\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\frac{b(3c^4d^2-38c^2de-25e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e}\\&-\frac{b(13c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e}\\&-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e}\\&-\frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2}\\&+\frac{b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^7e^{3/2}}\\&+\frac{2bd^{7/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{35e^2}\end{aligned}$$

```
[Out] -1/5*d*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arcs
ech(c*x))/e^2+1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-25*e^3)*arctan(
e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)/c^7/e^(3/2)+2/35*b*d^(7/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^
(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2-1/840*b*(13*c^2*d+25*e)*(e*x^2+d
)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e-1/42*b*(e*
x^2+d)^(5/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e+1/560
*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+
1)^(1/2)*(e*x^2+d)^(1/2)/c^6/e
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6436, 12, 587, 159, 163, 65, 223, 209, 95, 213}

$$\int x^3(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))\,dx = -\frac{d(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{5e^2}+\frac{(d+ex^2)^{7/2}(a+b\text{sech}^{-1}(cx))}{7e^2}$$
$$+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^7e^{3/2}}$$
$$+\frac{2bd^{7/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{35e^2}$$
$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e}$$
$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(13c^2d+25e)(d+ex^2)^{3/2}}{840c^4e}$$
$$+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(3c^4d^2-38c^2de-25e^2)\sqrt{d+ex^2}}{560c^6e}$$

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqr
t[1 - c^2*x^2]*Sqrt[d + e*x^2])/(560*c^6*e) - (b*(13*c^2*d + 25*e)*Sqrt[(1
+ c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(840*c^4*e)
 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2
))/(42*c^2*e) - (d*(d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^2) + ((d +
e*x^2)^(7/2)*(a + b*ArcSech[c*x]))/(7*e^2) + (b*(35*c^6*d^3 - 35*c^4*d^2*e
- 63*c^2*d*e^2 - 25*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]
*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^7*e^(3/2)) + (2*b*d^(7/2)*
Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1
- c^2*x^2])])/(35*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}\\&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{5/2}\left(-2d+5ex^2\right)}{35e^2x\sqrt{1-c^2x^2}}\,dx\\&=-\frac{d(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\text{sech}^{-1}(cx)\right)}{7e^2}\\&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{5/2}(-2d+5ex^2)}{x\sqrt{1-c^2x^2}}\,dx}{35e^2}\end{aligned}$$

$$= -\frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{(d+ex)^{5/2}(-2d+5ex)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{70e^2}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e} - \frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{(d+ex)^{3/2}\left(6c^2d^2-\frac{1}{2}e(13c^2d+25e)x\right)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{210c^2e^2}$$

$$= -\frac{b(13c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e} - \frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(-12c^4d^3-\frac{3}{4}e(3c^4d^2-38c^2de-25e^2)x\right)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{420c^4e^2}$$

$$= \frac{b(3c^4d^2-38c^2de-25e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e} - \frac{b(13c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e} - \frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{12c^6d^4+\frac{3}{8}e(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)x}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{420c^6e^2}$$

$$
\begin{aligned}
&= \frac{b(3c^4d^2-38c^2de-25e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e} \\
&- \frac{b(13c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e} \\
&- \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e} \\
&- \frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2} \\
&- \frac{\left(bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{35e^2} \\
&- \frac{\left(b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{1120c^6e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(3c^4d^2-38c^2de-25e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e} \\
&- \frac{b(13c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e} \\
&- \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e} \\
&- \frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2} \\
&- \frac{\left(2bd^4\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{-d+x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{35e^2} \\
&+ \frac{\left(b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1-c^2x^2}\right)}{560c^8e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(3c^4d^2-38c^2de-25e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e} \\
&- \frac{b(13c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e} \\
&- \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e} - \frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^2} \\
&+ \frac{(d+ex^2)^{7/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7e^2} + \frac{2bd^{7/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{35e^2} \\
&+ \frac{\left(b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{560c^8e}
\end{aligned}
$$

$$= \frac{b(3c^4d^2-38c^2de-25e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{560c^6e} - \frac{b(13c^2d+25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e} - \frac{d(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\operatorname{sech}^{-1}(cx))}{7e^2} + \frac{b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^7e^{3/2}} + \frac{2bd^{7/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{35e^2}$$

Mathematica [A] (verified)

Time = 37.50 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.75

$$\int x^3(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))\,dx = -\frac{\sqrt{d+ex^2}\left(48ac^6(2d-5ex^2)(d+ex^2)^2+be\sqrt{\frac{1-cx}{1+cx}}(1+cx)(75e^2+2c^2e(82d+25ex^2)+c^4(57d^2+106dex^2\right.}{1680c^6e^2} - \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{-1+c^2x^2}\left(-32c^7d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}(-35c^6d^3+35c^4d^2e+63c^2de^2+25e^3)\operatorname{arctanh}\left(\right.\right.}{560c^7e^2(-1+cx)}$$

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/1680*(Sqrt[d + e*x^2]*(48*a*c^6*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e*Sqrt
[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c^4*(
57*d^2 + 106*d*e*x^2 + 40*e^2*x^4)) + 48*b*c^6*(2*d - 5*e*x^2)*(d + e*x^2)^
2*ArcSech[c*x]))/(c^6*e^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[-1 + c^2*x^2
]*(-32*c^7*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] + S
qrt[e]*(-35*c^6*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 + 25*e^3)*ArcTanh[(Sqrt[e
]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(560*c^7*e^2*(-1 + c*x))
```

Maple [F]

$$\int x^{3}\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arcsech}\left(cx\right)\right)dx$$

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.47 (sec) , antiderivative size = 1989, normalized size of antiderivative = 4.76

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] [1/6720*(96*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(
-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e
- 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d
*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e
*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 192*(5*b*c^7*e^3
*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*l
og((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 +
 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^
5 + 2*(53*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e
^2 + 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7
*e^2), 1/3360*(48*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 - 35*b*c^4*
d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d
 - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^
4 + (c^2*d*e - e^2)*x^2 - d*e)) + 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 +
 b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*
a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 + 25
*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2), 1/6720*(192*b*c^7*s
qrt(-d)*d^3*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(
-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2
)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(-e)
*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2
*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1
```

```
)/(c^2*x^2)) + e^2) + 192*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*
e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e
*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 + 25*b*c^4*e^3)
*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2), 1/3360*(96*b*c^7*sqrt(-d)*d^3*
arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(35*b
*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*arctan(1/2*(
2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 96*(5*b*c^7*e^3*x^6 +
 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*
x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a
*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*
(53*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e^2 + 7
5*b*c^2*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^2)]
```

Sympy [F]

$$\int x^3(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int x^3(a+b\operatorname{asech}(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^3 (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx = \int (ex^2+d)^{\frac{3}{2}} (b \operatorname{arsech}(cx)+a)x^3 \, dx$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx = \int x^3 (e x^2+d)^{3/2} \left(a+b \operatorname{acosh}\left(\frac{1}{c x}\right)\right) \, dx$$

```
[In] int(x^3*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.141 $\int x(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 297

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx = -\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e} - \frac{b(15c^4d^2+10c^2de+3e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}} - \frac{bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{5e}$$

```
[Out] 1/5*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/e-1/5*b*d^(5/2)*arctanh((e*x^2+d)^(1
/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e-1/40*b*(1
5*c^4*d^2+10*c^2*d*e+3*e^2)*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(
1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5/e^(1/2)-1/20*b*(e*x^2+d)^(3/2)*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2-1/40*b*(7*c^2*d+3*e)*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {6434, 531, 457, 104, 159, 163, 65, 223, 209, 95, 213}

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(15c^4d^2+10c^2de+3e^2\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}}-\frac{bd^{5/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{5e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c^2}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{40c^4}$$

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -1/40*(b*(7*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^
2]*Sqrt[d + e*x^2])/c^4 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^
2*x^2]*(d + e*x^2)^(3/2))/(20*c^2) + ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]
))/(5*e) - (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(40*c^5*Sq
rt[e]) - (b*d^(5/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x
^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(5*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6434

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Dist[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{5/2}}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{5e}\\
&= \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{5/2}}{x\sqrt{1-c^2x^2}}\,dx}{5e}\\
&= \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{10e}\\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2}+\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e}\\
&\quad-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{\sqrt{d+ex}\left(-2c^2d^2-\frac{1}{2}e(7c^2d+3e)x\right)}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{20c^2e}\\
&= -\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4}\\
&\quad-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2}+\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e}\\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{2c^4d^3+\frac{1}{4}e(15c^4d^2+10c^2de+3e^2)x}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{20c^4e}
\end{aligned}$$

$$=-\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2}+\frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}+\frac{\left(bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{10e}+\frac{\left(b(15c^4d^2+10c^2de+3e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{80c^4}$$

$$=-\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2}+\frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}+\frac{\left(bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{-d+x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{5e}-\frac{\left(b(15c^4d^2+10c^2de+3e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1-c^2x^2}\right)}{40c^6}$$

$$=-\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2}+\frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}-\frac{bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{5e}-\frac{\left(b(15c^4d^2+10c^2de+3e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{40c^6}$$

$$=-\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2}+\frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e}-\frac{b(15c^4d^2+10c^2de+3e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}}-\frac{bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{5e}$$

Mathematica [A] (verified)

Time = 23.44 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.15

$$\int x(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))\,dx = \frac{\sqrt{d+ex^2}\left(8ac^4(d+ex^2)^2-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(3e+c^2(9d+2ex^2)\right)+8bc^4(d+ex^2)^2\operatorname{se}\right.}{40c^4e} + \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}(15c^4d^2+10c^2de+3e^2)\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+8c^7d^{5/}\right.}{40c^7e(-1+cx)\sqrt{d+ex^2}}$$

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^4*(d + e*x^2)^2 - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1
+ c*x)*(3*e + c^2*(9*d + 2*e*x^2)) + 8*b*c^4*(d + e*x^2)^2*ArcSech[c*x]))/(
40*c^4*e) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt
[-(c^2*d) - e]*Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[(c^2*(d + e*x
^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c
^2*d) - e])] + 8*c^7*d^(5/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*
x^2])/Sqrt[-d - e*x^2]]))/(40*c^7*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x\left(e\,x^2+d\right)^{\frac{3}{2}}(a+b\,\operatorname{arcsech}(cx))\,dx$$

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(181) = 362.

Time = 0.71 (sec) , antiderivative size = 1667, normalized size of antiderivative = 5.61

$$\int x(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] [1/160*(8*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
 d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(
c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*
e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^
```

```
2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(
-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*
c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)
) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 - (2*b*c^4*e^2*x^3
+ 3*(3*b*c^4*d*e + b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2
 + d))/(c^5*e), 1/80*(4*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4
- 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*s
qrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*
c^2*d*e + 3*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*
x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e
^2)*x^2 - d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x
^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(8*a*c^5*e^
2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 - (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e
+ b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e), -
1/160*(16*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt
(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2
 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(-e)*log(
8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*
e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) + e^2) - 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^
2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(8*a*c^5*e^2
*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 - (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e +
 b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e), -1
/80*(8*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*
x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 -
d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*arctan(1
/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) - 16*(b*c^5*e^2*x^4
 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x)) - 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*
d^2 - (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e + b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e)]
```

Sympy [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{asech}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
[Out] Integral(x*(a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(e*x^2 + d)^(5/2)*a/e + 1/15*b*(((3*e^2*x^4 + d*e*x^2 - 2*d^2)*x^3 + 5*
(d*e*x^4 + d^2*x^2)*x)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1
)/(e*x^3) - 15*integrate(1/15*(15*(c^2*e^2*x^4*log(c) - e^2*x^2*log(c))*x^3
 + 15*(c^2*d*e*x^4*log(c) - d*e*x^2*log(c))*x + ((3*(5*e^2*log(c) + e^2)*c^
2*x^4 - 2*c^2*d^2 + (c^2*d*e - 15*e^2*log(c))*x^2)*x^3 + 5*((3*d*e*log(c) +
 d*e)*c^2*x^4 + (c^2*d^2 - 3*d*e*log(c))*x^2)*x + 30*((c^2*e^2*x^4 - e^2*x^
2)*x^3 + (c^2*d*e*x^4 - d*e*x^2)*x)*log(sqrt(x)))*e^(1/2*log(c*x + 1) + 1/2
*log(-c*x + 1)) + 30*((c^2*e^2*x^4 - e^2*x^2)*x^3 + (c^2*d*e*x^4 - d*e*x^2)
*x)*log(sqrt(x)))*sqrt(e*x^2 + d)/(c^2*e*x^4 - e*x^2 + (c^2*e*x^4 - e*x^2)*
e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))), x))
```

Giac [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.142 $\int \frac{(d+ex^2)^{3/2}(a+b\,\mathrm{sech}^{-1}(cx))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx = \mathrm{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arcsech}(cx))}{x}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x,
x)
```

Sympy [N/A]

Not integrable

Time = 23.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{(a+b\operatorname{asech}(cx))\left(d+ex^2\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x}\,dx=\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{x}\,dx=\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x,x)

[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x, x)

3.143 $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \operatorname{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{x^3}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\ \text{arcsech}\,(cx))}{x^3}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^3
, x)
```

Sympy [N/A]

Not integrable

Time = 19.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**3,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 4.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^3, x)
```

3.144 $\int x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Int}\left(x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\text{integral}=\int x^2(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\ \operatorname{arcsech}\left(c x\right)\right)dx$$

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\int \left(ex^{2}+d\right)^{\frac{3}{2}}\left(b\operatorname{arsech}\left(cx\right)+a\right)x^{2}\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arcsech(c*x))*sqrt(e*x^2
+ d), x)
```

Sympy [N/A]

Not integrable

Time = 69.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\int x^{2}\left(a+b\operatorname{asech}\left(cx\right)\right)\left(d+ex^{2}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
[Out] Integral(x**2*(a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 (d + ex^2)^{3/2} (a + b\text{sech}^{-1}(cx)) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (d + ex^2)^{3/2} (a + b\text{sech}^{-1}(cx)) \, dx = \int (ex^2 + d)^{\frac{3}{2}} (b \,\text{arsech}\,(cx) + a) x^2 \, dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2 (d + ex^2)^{3/2} (a + b\text{sech}^{-1}(cx)) \, dx = \int x^2 \left(e\,x^2 + d\right)^{3/2} \left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.145 $\int (d+ex^2)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) dx = \text{Int}\left(\left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right), x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) dx = \int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) dx$$

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) dx$$

Mathematica [N/A]

Not integrable

Time = 8.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b \operatorname{arcsech}\left(cx\right)\right) dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} \left(b \operatorname{arsech}\left(cx\right)+a\right) dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 20.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(a+b \operatorname{asech}\left(cx\right)\right) \left(d+ex^2\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arsech}\left(cx\right)+a\right)dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.146 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx=\mathrm{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{sech}^{-1}(cx))}{x^2}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arcsech}(cx))}{x^2}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{sech}^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arsech}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^2
, x)
```

Sympy [N/A]

Not integrable

Time = 18.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**2,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^2} \, dx = \int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^2} \, dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^2, x)
```

3.147 $\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4} dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4}, x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4} dx = \int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4} dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4} dx$$

Mathematica [N/A]

Not integrable

Time = 18.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\ \text{arcsech}\,(cx))}{x^4}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^4
, x)
```

Sympy [N/A]

Not integrable

Time = 20.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^4} dx = \int \frac{(a+b\,\text{asech}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^4} dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**4,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^4} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^4} dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)}{x^4} dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 4.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^4, x)
```

3.148 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 409

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{x^6}\,dx = \frac{4b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3}$$
$$+\frac{b(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75dx}$$
$$+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25x^5}-\frac{(d+ex^2)^{5/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{5dx^5}$$
$$+\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{b(c^2d+e)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\mathrm{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75cd\sqrt{d+ex^2}}$$

```
[Out] -1/5*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/d/x^5+1/25*b*(e*x^2+d)^(3/2)*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/x^5+4/75*b*(c^2*d+2*e)*(1/(c*x
+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/x^3+1/75*b*(8*c
^4*d^2+23*c^2*d*e+23*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2
)*(e*x^2+d)^(1/2)/d/x+1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*EllipticE(c*x,
(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d/(1+e*x^
2/d)^(1/2)-1/75*b*(c^2*d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*EllipticF(c*x,(-e
/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/(e*x^2
+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {270, 6436, 12, 485, 594, 597, 538, 437, 435, 432, 430}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx = -\frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5}$$
$$-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75cd\sqrt{d+ex^2}}$$
$$+\frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d\sqrt{\frac{ex^2}{d}+1}}$$
$$+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25x^5}+\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{d+ex^2}}{75x^3}$$
$$+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}}{75dx}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^6,x]
```

```
[Out] (4*b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqr
t[d + e*x^2])/(75*x^3) + (b*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*d*x) + (b*Sqrt
[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*x^5
) - ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*d*x^5) + (b*c*(8*c^4*d^2 +
23*c^2*d*e + 23*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*Ell
ipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d +
 e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sq
rt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(75*c*d*Sqrt[d + e*
x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{\left(d+ex^2\right)^{5/2}}{5dx^6\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\left(d+ex^2\right)^{5/2}}{x^6\sqrt{1-c^2x^2}}\,dx}{5d}\\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5}\\
&\quad -\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}\left(4d\left(c^2d+2e\right)+e\left(c^2d+5e\right)x^2\right)}{x^4\sqrt{1-c^2x^2}}\,dx}{25d}
\end{aligned}
$$

$$= \frac{4b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25x^5} - \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{d(8c^4d^2+23c^2de+23e^2)+e(4c^4d^2+11c^2de+15e^2)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{75d}$$

$$= \frac{4b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3} + \frac{b(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75dx} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25x^5} - \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-de(4c^4d^2+11c^2de+15e^2)+c^2de(8c^4d^2+23c^2de+23e^2)x^2}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{75d^2}$$

$$= \frac{4b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3} + \frac{b(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75dx} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25x^5} - \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5dx^5} - \frac{\left(b(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{75d} + \frac{\left(bc^2(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{75d}$$

$$= \frac{4b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3} + \frac{b(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75dx} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25x^5} - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5} + \frac{\left(bc^2(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{75d\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(b(c^2d+e)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{75d\sqrt{d+ex^2}}$$

$$= \frac{4b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75x^3} + \frac{b(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{75dx} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25x^5} - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5dx^5} + \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)\left(8c^4d^2+19c^2de+15e^2\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75cd\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.98 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.52

$$\int\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^6}\,dx = \frac{-\frac{15a(d+ex^2)^3}{x^5} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2)\left(23e^2x^4+dex^2(11+23c^2x^2)+d^2(3+4c^2x^2+8c^4\ldots}{x^5}}{\ldots}$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^6,x]
```

[Out] ((-15*a*(d + e*x^2)^3)/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)*(23*e^2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)))/x^5 - (15*b*(d + e*x^2)^3*ArcSech[c*x])/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*(d + e*x^2)) - (I*(c*Sqrt[d] - I*Sqrt[e])^2*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*((8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*Sqrt[e]*((8*I)*c^3*d^(3/2) - 12*c^2*d*Sqrt[e] + (7*I)*c*Sqrt[d]*e - 15*e^(3/2))*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]))/c)/(75*d*Sqrt[d + e*x^2])

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsech}\,(cx))}{x^6}dx$$

[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x)

[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x)

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{sech}^{-1}(cx))}{x^6}\,dx = -\frac{15\,(bcde^2x^4+2\,bcd^2ex^2+bcd^3)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(15\,acde^2x^4+30\,acd^2ex^2+15\,acd^3-(3\,b\cdots}{\cdots}$$

[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")

[Out] -1/75*(15*(b*c*d*e^2*x^4 + 2*b*c*d^2*e*x^2 + b*c*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (15*a*c*d*e^2*x^4 + 30*a*c*d^2*e*x^2 + 15*a*c*d^3 - (3*b*c^2*d^3*x + (8*b*c^6*d^3 + 23*b*c^4*d^2*e + 23*b*c^2*d*e^2)*x^5 + (4*b*c^4*d^3 + 11*b*c^2*d^2*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) - ((8*b*c^8*d^3 + 23*b*c^6*d^2*e + 23*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (8*b*c^8*d^3 + (23*b*c^6 + 4*b*c^4)*d^2*e + (23*b*c^4 + 11*b*c^2)*d*e^2 + 15*b*e^3)*x^5*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c*d^2*x^5)

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(a+b\operatorname{asech}(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**6,x)
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**(3/2)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arsech}(cx)+a)}{x^6}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^6,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^6, x)
```

3.149 $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 556

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx = \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3}$$
$$+\frac{b(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675d^2x}$$
$$+\frac{b(30c^2d+11e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^5}$$
$$+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7}$$
$$-\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5}$$
$$+\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3675d^2\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{2b(c^2d+e)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675cd^2\sqrt{d+ex^2}}$$

```
[Out] -1/7*(e*x^2+d)^(5/2)*(a+b*arcsech(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*a
rcsech(c*x))/d^2/x^5+1/1225*b*(30*c^2*d+11*e)*(e*x^2+d)^(3/2)*(1/(c*x+1))^(
1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/x^5+1/49*b*(e*x^2+d)^(5/2)*(1/(c*x+
1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/x^7+1/3675*b*(120*c^4*d^2+159*
c^2*d*e-37*e^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d
)^(1/2)/d/x^3+1/3675*b*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*(1
```

```
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x+1/36
75*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*EllipticE(c*x,(-e/
c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/
d)^(1/2)-2/3675*b*(c^2*d+e)*(120*c^6*d^3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3
)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/
d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {277, 270, 6436, 12, 594, 597, 538, 437, 435, 432, 430}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{x^8}\,dx = \frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5} - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7} - \frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675cd^2\sqrt{d+ex^2}} + \frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3675d^2\sqrt{\frac{ex^2}{d}+1}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(30c^2d+11e)(d+ex^2)^{3/2}}{1225dx^5} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{d+ex^2}}{3675dx^3} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)\sqrt{d+ex^2}}{3675d^2x}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^8,x]

[Out] (b*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^3) + (b*(240*c^6*d^3 + 528*c^4
*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1
 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d^2*x) + (b*(30*c^2*d + 11*e)*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^5)
+ (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2)
)/(49*d*x^7) - ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(7*d*x^7) + (2*e*(d
 + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(35*d^2*x^5) + (b*c*(240*c^6*d^3 + 52
```

```
8*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*S
qrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[1 + (e*
x^2)/d]) - (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d*e^2 - 1
05*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[Ar
cSin[c*x], -(e/(c^2*d))])/(3675*c*d^2*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{5/2}\left(-5d+2ex^2\right)}{35d^2x^8\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{5/2}(-5d+2ex^2)}{x^8\sqrt{1-c^2x^2}}\,dx}{35d^2}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7}-\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{3/2}(-d(30c^2d+11e)-(5c^2d-14e)ex^2)}{x^6\sqrt{1-c^2x^2}}\,dx}{245d^2}$$

$$= \frac{b(30c^2d+11e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^5}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7}-\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}\left(-d(120c^4d^2+159c^2de-37e^2)-2e(15c^4d^2+18c^2de-35e^2)x^2\right)}{x^4\sqrt{1-c^2x^2}}\,dx}{1225d^2}$$

$$= \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3}+\frac{b(30c^2d+11e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^5}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7}-\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-d(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)-e(120c^6d^3+249c^4d^2e+71c^2de^2-210e^3)x^2}{x^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^2}$$

$$=\frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3}$$
$$+\frac{b(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675d^2x}$$
$$+\frac{b(30c^2d+11e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^5}$$
$$+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7}$$
$$-\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5}$$
$$-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{de(120c^6d^3+249c^4d^2e+71c^2de^2-210e^3)-c^2de(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x^2}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^3}$$

$$=\frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3}$$
$$+\frac{b(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675d^2x}$$
$$+\frac{b(30c^2d+11e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^5}$$
$$+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7}$$
$$-\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{35d^2x^5}$$
$$+\frac{\left(bc^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3675d^2}$$
$$-\frac{\left(2b(c^2d+e)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^2}$$

$$= \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675d^2x} + \frac{b(30c^2d+11e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^5} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7} - \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{35d^2x^5} + \frac{\left(bc^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{3675d^2\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(2b(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}}{3675d^2\sqrt{d+ex^2}}$$

$$= \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675d^2x} + \frac{b(30c^2d+11e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^5} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7} - \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{35d^2x^5} + \frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{3675d^2\sqrt{1+\frac{ex^2}{d}}} - \frac{2b(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}(\arcsin(c}{3675cd^2\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.47 (sec) , antiderivative size = 1187, normalized size of antiderivative = 2.13

$$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x^8}\,dx = \left(-\frac{ad}{7x^7}-\frac{8ae}{35x^5}-\frac{ae^2}{35dx^3}+\frac{2ae^3}{35d^2x}\right)\sqrt{d+ex^2} + \left(\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)}{3675d^2}+\frac{bd}{49x^7}+\frac{bcd}{49x^6}+\frac{b(30c^2d+61e)}{1225x^5} + \frac{bc(30c^2d+61e)}{1225x^4}+\frac{b(120c^4d^2+249c^2de+71e^2)}{3675dx^3}+\frac{bc(120c^4d^2+249c^2de+71e^2)}{3675dx^2} + \frac{b(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)}{3675d^2x}\right)\sqrt{\frac{1-cx}{1+cx}}\sqrt{d+ex^2} - \frac{b(5d-2ex^2)(d+ex^2)^{5/2}\operatorname{sech}^{-1}(cx)}{35d^2x^7}$$

$$+ \frac{-bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{\frac{1-cx}{1+cx}}\sqrt{\frac{e\left(-1+\frac{1-cx}{1+cx}\right)^2+c^2d\left(1+\frac{1-cx}{1+cx}\right)^2}{c^2\left(1+\frac{1-cx}{1+cx}\right)^2}} - \frac{ib\left(c\sqrt{d}-i\sqrt{e}\right)^2\sqrt{1+\frac{(c^2d+e)(}{\left(c\sqrt{d}-i\sqrt{e}\right)}}}{}}{}$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (-1/7*(a*d)/x^7 - (8*a*e)/(35*x^5) - (a*e^2)/(35*d*x^3) + (2*a*e^3)/(35*d^2
*x))*Sqrt[d + e*x^2] + ((b*c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 -
 247*e^3))/(3675*d^2) + (b*d)/(49*x^7) + (b*c*d)/(49*x^6) + (b*(30*c^2*d +
61*e))/(1225*x^5) + (b*c*(30*c^2*d + 61*e))/(1225*x^4) + (b*(120*c^4*d^2 +
249*c^2*d*e + 71*e^2))/(3675*d*x^3) + (b*c*(120*c^4*d^2 + 249*c^2*d*e + 71*
e^2))/(3675*d*x^2) + (b*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*
e^3))/(3675*d^2*x))*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[d + e*x^2] - (b*(5*d - 2
*e*x^2)*(d + e*x^2)^(5/2)*ArcSech[c*x])/(35*d^2*x^7) + (-(b*c*(240*c^6*d^3
+ 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[(
e*(-1 + (1 - c*x)/(1 + c*x))^2 + c^2*d*(1 + (1 - c*x)/(1 + c*x))^2)/(c^2*(1
 + (1 - c*x)/(1 + c*x))^2)]) - (I*b*(c*Sqrt[d] - I*Sqrt[e])^2*Sqrt[1 + ((c^
2*d + e)*(1 - c*x))/((c*Sqrt[d] - I*Sqrt[e])^2*(1 + c*x))]*Sqrt[1 + ((c^2*d
 + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]*((Sqrt[(c^2*d + e)/
(c*Sqrt[d] + I*Sqrt[e])^2]*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 2
47*e^3)*Sqrt[(1 - c*x)/(1 + c*x)]*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1
- c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/
(c*Sqrt[d] - I*Sqrt[e])^2])/Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sq
rt[e])^2*(1 + c*x))] + (2*Sqrt[e]*Sqrt[(c^2*d + e)/(c*Sqrt[d] + I*Sqrt[e])^
2]*((240*I)*c^5*d^(5/2) - 360*c^4*d^2*Sqrt[e] + (48*I)*c^3*d^(3/2)*e - 207*
```

```
c^2*d*e^(3/2) - (173*I)*c*Sqrt[d]*e^2 + 210*e^(5/2))*Sqrt[(1 - c*x)/(1 + c*
x)]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e
])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2])/Sq
rt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]))/(c*Sqrt
[(c*Sqrt[d] - I*Sqrt[e])/(c*Sqrt[d] + I*Sqrt[e])]*(1 + (1 - c*x)/(1 + c*x))
*Sqrt[(e*(-1 + (1 - c*x)/(1 + c*x))^2 + c^2*d*(1 + (1 - c*x)/(1 + c*x))^2)/
(c^2*(1 + (1 - c*x)/(1 + c*x))^2)]))/(3675*d^2)
```

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsech}\,(cx))}{x^8}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{sech}^{-1}(cx))}{x^8}\,dx = \frac{105\,(2\,bcde^3x^6 - bcd^2e^2x^4 - 8\,bcd^3ex^2 - 5\,bcd^4)\sqrt{ex^2+d}\,\log\left(\frac{cx\sqrt{-\frac{c^2}{c}}}{cx}\right.}{}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] 1/3675*(105*(2*b*c*d*e^3*x^6 - b*c*d^2*e^2*x^4 - 8*b*c*d^3*e*x^2 - 5*b*c*d^
4)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (2
10*a*c*d*e^3*x^6 - 105*a*c*d^2*e^2*x^4 - 840*a*c*d^3*e*x^2 - 525*a*c*d^4 +
(75*b*c^2*d^4*x + (240*b*c^8*d^4 + 528*b*c^6*d^3*e + 193*b*c^4*d^2*e^2 - 24
7*b*c^2*d*e^3)*x^7 + (120*b*c^6*d^4 + 249*b*c^4*d^3*e + 71*b*c^2*d^2*e^2)*x
^5 + 3*(30*b*c^4*d^4 + 61*b*c^2*d^3*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))
*sqrt(e*x^2 + d) + ((240*b*c^10*d^4 + 528*b*c^8*d^3*e + 193*b*c^6*d^2*e^2 -
 247*b*c^4*d*e^3)*x^7*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (240*b*c^10*d^4
 + 24*(22*b*c^8 + 5*b*c^6)*d^3*e + (193*b*c^6 + 249*b*c^4)*d^2*e^2 - (247*b
*c^4 - 71*b*c^2)*d*e^3 - 210*b*e^4)*x^7*elliptic_f(arcsin(c*x), -e/(c^2*d))
)*sqrt(d))/(c*d^3*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^8} dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**8,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^8} dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^8} dx = \int \frac{(ex^2+d)^{\frac{3}{2}} (b\,\text{arsech}\,(cx)+a)}{x^8} dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx))}{x^8} dx = \int \frac{(e\,x^2+d)^{3/2} \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{x^8} dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))))/x^8, x)
```

3.150 $\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 356

$$
\begin{aligned}
\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx &= \frac{b(19c^2d-9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2} \\
&- \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e^2} \\
&+ \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3} \\
&- \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} \\
&+ \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^3} \\
&- \frac{b(45c^4d^2-10c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^5e^{5/2}} \\
&- \frac{8bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^3}
\end{aligned}
$$

```
[Out] -2/3*d*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arcs
ech(c*x))/e^3-1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*arctan(e^(1/2)*(-c^2*x^
2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^5/e^(5/2)-8
/15*b*d^(5/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1
))^(1/2)*(c*x+1)^(1/2)/e^3-1/20*b*(e*x^2+d)^(3/2)*(1/(c*x+1))^(1/2)*(c*x+1)
^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2+d^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^
3+1/120*b*(19*c^2*d-9*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)
*(e*x^2+d)^(1/2)/c^4/e^2
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 6436, 12, 1629, 159, 163, 65, 223, 209, 95, 213}

$$\int \frac{x^5(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{d^2\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e^3} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(45c^4d^2-10c^2de+9e^2)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^5e^{5/2}} - \frac{8bd^{5/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^3} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e^2} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(19c^2d-9e)\sqrt{d+ex^2}}{120c^4e^2}$$

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (b*(19*c^2*d - 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sq
rt[d + e*x^2])/(120*c^4*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1
 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c^2*e^2) + (d^2*Sqrt[d + e*x^2]*(a + b*A
rcSech[c*x]))/e^3 - (2*d*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e^3) +
((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^3) - (b*(45*c^4*d^2 - 10*c^2*
d*e + 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^
2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^5*e^(5/2)) - (8*b*d^(5/2)*Sqrt[(1 + c*
x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])]
)/(15*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} \\
&\quad+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{5e^3} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}\left(8d^2-4dex^2+3e^2x^4\right)}{15e^3x\sqrt{1-c^2x^2}}\,dx
\end{aligned}
$$

$$= \frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{x\sqrt{1-c^2x^2}}\,dx}{15e^3}$$

$$= \frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sqrt{d+ex}(8d^2-4dex+3e^2x^2)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{30e^3}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e^2} + \frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(-16c^2d^2e+\frac{1}{2}(19c^2d-9e)e^2x\right)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{60c^2e^4}$$

$$= \frac{b(19c^2d-9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e^2} + \frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{16c^4d^3e+\frac{1}{4}e^2(45c^4d^2-10c^2de+9e^2)x}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{60c^4e^4}$$

$$= \frac{b(19c^2d-9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e^2} + \frac{d^2\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3} + \frac{(d+ex^2)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{5e^3} + \frac{\left(4bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{15e^3} + \frac{\left(b(45c^4d^2-10c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{240c^4e^2}$$

$$=\frac{b(19c^2d-9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e^2}+\frac{d^2\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^3}+\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{5e^3}+\frac{\left(8bd^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{-d+x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{15e^3}-\frac{\left(b(45c^4d^2-10c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1-c^2x^2}\right)}{120c^6e^2}$$

$$=\frac{b(19c^2d-9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e^2}+\frac{d^2\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^3}+\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{5e^3}-\frac{8bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^3}-\frac{\left(b(45c^4d^2-10c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{120c^6e^2}$$

$$=\frac{b(19c^2d-9e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{120c^4e^2}-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2e^2}+\frac{d^2\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^3}+\frac{(d+ex^2)^{5/2}(a+b\text{sech}^{-1}(cx))}{5e^3}-\frac{b(45c^4d^2-10c^2de+9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^5e^{5/2}}-\frac{8bd^{5/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{15e^3}$$

Mathematica [A] (verified)

Time = 23.56 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.03

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$
$$=\frac{\sqrt{d+ex^2}\left(8ac^4(8d^2-4dex^2+3e^2x^4)-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(9e+c^2(-13d+6ex^2)\right)+8bc^4(8d^2-4dex^2+3e^2x^4)\text{sech}^{-1}(cx)\right)}{120c^4e^3}$$
$$+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}(45c^4d^2-10c^2de+9e^2)\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+64c^7d^{5/2}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)\right)}{120c^7e^3(-1+cx)\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^4*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) - b*e*Sqrt[(1 - c
*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(-13*d + 6*e*x^2)) + 8*b*c^4*(8*d^2 - 4
*d*e*x^2 + 3*e^2*x^4)*ArcSech[c*x]))/(120*c^4*e^3) + (b*Sqrt[(1 - c*x)/(1 +
 c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(45*c^4*d^2
 - 10*c^2*d*e + 9*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e
]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 64*c^7*d^(5/2)*Sqrt
[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(120*c^
7*e^3*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^5(a+b\,\text{arcsech}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 1679, normalized size of antiderivative = 4.72

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-
```

```
(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b
*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e
^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt
(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 +
 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/
(c*x)) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^4*e
^2*x^3 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sq
rt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e
 + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(
e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*b*c^4
*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e
)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 +
(c^2*d*e - e^2)*x^2 - d*e)) + 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c
^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))
 + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^4*e^2*x^3
 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x
^2 + d))/(c^5*e^3), -1/480*(128*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*
e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(
c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b*c^4*d^2 - 10*b*c^2*d*e +
9*b*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^
2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^
2 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
1)/(c*x)) - 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^
4*e^2*x^3 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))
*sqrt(e*x^2 + d))/(c^5*e^3), -1/240*(64*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^
3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2
*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b*c^4*d^2 - 10*b*c^
2*d*e + 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^
2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2
)*x^2 - d*e)) - 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*sqrt(e
*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(24*a*c^5
*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^4*e^2*x^3 - (13*b*c^4*d
*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*
e^3)]
```

Sympy [F]

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^5(a+b\,\text{asech}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**5*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 (a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5 (a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^5}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 (a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^5 \left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.151 $\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 251

$$\begin{aligned}\int \frac{x^3(a+b\,\mathrm{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = &-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e} \\ &-\frac{d\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{e^2} \\ &+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{3e^2} \\ &+\frac{b(3c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3e^{3/2}} \\ &+\frac{2bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^2}\end{aligned}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^2+1/6*b*(3*c^2*d-e)*arctan(e^(1/2)
*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3/
e^(3/2)+2/3*b*d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2-d*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^2
-1/6*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c
^2/e
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 6436, 12, 587, 159, 163, 65, 223, 209, 95, 213}

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = -\frac{d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{3e^2} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(3c^2d-e)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3e^{3/2}} + \frac{2bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^2} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e}$$

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/6*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2
])/(c^2*e) - (d*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e^2 + ((d + e*x^2)^(3
/2)*(a + b*ArcSech[c*x]))/(3*e^2) + (b*(3*c^2*d - e)*Sqrt[(1 + c*x)^(-1)]*S
qrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^
3*e^(3/2)) + (2*b*d^(3/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d
 + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(3*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(-2d+ex^2\right)\sqrt{d+ex^2}}{3e^2x\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{d\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2} \\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\left(-2d+ex^2\right)\sqrt{d+ex^2}}{x\sqrt{1-c^2x^2}}\,dx}{3e^2} \\
&= -\frac{d\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2} \\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{(-2d+ex)\sqrt{d+ex}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{6e^2}
\end{aligned}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e} - \frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{2c^2d^2+\frac{1}{2}(3c^2d-e)ex}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6c^2e^2}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e} - \frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2} - \frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3e^2} - \frac{\left(b(3c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{12c^2e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e} - \frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2} - \frac{\left(2bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{-d+x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{3e^2} + \frac{\left(b(3c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{1-c^2x^2}\right)}{6c^4e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e} - \frac{d\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2} + \frac{2bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^2} + \frac{\left(b(3c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^4e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e} - \frac{d\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^2} + \frac{b(3c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3e^{3/2}} + \frac{2bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^2}$$

Mathematica [A] (verified)

Time = 22.96 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.62

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(be\sqrt{\frac{1-cx}{1+cx}}(1+cx)+2ac^2(2d-ex^2)+2bc^2(2d-ex^2)\operatorname{sech}^{-1}(cx)\right)}{6c^2e^2} - \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(-3(-c^2)^{3/2}\,d\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+\sqrt{-c^2}\sqrt{-c^2d-e}\,e^{3/2}}{6c^5e^2(-1+cx)\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/6*(Sqrt[d + e*x^2]*(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 2*a*c^2*(2
*d - e*x^2) + 2*b*c^2*(2*d - e*x^2)*ArcSech[c*x]))/(c^2*e^2) - (b*Sqrt[(1 -
 c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(-3*(-c^2)^(3/2)*d*Sqrt[-(c^2*d) - e]*Sq
rt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^
2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*e^(3/2
)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(Sqrt[-c^2]*Sqrt[e]*Sqrt[1 - c
^2*x^2])/(c*Sqrt[-(c^2*d) - e])] + 4*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcTan[(S
qrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(6*c^5*e^2*(-1 + c*x)*Sqrt[d
+ e*x^2])
```

Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arcsech}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(159) = 318.

Time = 0.48 (sec) , antiderivative size = 1389, normalized size of antiderivative = 5.53

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*b*c^3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(-e)*log(8*c^4
*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
)) + e^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*
x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*
d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((
c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 +
 (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2
*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) + 4*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e
*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*
e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 +
d))/(c^3*e^2), 1/24*(8*b*c^3*sqrt(-d)*d*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*
c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4
 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(-e)*log(8*c^4*e^2*x
^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^
4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e
^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 -
1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(4*b*c^3*sqrt(-
d)*d*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqr
t(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (3
*b*c^2*d - b*e)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2
 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)
*x^2 - d*e)) + 4*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(
c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c
^2*x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2)]
```

Sympy [F]

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^3(a + b\,\text{asech}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^3}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^3\,\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.152 $\int \frac{x\left(a+b\,\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e}$$

```
[Out] -b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*d^(1/2)*(1/(c*x+1))^
(1/2)*(c*x+1)^(1/2)/e-b*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2)
)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c/e^(1/2)+(a+b*arcsech(c*x))*(e*x^2+d)^(1
/2)/e
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6434, 531, 457, 132, 65, 223, 209, 12, 95, 213}

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e}$$

```
[In] Int[(x*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
 c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*Sqrt[e])
- (b*Sqrt[d]*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sq
rt[d]*Sqrt[1 - c^2*x^2])])/e
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6434

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Dist[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{e}\\
&= \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}}{x\sqrt{1-c^2x^2}}\,dx}{e}\\
&= \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{\sqrt{d+ex}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{2e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} \\
&\quad + \frac{1}{2}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right) \\
&\quad + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{d}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e} \\
&= \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} \\
&\quad - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1-c^2x^2}\right)}{c^2} \\
&\quad + \frac{\left(bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e} \\
&= \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{c^2} \\
&\quad + \frac{\left(bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{-d+x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{e} \\
&= \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}} \\
&\quad - \frac{b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 21.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.56

$$
\int \frac{x\left(a+b\,\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{e} + \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+c^3\sqrt{d}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d}\sqrt{}}{\sqrt{-}}\right.\right.}{c^3e(-1+cx)\sqrt{d+ex^2}}
$$

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqr
t[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2)
)/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*
d) - e])] + c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])
/Sqrt[-d - e*x^2]]))/(c^3*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x(a+b\,\mathrm{arcsech}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)

[Out] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(95) = 190$.

Time = 0.33 (sec) , antiderivative size = 1102, normalized size of antiderivative = 7.20

$$\int \frac{x\left(a+b\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \left[\frac{4\sqrt{ex^2+d}bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+bc\sqrt{d}\log\left(\frac{(c^4d^2-6c^2de+e^2)x^4-8(c^2d^2-de)x^2+4((c^3d-ce)x^3-2cdx)\sqrt{ex^2+d}\sqrt{d}\sqrt{-}}{x^4}\right.}{} \right.$$

$$-\frac{2bc\sqrt{-d}\arctan\left(-\frac{((c^3d-ce)x^3-2cdx)\sqrt{ex^2+d}\sqrt{-d}\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{2(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)-4\sqrt{ex^2+d}bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-4\sqrt{ex^2}}{}$$

$$-\frac{bc\sqrt{-d}\arctan\left(-\frac{((c^3d-ce)x^3-2cdx)\sqrt{ex^2+d}\sqrt{-d}\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{2(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)-2\sqrt{ex^2+d}bc\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-2\sqrt{ex^2+}}{2ce}$$

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c - b*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2))/(c*e), 1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c - 2*b*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)))/(c*e), -1/4*(2*b*c*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c

```
^2*d^2 - d*e)*x^2 - d^2)) - 4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + 1)/(c*x)) - 4*sqrt(e*x^2 + d)*a*c + b*sqrt(-e)*log(8*c^4*e
^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 +
 (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
 + e^2))/(c*e), -1/2*(b*c*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x
)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c
^2*d^2 - d*e)*x^2 - d^2)) - 2*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2)) + 1)/(c*x)) - 2*sqrt(e*x^2 + d)*a*c + b*sqrt(e)*arctan(1/2*(
2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)))/(c*e)]
```

Sympy [F]

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \int \frac{x(a + b\,\text{asech}\,(cx))}{\sqrt{d + ex^2}}\, dx$$

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x}{\sqrt{ex^2 + d}}\, dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*(sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/e - integrate((2*(
c^2*e*x^2 - e)*x*log(sqrt(x)) + (c^2*e*x^2*log(c) - e*log(c))*x + (2*(c^2*e
*x^2 - e)*x*log(sqrt(x)) + ((e*log(c) + e)*c^2*x^2 + c^2*d - e*log(c))*x)*e
^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^2*e*x^2 + (c^2*e*x^2 - e)*e^(1
/2*log(c*x + 1) + 1/2*log(-c*x + 1)) - e)*sqrt(e*x^2 + d)), x)) + sqrt(e*x^
2 + d)*a/e
```

Giac [F]

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x\,\left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.153 $\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx = \text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx = \int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

```
[In] Int[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Mathematica [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\operatorname{arcsech}\left(cx\right)}{x\sqrt{e\,x^{2}+d}}dx$$

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{\sqrt{ex^2+d}x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\text{asech}\left(cx\right)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asech(c*x))/(x*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{ex^2+d}x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

Mupad [N/A]

Not integrable

Time = 4.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

3.154 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\text{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsech}\,(cx)}{x^3\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arsech}\,(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 7.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asech}\,(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asech(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{ex^2 + d}x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 4.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.155 $\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\mathrm{Int}\left(\frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral}=\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\ \text{arcsech}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 3.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2(a+b\,\text{asech}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^2}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 4.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a + b\text{sech}^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^2\,\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.156 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\operatorname{Int}\left(\frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arcsech}\,(cx)}{\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arsech}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{sech}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asech}\,(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{sech}^{-1}(cx)}{\sqrt{d + ex^2}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 4.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a + b\text{sech}^{-1}(cx)}{\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

3.157 $\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 221

$$\int \frac{a+b\,\mathrm{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{dx} + \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\mathrm{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

```
[Out] -(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/d/x+b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(
-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/x+b*c*EllipticE(c*x,(-e/c^2/d)^(1/2))*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d/(1+e*x^2/d)^(1/2)-b*(c^2*d
+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^
2/d)^(1/2)/c/d/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used

= {270, 6436, 12, 486, 21, 434, 437, 435, 432, 430}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}} + \frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d\sqrt{\frac{ex^2}{d}+1}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx}$$

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(d
*x) - (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(d*x) + (b*c*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*
Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sq
rt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d*Sqrt[d + e*x^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
```

```
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx} - \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}}{dx^2\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}}{x^2\sqrt{1-c^2x^2}}\,dx}{d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{e-c^2ex^2}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx} - \frac{\left(be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\,dx}{d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx} + \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{d} - \frac{\left(b(c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{dx} + \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{d\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(b(c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{d+ex^2}}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{dx}$$
$$+ \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{b(c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.41 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.27

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx =$$
$$-\frac{a\left(\frac{d}{x}+ex\right)+bc\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)-\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2)}{x}+\frac{b(d+ex^2)\operatorname{sech}^{-1}(cx)}{x}+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{\left(c\sqrt{d}+i\sqrt{e}\right)(1+cx)}}\left(i\sqrt{d}+\sqrt{e}\right.}{}}{d\sqrt{d}}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((a*(d/x + e*x) + b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2) - (b*Sqrt[(1 -
 c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2))/x + (b*(d + e*x^2)*ArcSech[c*x])/x
+ (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d]
 + I*Sqrt[e])*(1 + c*x))]*(I*Sqrt[d] + Sqrt[e]*x)*((c*Sqrt[d] - I*Sqrt[e])*
EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2
*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + (2*I)
*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*S
qrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2
]))/(Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(
1 + c*x)))]*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 +
c*x))]))/(d*Sqrt[d + e*x^2]))
```

Maple [F]

$$\int \frac{a+b \operatorname{arcsech}(cx)}{x^2\sqrt{e\,x^2+d}}dx$$

[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x)

[Out] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.70

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{ex^2+d}bcd\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-\left(bc^2dx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-acd\right)\sqrt{ex^2+d}-\left(bc^4dxE(\arcsin(cx)\mid-\frac{e}{c^2d}\right)}{cd^2x}$$

[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(e*x^2 + d)*b*c*d*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a*c*d)*sqrt(e*x^2 + d) - (b*c^4*d*x*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (b*c^4*d + b*e)*x*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c*d^2*x)

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{asech}(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

[In] integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**(1/2),x)

[Out] Integral((a + b*asech(c*x))/(x**2*sqrt(d + e*x**2)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{b\operatorname{arsech}(cx)+a}{\sqrt{ex^2+d}x^2}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.158 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 346

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx &= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3} + \frac{b(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9d^2x} \\ &\quad - \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3} + \frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x} \\ &\quad + \frac{bc(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d^2\sqrt{1+\frac{ex^2}{d}}} \\ &\quad - \frac{2b(c^2d-3e)\left(c^2d+e\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9cd^2\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] -1/3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arcsech(c*x))*(e*x
^2+d)^(1/2)/d^2/x+1/9*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*
(e*x^2+d)^(1/2)/d/x^3+1/9*b*(2*c^2*d-5*e)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(
-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x+1/9*b*c*(2*c^2*d-5*e)*EllipticE(c*x
,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(1+e
*x^2/d)^(1/2)-2/9*b*(c^2*d-3*e)*(c^2*d+e)*EllipticF(c*x,(-e/c^2/d)^(1/2))*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {277, 270, 6436, 12, 594, 597, 538, 437, 435, 432, 430}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$= \frac{2e\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2x} - \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{3dx^3} - \frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d-3e)\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9cd^2\sqrt{d+ex^2}} + \frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(2c^2d-5e)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d^2\sqrt{\frac{ex^2}{d}+1}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(2c^2d-5e)\sqrt{d+ex^2}}{9d^2x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3}$$

```
[In] Int[(a + b*ArcSech[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*d*x^3) + (b*(2*c^2*d - 5*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^
2*x^2]*Sqrt[d + e*x^2])/(9*d^2*x) - (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/
(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(3*d^2*x) + (b*c*(2*
c^2*d - 5*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[A
rcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d - 3*e)
*(c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*Ellipti
cF[ArcSin[c*x], -(e/(c^2*d))])/(9*c*d^2*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
```

```
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d^2x} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{3d^2x^4\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d^2x} \\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{x^4\sqrt{1-c^2x^2}}\,dx}{3d^2} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3}-\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} \\
&\quad+\frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d^2x}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-d\left(2c^2d-5e\right)-\left(c^2d-6e\right)ex^2}{x^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3} \\
&+ \frac{b(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9d^2x} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} \\
&+ \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d^2x} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{d(c^2d-6e)e-c^2d(2c^2d-5e)ex^2}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^3} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3} \\
&+ \frac{b(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9d^2x} - \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} \\
&+ \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d^2x} + \frac{\left(bc^2(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{9d^2} \\
&- \frac{\left(2b(c^2d-3e)(c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3} \\
&+ \frac{b(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9d^2x} \\
&- \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} + \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d^2x} \\
&+ \frac{\left(bc^2(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{9d^2\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{\left(2b(c^2d-3e)(c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{9d^2\sqrt{d+ex^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx^3} \\
&+ \frac{b(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9d^2x} \\
&- \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3dx^3} + \frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d^2x} \\
&+ \frac{bc(2c^2d-5e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d^2\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{2b(c^2d-3e)(c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9cd^2\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.04 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.77

$$
\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx
$$

$$
= \frac{\frac{bd\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x^3} + \frac{bcd\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x^2} + \frac{b(2c^2d-5e)\sqrt{\frac{1-cx}{1+cx}}(d+ex^2)}{x} - \frac{3a(d-2ex^2)(d+ex^2)}{x^3} - \frac{3b(d-2ex^2)(d+ex^2)\operatorname{sech}^{-1}(cx)}{x^3} - \cdots}{\cdots}
$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] ((b*d*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^3 + (b*c*d*Sqrt[(1 - c*x)/(1
 + c*x)]*(d + e*x^2))/x^2 + (b*(2*c^2*d - 5*e)*Sqrt[(1 - c*x)/(1 + c*x)]*(d
 + e*x^2))/x - (3*a*(d - 2*e*x^2)*(d + e*x^2))/x^3 - (3*b*(d - 2*e*x^2)*(d
+ e*x^2)*ArcSech[c*x])/x^3 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[(c*(Sqrt[d]
+ I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*(I*Sqrt[d] + Sqrt[e]*x
)*((2*c^3*d^(3/2) - (2*I)*c^2*d*Sqrt[e] - 5*c*Sqrt[d]*e + (5*I)*e^(3/2))*El
lipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(
1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*((2*I
)*c^2*d - c*Sqrt[d]*Sqrt[e] - (6*I)*e)*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c
^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] +
I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*
(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]*Sqrt[(c*(Sqrt[d] - I*Sqrt
[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]))/(9*d^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\ \mathrm{arcsech}\,(cx)}{x^4\sqrt{e\,x^2+d}}dx$$

[In] int((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x)

[Out] int((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.71

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \frac{3\,(2\,bcdex^2-bcd^2)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(6\,acdex^2-3\,acd^2+(bc^2d^2x+(2\,bc^4d^2-5\,bc^2de)x^3\right)}{}$$

[In] integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] 1/9*(3*(2*b*c*d*e*x^2 - b*c*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (6*a*c*d*e*x^2 - 3*a*c*d^2 + (b*c^2*d^2*x + (2*b*c^4*d^2 - 5*b*c^2*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) + ((2*b*c^6*d^2 - 5*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (2*b*c^6*d^2 - (5*b*c^4 - b*c^2)*d*e - 6*b*e^2)*x^3*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c*d^3*x^3)

Sympy [F]

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{asech}\,(cx)}{x^4\sqrt{d+ex^2}}\,dx$$

[In] integrate((a+b*asech(c*x))/x**4/(e*x**2+d)**(1/2),x)

[Out] Integral((a + b*asech(c*x))/(x**4*sqrt(d + e*x**2)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{ex^2 + dx^4}}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.159 $\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 278

$$\int \frac{x^5(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2(a+b\text{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\text{sech}^{-1}(cx))}{3e^3} + \frac{b(9c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3e^{5/2}} + \frac{8bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/e^3+1/6*b*(9*c^2*d-e)*arctan(e^(1/2)
*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^3/
e^(5/2)+8/3*b*d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(
1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^3-d^2*(a+b*arcsech(c*x))/e^3/(e*x^2+d)^(1/
2)-2*d*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^3-1/6*b*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/c^2/e^2
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {272, 45, 6436, 12, 1629, 163, 65, 223, 209, 95, 213}

$$\int \frac{x^5(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{d^2(a+b\text{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}\,(a+b\text{sech}^{-1}(cx))}{3e^3} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(9c^2d-e)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3e^{5/2}} + \frac{8bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2}$$

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -1/6*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2
])/(c^2*e^2) - (d^2*(a + b*ArcSech[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt
[d + e*x^2]*(a + b*ArcSech[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcSech[c
*x]))/(3*e^3) + (b*(9*c^2*d - e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[
(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^3*e^(5/2)) + (8*b*d^
(3/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*S
qrt[1 - c^2*x^2])])/(3*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
```

```
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\,(a+b\operatorname{sech}^{-1}(cx))}{e^3} \\
&\quad+\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e^3} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-8d^2-4dex^2+e^2x^4}{3e^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx \\
&= -\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\,(a+b\operatorname{sech}^{-1}(cx))}{e^3} \\
&\quad+\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e^3}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-8d^2-4dex^2+e^2x^4}{x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3e^3} \\
&= -\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\,(a+b\operatorname{sech}^{-1}(cx))}{e^3} \\
&\quad+\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e^3} \\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{-8d^2-4dex+e^2x^2}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{6e^3} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2}-\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\
&\quad-\frac{2d\sqrt{d+ex^2}\,(a+b\operatorname{sech}^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e^3} \\
&\quad-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{8c^2d^2e+\frac{1}{2}(9c^2d-e)e^2x}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{6c^2e^4}
\end{aligned}
$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} - \frac{\left(4bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^3} - \frac{\left(b(9c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{12c^2e^2}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} - \frac{\left(8bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{-d+x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{3e^3} + \frac{\left(b(9c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{1-c^2x^2}\right)}{6c^4e^2}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} + \frac{8bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3} + \frac{\left(b(9c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{1}{1+\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^4e^2}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3} + \frac{b(9c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^3e^{5/2}} + \frac{8bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3}$$

Mathematica [A] (verified)

Time = 23.34 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.57

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \frac{-be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\,(d+ex^2)-2ac^2(8d^2+4dex^2-e^2x^4)-2bc^2(8d^2+4dex^2-}{6c^2e^3\sqrt{d+ex^2}}$$
$$-\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(-9(-c^2)^{3/2}\,d\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+\sqrt{-c^2}\sqrt{-c^2d-e}e^{3/2}\sqrt{}\right.}{6c^5e^3(-1+cx)\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (-(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)) - 2*a*c^2*(8*d^2 +
4*d*e*x^2 - e^2*x^4) - 2*b*c^2*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcSech[c*x])/
(6*c^2*e^3*Sqrt[d + e*x^2]) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2
]*(-9*(-c^2)^(3/2)*d*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2
*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e
])] + Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*e^(3/2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d +
 e)]*ArcSin[(Sqrt[-c^2]*Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[-(c^2*d) - e])]
+ 16*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-
d - e*x^2]]))/(6*c^5*e^3*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^5(a+b\,\text{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(184) = 368.

Time = 0.44 (sec) , antiderivative size = 1771, normalized size of antiderivative = 6.37

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*log(8*c^4
*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3
 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2
```

```
)) + e^2) + 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*x^2 +
d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 16*(b*c^3*d*e*x^2
+ b*c^3*d^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*
e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 -
 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*((9*b*c^2*d^2 - b*d*e +
 (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x
)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^
2*d*e - e^2)*x^2 - d*e)) + 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2
)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 8*(
b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8
*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt
(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 2*(2*a*c^3*e^2*x^4 - 8*a
*c^3*d*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/24*(32*(b*c^
3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*s
qrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*
d^2 - d*e)*x^2 - d^2)) + (9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*
sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^
2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2)) + e^2) + 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*
d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) +
4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^
2*d*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^
3*d*e^3), 1/12*(16*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*arctan(-1/2*((c^3*d
 - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^
2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (9*b*c^2*d^2 - b*d*e + (9*
b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sq
rt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*
e - e^2)*x^2 - d*e)) + 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sq
rt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*
c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^2*d*e*x
)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3
)]
```

Sympy [F]

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^5(a+b\operatorname{asech}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**5*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^5}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{x^5\,\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{3/2}}\,dx$$

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.160 $\int \frac{x^3\left(a+b\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 177

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{3/2}} - \frac{2b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e^2}$$

```
[Out] -b*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(
c*x+1)^(1/2)/c/e^(3/2)-2*b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/
2))*d^(1/2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2+d*(a+b*arcsech(c*x))/e^2/(e
*x^2+d)^(1/2)+(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {272, 45, 6436, 12, 587, 163, 65, 223, 209, 95, 213}

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^2} + \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{3/2}} - \frac{2b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e^2}$$

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (d*(a + b*ArcSech[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cSech[c*x]))/e^2 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sq
```

```
rt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*e^(3/2)) - (2*b*Sqrt[d]*Sqrt[(1 +
 c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2
])])/e^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2}\\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{2d+ex^2}{e^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx\\
&= \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{2d+ex^2}{x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{e^2}
\end{aligned}$$

$$= \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{2d+ex}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e^2}$$

$$= \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{e^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e}$$

$$= \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} + \frac{\left(2bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{-d+x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{e^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{1-c^2x^2}\right)}{c^2e}$$

$$= \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} - \frac{2b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{c^2e}$$

$$= \frac{d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{3/2}} - \frac{2b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 23.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.41

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\frac{\left(2d+ex^2\right)\left(a+b\text{sech}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right)+2c^3\sqrt{d}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d}\sqrt{}}{\sqrt{-}}\right.\right.}{c^3e^2(-1+cx)\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*d + e*x^2)*(a + b*ArcSech[c*x]))/(e^2*Sqrt[d + e*x^2]) + (b*Sqrt[(1 - c
*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqr
t[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt
[-c^2]*Sqrt[-(c^2*d) - e])] + 2*c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d
]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(c^3*e^2*(-1 + c*x)*Sqrt[d + e*x^2
])
```

Maple [F]

$$\int \frac{x^3(a+b\ \text{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(117) = 234.

Time = 0.36 (sec) , antiderivative size = 1311, normalized size of antiderivative = 7.41

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*e*x^2 + b*d)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8
*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 +
 d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(b*c*e*x^2 + 2*b*c*d
)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(
b*c*e*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
```

```
(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 4*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x
^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*((b*e*x^2 + b*d)*sqrt(e)*arctan(1/2*(2
*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^
2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) - 2*(b*c*e*x^2 + 2*b*c*d
)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*
c*e*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-
(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2
 + d))/(c*e^3*x^2 + c*d*e^2), -1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(-d)*arctan(-
1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 -
 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x^2 + b*d)
*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x
^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(b*c*e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(a*c*e*x^2 + 2*a*c*d)*s
qrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*(2*(b*c*e*x^2 + b*c*d)*sqrt(-d)
*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(
c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x
^2 + b*d)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*
sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 -
 d*e)) - 2*(b*c*e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 -
1)/(c^2*x^2)) + 1)/(c*x)) - 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d))/(c*e^3
*x^2 + c*d*e^2)]
```

Sympy [F]

$$\int \frac{x^3(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{x^3(a + b\operatorname{asech}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**3*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.161 $\int \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}e}$$

```
[Out] b*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*
x+1)^(1/2)/e/d^(1/2)+(-a-b*arcsech(c*x))/e/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6434, 531, 457, 95, 213}

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}e} - \frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSech[c*x])/(e*Sqrt[d + e*x^2])) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[
1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(Sqrt[d]*e)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6434

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
 x] + Dist[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex^2}}\,dx}{e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{-d+x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{e}$$

$$= -\frac{a + b\text{sech}^{-1}(cx)}{e\sqrt{d + ex^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1 + cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}e}$$

Mathematica [A] (verified)

Time = 21.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.55

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = -\frac{a + b\text{sech}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1 - c^2x^2}\sqrt{-d - ex^2}\arctan\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)}{\sqrt{d}e(-1 + cx)\sqrt{d + ex^2}}$$

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
[Out] -((a + b*ArcSech[c*x])/(e*Sqrt[d + e*x^2])) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*
Sqrt[1 - c^2*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[
-d - e*x^2]])/(Sqrt[d]*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x(a + b\ \text{arcsech}\,(cx))}{(e\,x^2 + d)^{\frac{3}{2}}}dx$$

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(57) = 114.

Time = 0.29 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.36

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \left[-\frac{4\sqrt{ex^2 + d}bd\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + 4\sqrt{ex^2 + d}ad - (bex^2 + bd)\sqrt{d}\log\left(\frac{(c^4d}{}\right.}{4\,(de^2x^2 + d^2}\right.$$

$$-\frac{2\sqrt{ex^2 + d}bd\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + 2\sqrt{ex^2 + d}ad - (bex^2 + bd)\sqrt{-d}\arctan\left(-\frac{((c^3d-ce)x^3-2\,cdx)\sqrt{ex^2+d}\sqrt{-d}}{2\,(c^2dex^4+(c^2d^2-de)x^2-d}\right.}{2\,(de^2x^2 + d^2e)}$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(
c*x)) + 4*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(d)*log(((c^4*d^2 - 6*c
^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)
*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4))/(d*e
^2*x^2 + d^2*e), -1/2*(2*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + 1)/(c*x)) + 2*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(-d)*ar
ctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2
*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)))/(d*e^2*x^2
 + d^2*e)]
```

Sympy [F]

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{x(a + b\,\text{asech}\,(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d
)^(3/2), x) - a/(sqrt(e*x^2 + d)*e)
```

Giac [F]

$$\int \frac{x(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.162 $\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \mathrm{arcsech}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arsech}\left(cx\right)+a}{\left(ex^{2}+d\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)
```

Sympy [N/A]

Not integrable

Time = 18.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{x\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*asech(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 4.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\,(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

3.163 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \mathrm{arcsech}\left(cx\right)}{x^{3}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arsech}\left(cx\right)+a}{\left(ex^{2}+d\right)^{\frac{3}{2}}x^{3}}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

Sympy [N/A]

Not integrable

Time = 77.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\text{asech}\,(cx)}{x^3\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x**3*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.164 $\int \frac{x^4\left(a+b\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\ \operatorname{arcsech}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^4*arcsech(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [N/A]

Not integrable

Time = 40.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^4(a+b\operatorname{asech}(cx))}{\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^4}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,dx$$

```
[In] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.165 $\int \frac{x^2\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\operatorname{Int}\left(\frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.97 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.87 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\ \text{arcsech}\left(cx\right))}{\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(b\,\text{arsech}\left(cx\right)+a)x^2}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\,\text{asech}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**2*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.166 $\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

```
[Out] x*(a+b*arcsech(c*x))/d/(e*x^2+d)^(1/2)+b*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {197, 6426, 12, 432, 430}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSech[c*x]))/(d*Sqrt[d + e*x^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt
[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d*Sq
rt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 6426

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x
] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt
[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ
[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx\\
&= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d}\\
&= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{d+ex^2}}\\
&= \frac{x\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 50.61 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.63

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\operatorname{sech}^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{2ib\sqrt{\frac{1-cx}{1+cx}}\sqrt{\frac{(c\sqrt{d}+i\sqrt{e})(1+cx)}{(c\sqrt{d}-i\sqrt{e})(-1+cx)}}\left(-i\sqrt{d}+\sqrt{e}x\right)\sqrt{-\frac{-1+\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{1-cx}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+\frac{ic\sqrt{d}}{\sqrt{e}}-cx+\frac{i\sqrt{e}x}{\sqrt{d}}}{2-2cx}}\right)\right.}{d\left(c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{1+\frac{ic\sqrt{d}}{\sqrt{e}}-cx+\frac{i\sqrt{e}x}{\sqrt{d}}}{1-cx}}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSech[c*x]))/(d*Sqrt[d + e*x^2]) + ((2*I)*b*Sqrt[(1 - c*x)/(1 +
 c*x)]*Sqrt[((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] - I*Sqrt[e])*(-
1 + c*x))]*((-I)*Sqrt[d] + Sqrt[e]*x)*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] +
c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticF[ArcSin[Sqrt[(1 + (I*c*Sq
rt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(2 - 2*c*x)]], ((-4*I)*c*Sqrt
[d]*Sqrt[e])/(c*Sqrt[d] - I*Sqrt[e])^2])/(d*(c*Sqrt[d] + I*Sqrt[e])*Sqrt[(1
 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(1 - c*x)]*Sqrt[d +
 e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\operatorname{arcsech}(cx)}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{\sqrt{ex^2+d}bcdx\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\sqrt{ex^2+d}acdx+(bex^2+bd)\sqrt{d}F(\arcsin(cx)}{cd^2ex^2+cd^3}$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(e*x^2 + d)*b*c*d*x*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)
) + sqrt(e*x^2 + d)*a*c*d*x + (b*e*x^2 + b*d)*sqrt(d)*elliptic_f(arcsin(c*x
), -e/(c^2*d)))/(c*d^2*e*x^2 + c*d^3)
```

Sympy [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^{3/2}}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^{3/2}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^
(3/2), x) + a*x/(sqrt(e*x^2 + d)*d)
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^{3/2}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^{3/2}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^{3/2}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(3/2),x)
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

3.167 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 249

$$\begin{aligned}\int \frac{a+b\text{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx &= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} \\ &\quad - \frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{1+\frac{ex^2}{d}}} \\ &\quad - \frac{b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd^2\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] (-a-b*arcsech(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arcsech(c*x))/d^2/(e*x^2
+d)^(1/2)+b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1
/2)/d^2/x+b*c*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/
2)*(e*x^2+d)^(1/2)/d^2/(1+e*x^2/d)^(1/2)-b*(c^2*d+2*e)*EllipticF(c*x,(-e/c^
2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d^2/(e*x^2+
d)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {277, 197, 6436, 12, 597, 538, 437, 435, 432, 430}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = -\frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} -\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd^2\sqrt{d+ex^2}} +\frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{\frac{ex^2}{d}+1}} +\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x}$$

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(d
^2*x) - (a + b*ArcSech[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcSech[
c*x]))/(d^2*Sqrt[d + e*x^2]) + (b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt
[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d^2*Sqrt[1 + (e*x^2)/d])
 - (b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*
EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d^2*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-d-2ex^2}{d^2x^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{-d-2ex^2}{x^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{2de-c^2dex^2}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^3}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{d^2} - \frac{\left(b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\text{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\text{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{d^2\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d^2\sqrt{d+ex^2}}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\operatorname{sech}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{cd^2\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.73 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.01

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2)}{x} - \frac{a(d+2ex^2)}{x} - \frac{b(d+2ex^2)\operatorname{sech}^{-1}(cx)}{x} + \frac{b\sqrt{\frac{1-cx}{1+cx}}\left(-c^2(d+ex^2)+\frac{(1+cx)\sqrt{\frac{c(\cdots}{(c\sqrt{d}\cdots}}}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2))/x - (a*(d + 2*e*x^2))/
x - (b*(d + 2*e*x^2)*ArcSech[c*x])/x + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*
(d + e*x^2)) + ((1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*
Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[
e])*(1 + c*x))]*((-I)*(c*Sqrt[d] - I*Sqrt[e])^2*EllipticE[I*ArcSinh[Sqrt[((
c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] +
 I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*(c*Sqrt[d] - (2*I)*Sqrt[e])*Sq
rt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt
[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))
/Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 +
c*x)))]))/c)/(d^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\operatorname{arcsech}\left(cx\right)}{x^2\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx =$$

$$-\frac{(2\,bcdex^2+bcd^2)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(2\,acdex^2+acd^2-(bc^2dex^3+bc^2d^2x)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right)\sqrt{ex^2}}{cd^3}$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -((2*b*c*d*e*x^2 + b*c*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c
^2*x^2)) + 1)/(c*x)) + (2*a*c*d*e*x^2 + a*c*d^2 - (b*c^2*d*e*x^3 + b*c^2*d^
2*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) - ((b*c^4*d*e*x^3 + b*
c^4*d^2*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((b*c^4*d*e + 2*b*e^2)*x^3
 + (b*c^4*d^2 + 2*b*d*e)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(
c*d^3*e*x^3 + c*d^4*x)
```

Sympy [F]

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\,\text{asech}\,(cx)}{x^2\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asech(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2 (d + ex^2)^{3/2}} dx = \int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^2} dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^2 (d + ex^2)^{3/2}} dx = \int \frac{a + b \operatorname{acosh}\left(\frac{1}{c x}\right)}{x^2 (e x^2 + d)^{3/2}} dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*acosh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.168 $\int \frac{x^5\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 272

$$\begin{aligned}\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx &= -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2(c^2d+e)\sqrt{d+ex^2}} \\ &\quad -\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3(d+ex^2)^{3/2}}+\frac{2d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3} \\ &\quad -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{5/2}}-\frac{8b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3}\end{aligned}$$

```
[Out] -1/3*d^2*(a+b*arcsech(c*x))/e^3/(e*x^2+d)^(3/2)-b*arctan(e^(1/2)*(-c^2*x^2+
1)^(1/2)/c/(e*x^2+d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c/e^(5/2)-8/3*b
*arctanh((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*d^(1/2)*(1/(c*x+1))^(1
/2)*(c*x+1)^(1/2)/e^3+2*d*(a+b*arcsech(c*x))/e^3/(e*x^2+d)^(1/2)-1/3*b*d*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)/(e*x^2+d)^(1
/2)+(a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {272, 45, 6436, 12, 1628, 163, 65, 223, 209, 95, 213}

$$\int \frac{x^5\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=-\frac{d^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{e^3}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{5/2}}-\frac{8b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3}-\frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{d+ex^2}}$$

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(e^2*(c^2*d
 + e)*Sqrt[d + e*x^2]) - (d^2*(a + b*ArcSech[c*x]))/(3*e^3*(d + e*x^2)^(3/2
)) + (2*d*(a + b*ArcSech[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a
 + b*ArcSech[c*x]))/e^3 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqr
t[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*e^(5/2)) - (8*b*Sqrt[d]*Sq
rt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 -
c^2*x^2])])/(3*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{8d^2+12dex^2+3e^2x^4}{3e^3x\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx$$

$$=-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{8d^2+12dex^2+3e^2x^4}{x\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e^3}$$

$$=-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{8d^2+12dex+3e^2x^2}{x\sqrt{1-c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e^3}$$

$$=-\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{d+ex^2}}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{4d^2(c^2d+e)+\frac{3}{2}de(c^2d+e)x}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3de^3\left(c^2d+e\right)}$$

$$=-\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{d+ex^2}}-\frac{d^2\left(a+b\text{sech}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{sech}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)}{e^3}+\frac{\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^3}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e^2}$$

$$= -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2(c^2d+e)\sqrt{d+ex^2}} - \frac{d^2(a+b\text{sech}^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\text{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3} + \frac{\left(8bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{-d+x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{3e^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx, x, \sqrt{1-c^2x^2}\right)}{c^2e^2}$$

$$= -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2(c^2d+e)\sqrt{d+ex^2}} - \frac{d^2(a+b\text{sech}^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\text{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3} - \frac{8b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{1+\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\right)}{c^2e^2}$$

$$= -\frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e^2(c^2d+e)\sqrt{d+ex^2}} - \frac{d^2(a+b\text{sech}^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\text{sech}^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\text{sech}^{-1}(cx))}{e^3} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{5/2}} - \frac{8b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^3}$$

Mathematica [A] (verified)

Time = 23.42 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.28

$$\int \frac{x^5(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-bde\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2) + a(c^2d+e)(8d^2+12dex^2+3e^2x^4) + b(c^2d+e)}{3e^3(c^2d+e)(d+ex^2)^{3/2}} + \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\left(3\sqrt{-c^2}\sqrt{-c^2d-e}\sqrt{e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\arcsin\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{-c^2d-e}}\right) + 8c^3\sqrt{d}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d}\sqrt{}}{\sqrt{-}}\right.\right.}{3c^3e^3(-1+cx)\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(b*d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)) + a*(c^2*d + e)*(
8*d^2 + 12*d*e*x^2 + 3*e^2*x^4) + b*(c^2*d + e)*(8*d^2 + 12*d*e*x^2 + 3*e^2
```

```
*x^4)*ArcSech[c*x])/(3*e^3*(c^2*d + e)*(d + e*x^2)^(3/2)) + (b*Sqrt[(1 - c*
x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(3*Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sq
rt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqr
t[-c^2]*Sqrt[-(c^2*d) - e])] + 8*c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[
d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(3*c^3*e^3*(-1 + c*x)*Sqrt[d + e*
x^2])
```

Maple [F]

$$\int \frac{x^5(a+b\ \mathrm{arcsech}\left(cx\right))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(180) = 360$.

Time = 0.47 (sec) , antiderivative size = 2415, normalized size of antiderivative = 8.88

$$\int \frac{x^5\left(a+b\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
 + b*d*e^2)*x^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*
d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sq
rt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(8*b*c^3*d^3 + 8*b*c*d^2*e
 + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e
*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 8*(b*c^3*d^
3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x
^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 +
4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)
/(c^2*x^2)) + 8*d^2)/x^4) - 4*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 +
 a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 - (b*c^2*d*e^2*x^3 + b*c^2
*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c
*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/6*(
3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e
^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*s
qrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 -
d*e)) - 2*(8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(
b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/
(c^2*x^2)) + 1)/(c*x)) - 4*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)
```

```
*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e +
 e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*
x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(8*a*c^3*
d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d
*e^2)*x^2 - (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*
(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/12*(16*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e
^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*(
(c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^
2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*lo
g(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^
4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(
c^2*x^2)) + e^2) - 4*(8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)
*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^
2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3
*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 - (b*c^2*d*e^2*x^3
 + b*c^2*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3
*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2)
, -1/6*(8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d
^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*s
qrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*
d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e
 + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d
- e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*e^2*x^4
 + (c^2*d*e - e^2)*x^2 - d*e)) - 2*(8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*
e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log(
(c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(8*a*c^3*d^3 + 8*a*c*d^
2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 - (b
*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2
+ d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 +
 c*d*e^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\,\operatorname{arsech}(cx) + a)x^5}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x^5\,\left(a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.169 $\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 179

$$\int \frac{x^3(a+b\,\mathrm{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e(c^2d+e)\sqrt{d+ex^2}}+\frac{d(a+b\,\mathrm{sech}^{-1}(cx))}{3e^2(d+ex^2)^{3/2}}-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3\sqrt{d}e^2}$$

```
[Out] 1/3*d*(a+b*arcsech(c*x))/e^2/(e*x^2+d)^(3/2)+2/3*b*arctanh((e*x^2+d)^(1/2)/
d^(1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/e^2/d^(1/2)+(-a
-b*arcsech(c*x))/e^2/(e*x^2+d)^(1/2)+1/3*b*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*
(-c^2*x^2+1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {272, 45, 6436, 12, 587, 157, 95, 213}

$$\int \frac{x^3(a+b\,\mathrm{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=-\frac{a+b\,\mathrm{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\,\mathrm{sech}^{-1}(cx))}{3e^2(d+ex^2)^{3/2}}+\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3\sqrt{d}e^2}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3e(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*e*(c^2*d + e)*S
qrt[d + e*x^2]) + (d*(a + b*ArcSech[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a +
 b*ArcSech[c*x])/(e^2*Sqrt[d + e*x^2]) + (2*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 +
 c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(3*Sqrt[d]*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-2d-3ex^2}{3e^2x\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx \\
&= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-2d-3ex^2}{x\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e^2} \\
&= \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} \\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{-2d-3ex}{x\sqrt{1-c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e^2} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} \\
&\quad-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{d(c^2d+e)}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3de^2\left(c^2d+e\right)} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} \\
&\quad-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^2}
\end{aligned}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{-d+x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{3e^2}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3e\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{d\left(a+b\text{sech}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{sech}^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3\sqrt{d}e^2}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.22

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{be\sqrt{\frac{1-cx}{1+cx}}(1+cx)\left(d+ex^2\right) - a(c^2d+e)\left(2d+3ex^2\right) - b(c^2d+e)\left(2d+3ex^2\right)}{3e^2\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}} - \frac{2b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\sqrt{-d-ex^2}\,\text{arctan}\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)}{3\sqrt{d}e^2(-1+cx)\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2) - a*(c^2*d + e)*(2*d +
 3*e*x^2) - b*(c^2*d + e)*(2*d + 3*e*x^2)*ArcSech[c*x])/(3*e^2*(c^2*d + e)*
(d + e*x^2)^(3/2)) - (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Sqrt[
-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(3*Sqrt[d
]*e^2*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^3(a+b\,\text{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(117) = 234.

Time = 0.35 (sec) , antiderivative size = 786, normalized size of antiderivative = 4.39

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{2\left(2\,bc^2d^3+2\,bd^2e+3\left(bc^2d^2e+bde^2\right)x^2\right)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(b\cdots}{\cdots}\right.$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e*x
^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 +
(b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(d
)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d
- c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)
) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^
2 - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^
2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3
 + d^2*e^4)*x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2
*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c
*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*x^4
+ (c^2*d^2 - d*e)*x^2 - d^2)) - (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)
/(c*x)) - (2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 - (b*c*d
*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c
^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4
)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*inte
grate(x^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5
/2), x)
```

Giac [F]

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x^3\,\left(a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.170 $\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=-\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}}-\frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3d^{3/2}e}$$

```
[Out] 1/3*(-a-b*arcsech(c*x))/e/(e*x^2+d)^(3/2)+1/3*b*arctanh((e*x^2+d)^(1/2)/d^(
1/2)/(-c^2*x^2+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/d^(3/2)/e-1/3*b*(1
/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(1/2
)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6434, 531, 457, 98, 95, 213}

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=-\frac{a+b\text{sech}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3d^{3/2}e}-\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}}$$

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(d*(c^2*d + e
)*Sqrt[d + e*x^2]) - (a + b*ArcSech[c*x])/(3*e*(d + e*x^2)^(3/2)) + (b*Sqrt
[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^
2*x^2])])/(3*d^(3/2)*e)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 6434

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSech[c*x])/(2*e*(p + 1))),
```

```
x] + Dist[b*(Sqrt[1 + c*x]/(2*e*(p + 1)))*Sqrt[1/(1 + c*x)], Int[(d + e*x^
2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e,
 p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\text{sech}^{-1}(cx)}{3e(d+ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}(d+ex^2)^{3/2}}\,dx}{3e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{3e(d+ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e}$$

$$= -\frac{a+b\text{sech}^{-1}(cx)}{3e(d+ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}} - \frac{a+b\text{sech}^{-1}(cx)}{3e(d+ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6de}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}} - \frac{a+b\text{sech}^{-1}(cx)}{3e(d+ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{1}{-d+x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{3de}$$

$$= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}} - \frac{a+b\text{sech}^{-1}(cx)}{3e(d+ex^2)^{3/2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3d^{3/2}e}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.32

$$\int \frac{x(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-ad(c^2d+e) - be\sqrt{\frac{1-cx}{1+cx}}(1+cx)(d+ex^2) - bd(c^2d+e)\,\text{sech}^{-1}(cx)}{3de(c^2d+e)(d+ex^2)^{3/2}} - \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)}{3d^{3/2}e(-1+cx)\sqrt{d+ex^2}}$$

```
[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(a*d*(c^2*d + e)) - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2) -
 b*d*(c^2*d + e)*ArcSech[c*x])/(3*d*e*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b*S
qrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]
*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(3*d^(3/2)*e*(-1 + c*x)*Sqrt[d + e*x
^2])
```

Maple [F]

$$\int \frac{x(a+b\,\mathrm{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(94) = 188$.

Time = 0.34 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.49

$$\int \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{4\,(bc^2d^3+bd^2e)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(bc^2d^3+(bc^2de^2+be^3)x^4+\cdots}{\cdots}\right.$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(b*c^2*d^3 + b*d^2*e)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1
)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e
 + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*
x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 +
d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*
d^2*e + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(
e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4
*e^2 + d^3*e^3)*x^2), 1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e
 + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 -
 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*e*
x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 2*(b*c^2*d^3 + b*d^2*e)*sqrt(e*x^2 + d)
*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(a*c^2*d^3 + a*d^2
*e + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x
^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^
2 + d^3*e^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\,\operatorname{arsech}(cx) + a)x}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d
)^(5/2), x) - 1/3*a/((e*x^2 + d)^(3/2)*e)
```

Giac [F]

$$\int \frac{x(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\,\operatorname{arsech}(cx) + a)x}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x\left(a + b\,\operatorname{acosh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.171 $\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 20.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsech}\,(cx)}{x\,(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx=\int \frac{b\,\mathrm{arsech}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^
2*e*x^3 + d^3*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*acosh(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

3.172 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\text{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 24.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\operatorname{arcsech}\left(cx\right)}{x^3\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^
2*e*x^5 + d^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arsech}\,(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\,(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*acosh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.173 $\int \frac{x^6\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\operatorname{Int}\left(\frac{x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Defer[Int][(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 20.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^6(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^6(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Integrate[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^6(a+b\ \text{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^6(a+b\text{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^6*arcsech(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(x**6*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^6\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{(b\operatorname{arsech}(cx)+a)x^6}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 5.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^6(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^6\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((x^6*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^6*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.174 $\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\operatorname{Int}\left(\frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Defer[Int][(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^4\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\,\operatorname{arcsech}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^4*arcsech(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}}\, dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}}\, dx = \int \frac{(b\,\text{arsech}\,(cx) + a)x^4}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 4.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

```
[In] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^4*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.175 $\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 246

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}} + \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3de\left(c^2d+e\right)\sqrt{1+\frac{ex^2}{d}}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3cde\sqrt{d+ex^2}}$$

```
[Out] 1/3*x^3*(a+b*arcsech(c*x))/d/(e*x^2+d)^(3/2)-1/3*b*x*(1/(c*x+1))^(1/2)*(c*x
+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(1/2)-1/3*b*c*EllipticE(
c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d/e/(
c^2*d+e)/(1+e*x^2/d)^(1/2)+1/3*b*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1)
)^(1/2)*(c*x+1)^(1/2)*(1+e*x^2/d)^(1/2)/c/d/e/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

= {270, 6436, 12, 482, 434, 437, 435, 432, 430}

$$\int \frac{x^2\left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx = \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3cde\sqrt{d+ex^2}} - \frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3de\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}} - \frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d\,(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(d*(c^2*d +
 e)*Sqrt[d + e*x^2]) + (x^3*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) -
 (b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c
*x], -(e/(c^2*d))])/(3*d*e*(c^2*d + e)*Sqrt[1 + (e*x^2)/d]) + (b*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^
2*d))])/(3*c*d*e*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^2}{3d\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d}$$

$$= -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\,(c^2d+e)\,\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}\,dx}{3d\,(c^2d+e)}$$

$$= -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\,(c^2d+e)\,\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3de} - \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3de\,(c^2d+e)}$$

$$= -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\,(c^2d+e)\,\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}} - \frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{3de\,(c^2d+e)\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3de\sqrt{d+ex^2}}$$

$$= -\frac{bx\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d\,(c^2d+e)\,\sqrt{d+ex^2}} + \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}} - \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3de\,(c^2d+e)\sqrt{1+\frac{ex^2}{d}}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3cde\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.98

$$\int \frac{x^2\left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx = \frac{ax^3 - \frac{b\sqrt{\frac{1-cx}{1+cx}}(-cd+ex)(d+ex^2)}{e(c^2d+e)} + bx^3\operatorname{sech}^{-1}(cx) + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)\sqrt{\frac{c\left(\sqrt{d}+i\sqrt{e}x\right)}{\left(c\sqrt{d}+i\sqrt{e}\right)(1+cx)}}\sqrt{\frac{c(i\cdots}{(ic\sqrt{d}\cdots}}\cdots}{\cdots}}{\cdots}$$

```
[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (a*x^3 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*d) + e*x)*(d + e*x^2))/(e*(c^2*d
 + e)) + b*x^3*ArcSech[c*x] + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[(
```

```
c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(I*
Sqrt[d] + Sqrt[e]*x))/((I*c*Sqrt[d] + Sqrt[e])*(1 + c*x))]*(d + e*x^2)*((I*
c*Sqrt[d] + Sqrt[e])*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*S
qrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] -
I*Sqrt[e])^2] - 2*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/
((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[
d] - I*Sqrt[e])^2]))/(c*(c*Sqrt[d] + I*Sqrt[e])*e*Sqrt[((I*c*Sqrt[d] + Sqrt
[e])*(-1 + c*x))/(((-I)*c*Sqrt[d] + Sqrt[e])*(1 + c*x))]))/(3*d*(d + e*x^2)
^(3/2))
```

Maple [F]

$$\int \frac{x^2(a+b\operatorname{arcsech}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(166) = 332$.

Time = 0.10 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.36

$$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{(bc^3d^2e+bcde^2)\sqrt{ex^2+d}\,x^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left((ac^3d^2e+acde^2)x^3-(bc\right.}{}$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((b*c^3*d^2*e + b*c*d*e^2)*sqrt(e*x^2 + d)*x^3*log((c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) + 1)/(c*x)) + ((a*c^3*d^2*e + a*c*d*e^2)*x^3 - (b*c^2*d*e^2
*x^4 + b*c^2*d^2*e*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) - (
(b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -
e/(c^2*d)) - (b*c^4*d^3 + (b*c^4*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^4*d^
2*e + b*d*e^2)*x^2)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(d))/(c^3*d^5*
e + c*d^4*e^2 + (c^3*d^3*e^3 + c*d^2*e^4)*x^4 + 2*(c^3*d^4*e^2 + c*d^3*e^3)
*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}} \, dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^2}{\left(ex^2 + d\right)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x^
2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}} \, dx = \int \frac{(b \operatorname{arsech}(cx) + a)x^2}{\left(ex^2 + d\right)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \left(a + b\text{sech}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}} \, dx = \int \frac{x^2 \left(a + b \operatorname{acosh}\left(\frac{1}{c x}\right)\right)}{\left(e x^2 + d\right)^{5/2}} \, dx$$

```
[In] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^2*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.176 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 266

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2(c^2d+e)\sqrt{d+ex^2}} + \frac{x(a+b\operatorname{sech}^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\operatorname{sech}^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2(c^2d+e)\sqrt{1+\frac{ex^2}{d}}} + \frac{2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3cd^2\sqrt{d+ex^2}}$$

```
[Out] 1/3*x*(a+b*arcsech(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arcsech(c*x))/d^2/(e*
x^2+d)^(1/2)+1/3*b*e*x*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/d
^2/(c^2*d+e)/(e*x^2+d)^(1/2)+1/3*b*c*EllipticE(c*x,(-e/c^2/d)^(1/2))*(1/(c*
x+1))^(1/2)*(c*x+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*d+e)/(1+e*x^2/d)^(1/2)+2
/3*b*EllipticF(c*x,(-e/c^2/d)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(1+e*x
^2/d)^(1/2)/c/d^2/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {198, 197, 6426, 12, 541, 538, 437, 435, 432, 430}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3cd^2\sqrt{d+ex^2}}+\frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}}+\frac{bex\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*e*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d^2*(c^2*d
+ e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) +
(2*x*(a + b*ArcSech[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*c*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*
d^2*(c^2*d + e)*Sqrt[1 + (e*x^2)/d]) + (2*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c
*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*c*d^2*Sqrt
[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 6426

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x
] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt
[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ
```

```
[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\text{integral} = \frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{3d+2ex^2}{3d^2\sqrt{1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx$$

$$=\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{3d+2ex^2}{\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}\,dx}{3d^2}$$

$$=\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{-d(3c^2d+2e)-c^2dex^2}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^3\left(c^2d+e\right)}$$

$$=\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^2}+\frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3d^2\left(c^2d+e\right)}$$

$$=\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{\left(bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{3d^2\left(c^2d+e\right)\sqrt{1+\frac{ex^2}{d}}}+\frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^2\sqrt{d+ex^2}}$$

$$=\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\text{sech}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\text{sech}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2\left(c^2d+e\right)\sqrt{1+\frac{ex^2}{d}}}+\frac{2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3cd^2\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.79 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.94

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{\frac{b\sqrt{\frac{1-cx}{1+cx}}(-cd+ex)(d+ex^2)}{c^2d+e}+ax(3d+2ex^2)+bx(3d+2ex^2)\,\text{sech}^{-1}(cx)-\frac{ib\sqrt{\frac{1-cx}{1+cx}}(1+cx)\sqrt{}}{}}{}$$

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^(5/2),x]

[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*d) + e*x)*(d + e*x^2))/(c^2*d + e) + a*x *(3*d + 2*e*x^2) + b*x*(3*d + 2*e*x^2)*ArcSech[c*x] - (I*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*(d + e*x^2)*((c*Sqrt[d] - I*Sqrt[e])*EllipticE[I*ArcSinh[Sqrt[((c ^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] - 2*(3*c*Sqrt[d] + (2*I)*Sqrt[e])*E llipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2* (1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(c*(c* Sqrt[d] + I*Sqrt[e])*Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]))/(3*d^2*(d + e*x^2)^(3/2))

Maple [F]

$$\int \frac{a+b\,\text{arcsech}\,(cx)}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

[In] int((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)

[Out] int((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(182) = 364.

Time = 0.11 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.49

$$\int \frac{a+b\text{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{(2\,(bc^3d^2e+bcde^2)x^3+3\,(bc^3d^3+bcd^2e)x)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)+\left(2\,(ac\right.}{}$$

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")

```
[Out] 1/3*((2*(b*c^3*d^2*e + b*c*d*e^2)*x^3 + 3*(b*c^3*d^3 + b*c*d^2*e)*x)*sqrt(e
*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (2*(a*c^3*d
^2*e + a*c*d*e^2)*x^3 + 3*(a*c^3*d^3 + a*c*d^2*e)*x + (b*c^2*d*e^2*x^4 + b*
c^2*d^2*e*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d) + ((b*c^4*d*
e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -e/(c^2*d)
) - (((b*c^4 - 3*b*c^2)*d*e^2 - 2*b*e^3)*x^4 + (b*c^4 - 3*b*c^2)*d^3 - 2*b*
d^2*e + 2*((b*c^4 - 3*b*c^2)*d^2*e - 2*b*d*e^2)*x^2)*elliptic_f(arcsin(c*x)
, -e/(c^2*d)))*sqrt(d))/(c^3*d^6 + c*d^5*e + (c^3*d^4*e^2 + c*d^3*e^3)*x^4
+ 2*(c^3*d^5*e + c*d^4*e^2)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^{5/2}} \, dx = \text{Timed out}$$

```
[In] integrate((a+b*asech(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^{5/2}} \, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(l
og(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{a + b\text{sech}^{-1}(cx)}{(d + ex^2)^{5/2}} \, dx = \int \frac{b\,\text{arsech}\,(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(5/2),x)
[Out] int((a + b*acosh(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

3.177 $\int (fx)^m (d+ex^2)^3 (a+b\text{sech}^{-1}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 596

$$\int (fx)^m (d+ex^2)^3 (a+b\text{sech}^{-1}(cx))\,dx =$$

$$-\frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right.}{c^6f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}$$

$$-\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))(fx)^{3+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^4f^3(4+m)(5+m)(6+m)(7+m)}$$

$$-\frac{be^3(fx)^{5+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^5(6+m)(7+m)}$$

$$+\frac{d^3(fx)^{1+m}(a+b\text{sech}^{-1}(cx))}{f(1+m)}+\frac{3d^2e(fx)^{3+m}(a+b\text{sech}^{-1}(cx))}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}(a+b\text{sech}^{-1}(cx))}{f^5(5+m)}+\frac{e^3(fx)^{7+m}(a+b\text{sech}^{-1}(cx))}{f^7(7+m)}$$

$$+\frac{b\left(\frac{c^6d^3(2+m)(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{(3+m)(5+m)(7+m)}\right)(f}{c^6f(1+m)(2+m)(4+m)(6+m)}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arcsech(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arcsech
(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arcsech(c*x))/f^5/(5+m)+e^3*(f*x)
^(7+m)*(a+b*arcsech(c*x))/f^7/(7+m)+b*(c^6*d^3*(2+m)*(4+m)*(6+m)/(1+m)+e*(1
+m)*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m
^3+179*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*(f*x)^(1+m)*hypergeom([1/2, 1
/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^6/f/(1+m)/
(2+m)/(4+m)/(6+m)-b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3
*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*(f*x)^(1+m)*(1/(c*x+1))^(1/2)*(c*x
```

```
+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^6/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)-
b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*m+42))*(f*x)^(3+m)*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/f^3/(4+m)/(5+m)/(6+m)/(7+m)-b*e^3*(f*x)^(5+
m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/f^5/(6+m)/(7+m)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {276, 6436, 1823, 1281, 470, 371}

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\operatorname{sech}^{-1}(cx)\right) dx = \frac{d^3(fx)^{m+1}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(m+1)}$$
$$+\frac{3d^2e(fx)^{m+3}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^5(m+5)}$$
$$+\frac{e^3(fx)^{m+7}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^7(m+7)}-\frac{be^3\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}$$
$$-\frac{be^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(fx)^{m+3}\left(3c^2d(m^2+13m+42)+e(m+5)^2\right)}{c^4f^3(m+4)(m+5)(m+6)(m+7)}$$
$$-\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(fx)^{m+1}\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)+3c^2de(m+3)^2(m^2\right.}{c^6f(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)}$$
$$+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(fx)^{m+1}\left(\frac{e\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)+3c^2de(m+3)^2(m^2+13m+42)+e^2(m^2+8m+15)^2\right)}{c^6(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)}+\frac{d^3}{(m+\right.}{f}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*
c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*(f*x)^(1 + m)*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^6*f*(2 + m)*(3 + m)*(4 + m)*(5
+ m)*(6 + m)*(7 + m))) - (b*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*(
f*x)^(3 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^4*f^3
*(4 + m)*(5 + m)*(6 + m)*(7 + m)) - (b*e^3*(f*x)^(5 + m)*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^2*f^5*(6 + m)*(7 + m)) + (d^3*(f*x)^
(1 + m)*(a + b*ArcSech[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 + m)*(a + b*A
rcSech[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcSech[c*x]))/
(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcSech[c*x]))/(f^7*(7 + m)) + (b
*(d^3/(1 + m)^2 + (e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13
*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/(c^6*(2 + m)
*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(-1
)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/f
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
```

```
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{3d^2e(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(fx)^m\left(\frac{d^3}{1+m}+\frac{3d^2ex^2}{3+m}+\frac{3de^2x^4}{5+m}+\frac{e^3x^6}{7+m}\right)}{\sqrt{1-c^2x^2}}\,dx \\
&=-\frac{be^3(fx)^{5+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^5(6+m)(7+m)} \\
&\quad+\frac{d^3(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{3d^2e(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad-\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(fx)^m\left(-\frac{c^2d^3(6+m)}{1+m}-\frac{3c^2d^2e(6+m)x^2}{3+m}-\frac{e^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)x^4}{(5+m)(7+m)}\right)}{\sqrt{1-c^2x^2}}\,dx}{c^2(6+m)} \\
&=-\frac{be^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)(fx)^{3+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^4f^3(4+m)(5+m)(6+m)(7+m)} \\
&\quad-\frac{be^3(fx)^{5+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^5(6+m)(7+m)} \\
&\quad+\frac{d^3(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{3d^2e(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(fx)^m\left(\frac{c^4d^3(4+m)(6+m)}{1+m}+\frac{e\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638m+179m^2+\right.\right.}{(3+m)(5+m)(7+m)}}{\sqrt{1-c^2x^2}}}{c^4(4+m)(6+m)}
\end{aligned}$$

$$=$$

$$-\frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m}{c^6f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}$$

$$-\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))(fx)^{3+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^4f^3(4+m)(5+m)(6+m)(7+m)}$$

$$-\frac{be^3(fx)^{5+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^5(6+m)(7+m)}$$

$$+\frac{d^3(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{3d^2e(fx)^{3+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^7(7+m)}$$

$$+\frac{\left(b\left(\frac{c^4d^3(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^2(2+m)(3+m)(5+m)(7+m)}\right.\right.}{c^4(4+m)(6+m)}$$

$$=$$

$$-\frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m}{c^6f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}$$

$$-\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))(fx)^{3+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^4f^3(4+m)(5+m)(6+m)(7+m)}$$

$$-\frac{be^3(fx)^{5+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^5(6+m)(7+m)}$$

$$+\frac{d^3(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{3d^2e(fx)^{3+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^7(7+m)}$$

$$+\frac{b\left(\frac{d^3}{(1+m)^2}+\frac{e\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^6(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}\right)(fx)^{1+m}\sqrt{\frac{}{1+}}}{f}$$

Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.74

$$\int (fx)^m (d+ex^2)^3 (a+b\operatorname{sech}^{-1}(cx))\,dx$$
$$= x(fx)^m \left(\frac{ad^3}{1+m}+\frac{3ad^2ex^2}{3+m}+\frac{3ade^2x^4}{5+m}+\frac{ae^3x^6}{7+m}+\frac{bd^3\operatorname{sech}^{-1}(cx)}{1+m}+\frac{3bd^2ex^2\operatorname{sech}^{-1}(cx)}{3+m}+\frac{3bde^2x^4\operatorname{sech}^{-1}(cx)}{5+m}+\frac{be^3x^6\operatorname{sech}^{-1}(cx)}{7+m} - \frac{bd^3\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2(-1+cx)} - \frac{3bd^2ex^2\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2(-1+cx)} - \frac{3bde^2x^4\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2(-1+cx)} - \frac{be^3x^6\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7+m}{2},\frac{9+m}{2},c^2x^2\right)}{(7+m)^2(-1+cx)}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSech[c*x]),x]
```

```
[Out] x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5 +
 m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcSech[c*x])/(1 + m) + (3*b*d^2*e*x^2*A
rcSech[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcSech[c*x])/(5 + m) + (b*e^3*x^6*Ar
cSech[c*x])/(7 + m) - (b*d^3*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hy
pergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*(-1 + c*x))
 - (3*b*d^2*e*x^2*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometri
c2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/((3 + m)^2*(-1 + c*x)) - (3*b*d*e
^2*x^4*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (
5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^2*(-1 + c*x)) - (b*e^3*x^6*Sqrt[(1
- c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (7 + m)/2, (9 +
m)/2, c^2*x^2])/((7 + m)^2*(-1 + c*x)))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 (a+b\ \mathrm{arcsech}\,(cx))\,dx$$

[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x)

[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x)

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^3 (b\,\mathrm{arsech}\,(cx)+a)(fx)^m\,dx$$

[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsech(c*x))*(f*x)^m, x)

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx = \int (fx)^m (a+b\,\mathrm{asech}\,(cx)) \left(d+ex^2\right)^3\,dx$$

[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*asech(c*x)),x)

[Out] Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2)**3, x)

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\mathrm{sech}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^3 (b\,\mathrm{arsech}\,(cx)+a)(fx)^m\,dx$$

[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((m^3 + 9*m^2 + 23*m + 15)*b*e^3*f^m*x^7*x^m + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2*f^m*x^5*x^m + 3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3*x^m + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*f^m*x*x^m)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - ((m^3 + 9*m^2 + 23*m + 15)*b*e^3*f^m*x^7*x^m + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2*f^m*x^5*x^m + 3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3*x^m + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*f^m*x*x^m)*log(x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)

```
- integrate((b*c^2*e^3*f^m*(m + 7)*x^2*log(c) - (e^3*f^m*(m + 7)*log(c) -
e^3*f^m)*b)*x^6*x^m/(c^2*(m + 7)*x^2 - m - 7), x) - integrate(3*(b*c^2*d*e^
2*f^m*(m + 5)*x^2*log(c) - (d*e^2*f^m*(m + 5)*log(c) - d*e^2*f^m)*b)*x^4*x^
m/(c^2*(m + 5)*x^2 - m - 5), x) - integrate(3*(b*c^2*d^2*e*f^m*(m + 3)*x^2*
log(c) - (d^2*e*f^m*(m + 3)*log(c) - d^2*e*f^m)*b)*x^2*x^m/(c^2*(m + 3)*x^2
 - m - 3), x) - integrate((b*c^2*d^3*f^m*(m + 1)*x^2*log(c) - (d^3*f^m*(m +
 1)*log(c) - d^3*f^m)*b)*x^m/(c^2*(m + 1)*x^2 - m - 1), x) + integrate(((m^
3 + 9*m^2 + 23*m + 15)*b*c^2*e^3*f^m*x^8*x^m + 3*(m^3 + 11*m^2 + 31*m + 21)
*b*c^2*d*e^2*f^m*x^6*x^m + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^2*e*f^m*x^4
*x^m + (m^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*f^m*x^2*x^m)/((m^4 + 16*m^3 +
86*m^2 + 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 + ((m^4 + 16*m^3 + 86*m^2 + 17
6*m + 105)*c^2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105)*sqrt(c*x + 1)*sqr
t(-c*x + 1) - 86*m^2 - 176*m - 105), x)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b\,\text{arsech}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (f\,x)^m\,\left(e\,x^2+d\right)^3\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

```
[In] int((f*x)^m*(d + e*x^2)^3*(a + b*acosh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^3*(a + b*acosh(1/(c*x))), x)
```

3.178 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 372

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx = -\frac{be(e(3+m)^2+2c^2d(20+9m+m^2))\,(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^4f(2+m)(3+m)(4+m)(5+m)} - \frac{be^2(fx)^{3+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f^3(4+m)(5+m)} + \frac{d^2(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(5+m)} + \frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))\,(fx)^{1+m}\sqrt{\frac{1}{1+cx}}}{c^4f(1+m)^2(2+m)(3+m)(4+m)(5+m)}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arcsech(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arcsech(c
*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arcsech(c*x))/f^5/(5+m)+b*(c^4*d^2*(2+m
)*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*(f*x)^(1+m)
*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^
(1/2)/c^4/f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)-b*e*(e*(3+m)^2+2*c^2*d*(m^2+9*m
+20))*(f*x)^(1+m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/f/
(4+m)/(5+m)/(m^2+5*m+6)-b*e^2*(f*x)^(3+m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(
-c^2*x^2+1)^(1/2)/c^2/f^3/(4+m)/(5+m)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {276, 6436, 12, 1281, 470, 371}

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$
$$= \frac{d^2(fx)^{m+1}\left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}$$
$$+\frac{e^2(fx)^{m+5}\left(a+b\text{sech}^{-1}(cx)\right)}{f^5(m+5)}-\frac{be^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(fx)^{m+3}}{c^2f^3(m+4)(m+5)}$$
$$+\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(fx)^{m+1}\left(\frac{e(2c^2d(m^2+9m+20)+e(m+3)^2)}{c^4(m+2)(m+3)(m+4)(m+5)}+\frac{d^2}{(m+1)^2}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f}$$
$$-\frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(fx)^{m+1}\left(2c^2d(m^2+9m+20)+e(m+3)^2\right)}{c^4f(m+2)(m+3)(m+4)(m+5)}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2))*(f*x)^(1 + m)*Sqrt[(1 + c*x
)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^4*f*(2 + m)*(3 + m)*(4 + m)*(5
+ m))) - (b*e^2*(f*x)^(3 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c
^2*x^2])/(c^2*f^3*(4 + m)*(5 + m)) + (d^2*(f*x)^(1 + m)*(a + b*ArcSech[c*x]
))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a + b*ArcSech[c*x]))/(f^3*(3 + m)) +
 (e^2*(f*x)^(5 + m)*(a + b*ArcSech[c*x]))/(f^5*(5 + m)) + (b*(d^2/(1 + m)^2
 + (e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2)))/(c^4*(2 + m)*(3 + m)*(4 + m
)*(5 + m)))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric
2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d^2(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(1+m)} \\ &\quad + \frac{2de(fx)^{3+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^5(5+m)} \\ &\quad + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(d^2(15+8m+m^2)+2de(5+6m+m^2)\,x^2+e^2(3+4m+m^2)\,x^4\right)}{(1+m)(3+m)(5+m)\sqrt{1-c^2x^2}}\,dx\end{aligned}$$

$$= \frac{d^2 (fx)^{1+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2 (fx)^{5+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f^5(5+m)} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(fx)^m (d^2(15+8m+m^2)+2de(5+6m+m^2)x^2+e^2(3+4m+m^2)x^4)}{\sqrt{1-c^2x^2}} dx}{15+23m+9m^2+m^3}$$

$$= -\frac{be^2 (fx)^{3+m} \sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2 f^3 (4+m)(5+m)} + \frac{d^2 (fx)^{1+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2 (fx)^{5+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f^5(5+m)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(fx)^m (-c^2d^2(3+m)(4+m)(5+m)-e(1+m)(e(3+m)^2+2c^2d(20+9m+m^2))x^2)}{\sqrt{1-c^2x^2}} dx}{c^2(4+m)\left(15+23m+9m^2+m^3\right)}$$

$$= -\frac{be(e(3+m)^2+2c^2d(20+9m+m^2))(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^4 f(2+m)(4+m)\left(15+8m+m^2\right)} - \frac{be^2 (fx)^{3+m} \sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2 f^3 (4+m)(5+m)} + \frac{d^2 (fx)^{1+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2 (fx)^{5+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f^5(5+m)} + -\frac{\left(b(-c^4d^2(2+m)(3+m)(4+m)(5+m) - e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)\right)\sqrt{1}}{c^4(2+m)(4+m)\left(15+23m+9m^2+m^3\right)}$$

$$= -\frac{be(e(3+m)^2+2c^2d(20+9m+m^2))(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^4 f(2+m)(4+m)\left(15+8m+m^2\right)} - \frac{be^2 (fx)^{3+m} \sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2 f^3 (4+m)(5+m)} + \frac{d^2 (fx)^{1+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2 (fx)^{5+m} \left(a + b\text{sech}^{-1}(cx)\right)}{f^5(5+m)} + \frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m) + e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))(fx)^{1+}}{c^4 f(1+m)(2+m)(4+m)\left(15+23m+9m^2\right.}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.87

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$
$$= x(fx)^m \left(\frac{ad^2}{1+m}+\frac{2adex^2}{3+m}+\frac{ae^2x^4}{5+m}+\frac{bd^2\text{sech}^{-1}(cx)}{1+m}+\frac{2bdex^2\text{sech}^{-1}(cx)}{3+m} + \frac{be^2x^4\text{sech}^{-1}(cx)}{5+m}-\frac{bd^2\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2(-1+cx)} - \frac{2bdex^2\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2(-1+cx)} - \frac{be^2x^4\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2(-1+cx)}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
(b*d^2*ArcSech[c*x])/(1 + m) + (2*b*d*e*x^2*ArcSech[c*x])/(3 + m) + (b*e^2*
x^4*ArcSech[c*x])/(5 + m) - (b*d^2*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x
^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*(-1 +
 c*x)) - (2*b*d*e*x^2*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeom
etric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/((3 + m)^2*(-1 + c*x)) - (b*e
^2*x^4*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (
5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^2*(-1 + c*x)))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 \left(a+b\ \text{arcsech}\left(cx\right)\right) dx$$

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^2 (b\,\text{arsech}\,(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arcsech(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (fx)^m \left(a+b\,\text{asech}\,(cx)\right)\left(d+ex^2\right)^2\,dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*asech(c*x)),x)
[Out] Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2)**2, x)
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^2 (b\,\text{arsech}\,(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d
^2/(f*(m + 1)) + (((m^2 + 4*m + 3)*b*e^2*f^m*x^5*x^m + 2*(m^2 + 6*m + 5)*b*
d*e*f^m*x^3*x^m + (m^2 + 8*m + 15)*b*d^2*f^m*x*x^m)*log(sqrt(c*x + 1)*sqrt(
-c*x + 1) + 1) - ((m^2 + 4*m + 3)*b*e^2*f^m*x^5*x^m + 2*(m^2 + 6*m + 5)*b*d
*e*f^m*x^3*x^m + (m^2 + 8*m + 15)*b*d^2*f^m*x*x^m)*log(x))/(m^3 + 9*m^2 + 2
3*m + 15) - integrate((b*c^2*e^2*f^m*(m + 5)*x^2*log(c) - (e^2*f^m*(m + 5)*
log(c) - e^2*f^m)*b)*x^4*x^m/(c^2*(m + 5)*x^2 - m - 5), x) - integrate(2*(b
*c^2*d*e*f^m*(m + 3)*x^2*log(c) - (d*e*f^m*(m + 3)*log(c) - d*e*f^m)*b)*x^2
*x^m/(c^2*(m + 3)*x^2 - m - 3), x) - integrate((b*c^2*d^2*f^m*(m + 1)*x^2*l
og(c) - (d^2*f^m*(m + 1)*log(c) - d^2*f^m)*b)*x^m/(c^2*(m + 1)*x^2 - m - 1)
, x) + integrate(((m^2 + 4*m + 3)*b*c^2*e^2*f^m*x^6*x^m + 2*(m^2 + 6*m + 5)
*b*c^2*d*e*f^m*x^4*x^m + (m^2 + 8*m + 15)*b*c^2*d^2*f^m*x^2*x^m)/((m^3 + 9*
m^2 + 23*m + 15)*c^2*x^2 - m^3 + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 -
 9*m^2 - 23*m - 15)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 9*m^2 - 23*m - 15), x)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2 (b\,\operatorname{arsech}(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx = \int (f\,x)^m \left(e\,x^2+d\right)^2 \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)^2*(a + b*acosh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^2*(a + b*acosh(1/(c*x))), x)
```

3.179 $\int (fx)^m (d+ex^2) \left(a+b\text{sech}^{-1}(cx)\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 206

$$\int (fx)^m (d+ex^2) \left(a+b\text{sech}^{-1}(cx)\right) dx = -\frac{be(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f(2+m)(3+m)} + \frac{d(fx)^{1+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\text{sech}^{-1}(cx)\right)}{f^3(3+m)} + \frac{b(e(1+m)^2+c^2d(2+m)(3+m))(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{c^2f(1+m)^2(2+m)(3+m)}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arcsech(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arcsech(c*x))/f
^3/(3+m)+b*(e*(1+m)^2+c^2*d*(2+m)*(3+m))*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/
2*m],[3/2+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^2/f/(1+m)^2/(2+
m)/(3+m)-b*e*(f*x)^(1+m)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)
/c^2/f/(2+m)/(3+m)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {14, 6436, 12, 470, 371}

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$
$$= \frac{d(fx)^{m+1} \left(a+b\text{sech}^{-1}(cx)\right)}{f(m+1)} + \frac{e(fx)^{m+3} \left(a+b\text{sech}^{-1}(cx)\right)}{f^3(m+3)}$$
$$+ \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(fx)^{m+1} \left(\frac{e}{c^2(m+2)(m+3)}+\frac{d}{(m+1)^2}\right) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f}$$
$$- \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(fx)^{m+1}}{c^2f(m+2)(m+3)}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*e*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/
(c^2*f*(2 + m)*(3 + m))) + (d*(f*x)^(1 + m)*(a + b*ArcSech[c*x]))/(f*(1 + m
)) + (e*(f*x)^(3 + m)*(a + b*ArcSech[c*x]))/(f^3*(3 + m)) + (b*(d/(1 + m)^2
 + e/(c^2*(2 + m)*(3 + m)))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x
]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6436

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x]] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p +
 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(fx)^m\left(d(3+m)+e(1+m)x^2\right)}{(1+m)(3+m)\sqrt{1-c^2x^2}}\,dx \\
&= \frac{d(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(fx)^m\left(d(3+m)+e(1+m)x^2\right)}{\sqrt{1-c^2x^2}}\,dx}{3+4m+m^2} \\
&= -\frac{be(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f(2+m)(3+m)} \\
&\quad+\frac{d(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{\left(b\left(\frac{e(1+m)^2}{c^2(2+m)}+d(3+m)\right)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(fx)^m}{\sqrt{1-c^2x^2}}\,dx}{3+4m+m^2} \\
&= -\frac{be(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{c^2f(2+m)(3+m)} \\
&\quad+\frac{d(fx)^{1+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{b\left(\frac{e(1+m)^2}{c^2(2+m)}+d(3+m)\right)(fx)^{1+m}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f(1+m)\left(3+4m+m^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\operatorname{sech}^{-1}(cx)\right) \, dx$$
$$= x(fx)^m \left(-\frac{bd\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2(-1+cx)} + \frac{\frac{(3+m)\left(d(3+m)+e(1+m)x^2\right)\left(a+b\operatorname{sech}^{-1}(cx)\right)}{1+m} - \frac{bex^2\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{-1+cx}}{(3+m)^2}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] x*(f*x)^m*(-((b*d*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometri
c2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*(-1 + c*x))) + (((3 +
m)*(d*(3 + m) + e*(1 + m)*x^2)*(a + b*ArcSech[c*x]))/(1 + m) - (b*e*x^2*Sqr
t[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (3 + m)/2,
(5 + m)/2, c^2*x^2])/(-1 + c*x))/(3 + m)^2)
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right) \left(a+b\,\operatorname{arcsech}\left(cx\right)\right) dx$$

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\operatorname{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)\left(b\operatorname{arsech}\left(cx\right)+a\right)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m (d+ex^2) (a+b\text{sech}^{-1}(cx)) \, dx = \int (fx)^m (a+b\,\text{asech}\,(cx)) (d+ex^2) \, dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*asech(c*x)),x)
[Out] Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int (fx)^m (d+ex^2) (a+b\text{sech}^{-1}(cx)) \, dx = \int (ex^2+d)(b\,\text{arsech}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + ((b*e*f^m*(m + 1)
*x^3*x^m + b*d*f^m*(m + 3)*x*x^m)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - (
b*e*f^m*(m + 1)*x^3*x^m + b*d*f^m*(m + 3)*x*x^m)*log(x))/(m^2 + 4*m + 3) -
integrate((b*c^2*e*f^m*(m + 3)*x^2*log(c) - (e*f^m*(m + 3)*log(c) - e*f^m)*
b)*x^2*x^m/(c^2*(m + 3)*x^2 - m - 3), x) - integrate((b*c^2*d*f^m*(m + 1)*x
^2*log(c) - (d*f^m*(m + 1)*log(c) - d*f^m)*b)*x^m/(c^2*(m + 1)*x^2 - m - 1)
, x) + integrate((b*c^2*e*f^m*(m + 1)*x^4*x^m + b*c^2*d*f^m*(m + 3)*x^2*x^m
)/((m^2 + 4*m + 3)*c^2*x^2 + ((m^2 + 4*m + 3)*c^2*x^2 - m^2 - 4*m - 3)*sqrt
(c*x + 1)*sqrt(-c*x + 1) - m^2 - 4*m - 3), x)
```

Giac [F]

$$\int (fx)^m (d+ex^2) (a+b\text{sech}^{-1}(cx)) \, dx = \int (ex^2+d)(b\,\text{arsech}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\operatorname{sech}^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right) \left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*acosh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*acosh(1/(c*x))), x)
```

3.180 $\int \frac{(fx)^m\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\,\operatorname{arcsech}\left(cx\right)\right)}{e\,x^2+d}dx$$

```
[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x)
[Out] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(b\,\text{arsech}\left(cx\right)+a)(fx)^m}{ex^2+d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 8.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m \left(a+b\,\text{asech}\left(cx\right)\right)}{d+ex^2}\,dx$$

```
[In] integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d),x)
[Out] Integral((f*x)**m*(a + b*asech(c*x))/(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 3.92 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{{\left(f\,x\right)}^m\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

```
[In] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2),x)
[Out] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2), x)
```

3.181 $\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\text{Int}\left(\frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2} \, dx = \int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2} \, dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b \operatorname{arcsech}(cx))}{(e\,x^2+d)^2}dx$$

```
[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
[Out] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2} \, dx = \int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{(ex^2+d)^2} \, dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{(d+ex^2)^2} \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\text{arsech}\left(cx\right)+a)(fx)^m}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\text{arsech}\left(cx\right)+a)(fx)^m}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{{\left(f\,x\right)}^m\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

```
[In] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.182 $\int (fx)^m (d+ex^2)^{3/2} (a+b\text{sech}^{-1}(cx)) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \text{Int}\Big((fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right),x\Big)$$

```
[Out] Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

```
[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Defer[Int][(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\text{integral} = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^{m} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\operatorname{arcsech}\left(cx\right)\right)dx$$

```
[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\int { {\left(ex^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsech}\left(cx\right) + a\right)} \left(fx\right)^{m} \,d x }$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)*(f*
x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx=\text{Timed out}$$

```
[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\text{arsech}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 4.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{sech}^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right)^{3/2} \left(a+b\,\text{acosh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acosh(1/(c*x))), x)
```

3.183 $\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right) dx = \text{Int}\left((fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right), x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right) dx = \int (fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right) dx$$

```
[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] Defer[Int][(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

Rubi steps

$$\text{integral} = \int (fx)^m \sqrt{d+ex^2} \left(a+b\text{sech}^{-1}(cx)\right) dx$$

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx$$

```
[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m\left(a+b\ \text{arcsech}\left(cx\right)\right)\sqrt{e\,x^2+d}dx$$

```
[In] int((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arsech}\left(cx\right)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 8.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{sech}^{-1}(cx)\right)\,dx=\int (fx)^m\left(a+b\,\text{asech}\left(cx\right)\right)\sqrt{d+ex^2}\,dx$$

```
[In] integrate((f*x)**m*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\operatorname{arsech}(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\operatorname{arsech}(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\operatorname{sech}^{-1}(cx)\right)\,dx=\int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acosh(1/(c*x))), x)
```

3.184 $\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \left(a+b\,\mathrm{arcsech}\left(cx\right)\right)}{\sqrt{e\,x^2+d}}dx$$

```
[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arsech}\left(cx\right)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m \left(a+b\,\mathrm{asech}\left(cx\right)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*asech(c*x))/sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.185 $\int \frac{(fx)^m\left(a+b\,\mathbf{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\left(a+b\,\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(fx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2),x]

[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Maple [N/A] (verified)

Not integrable

Time = 1.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m (a+b\ \mathrm{arcsech}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)

[Out] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arsech}\,(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Sympy [N/A]

Not integrable

Time = 22.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \int \frac{(fx)^m \left(a+b\operatorname{asech}(cx)\right)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral((f*x)**m*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsech}(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\operatorname{sech}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arsech}(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a+b\text{sech}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{{\left(f\,x\right)}^m\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

```
[In] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int(((f*x)^m*(a + b*acosh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.186 $\int \frac{x^{11}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 473

$$\begin{aligned}\int \frac{x^{11}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = &-\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}+\frac{7b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}\\&-\frac{13b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/2}}{150c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}+\frac{3b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{7/2}}{70c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}\\&-\frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{9/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^{12}}\\&+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3c^{12}}\\&-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{10c^{12}}\\&+\frac{4b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}\end{aligned}$$

```
[Out] 1/3*(-c^4*x^4+1)^(3/2)*(a+b*arcsech(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b
*arcsech(c*x))/c^12+7/90*b*(c^2*x^2+1)^(3/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(-1+
1/c/x)^(1/2)/(1+1/c/x)^(1/2)-13/150*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+1)^(1/2)/
c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+3/70*b*(c^2*x^2+1)^(7/2)*(-c^2*x^2+
1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/90*b*(c^2*x^2+1)^(9/2)*(
-c^2*x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+4/15*b*arctanh((c
^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)
-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(-1+1/c/x)^(1/2)/(1+1/c
/x)^(1/2)-1/2*(a+b*arcsech(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {272, 45, 6444, 12, 6874, 862, 52, 65, 214, 797}

$$\int \frac{x^{11}(a+b\text{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{(1-c^4x^4)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}} + \frac{(1-c^4x^4)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} - \frac{\sqrt{1-c^4x^4}(a+b\text{sech}^{-1}(cx))}{2c^{12}} + \frac{4b\sqrt{1-c^2x^2}\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{15c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{b\sqrt{1-c^2x^2}(c^2x^2+1)^{9/2}}{90c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} + \frac{3b\sqrt{1-c^2x^2}(c^2x^2+1)^{7/2}}{70c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{13b\sqrt{1-c^2x^2}(c^2x^2+1)^{5/2}}{150c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} + \frac{7b\sqrt{1-c^2x^2}(c^2x^2+1)^{3/2}}{90c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{15c^{13}x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

```
[In] Int[(x^11*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]

[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt
[1 + 1/(c*x)]*x) + (7*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(90*c^13*Sqr
t[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (13*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2
)^(5/2))/(150*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) + (3*b*Sqrt[1 -
c^2*x^2]*(1 + c^2*x^2)^(7/2))/(70*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]
*x) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(9/2))/(90*c^13*Sqrt[-1 + 1/(c*x)]
*Sqrt[1 + 1/(c*x)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcSech[c*x]))/(2*c^12) +
 ((1 - c^4*x^4)^(3/2)*(a + b*ArcSech[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)
*(a + b*ArcSech[c*x]))/(10*c^12) + (4*b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 +
c^2*x^2]])/(15*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6444

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcSech[c*x], v, x] + Dist[b*(Sqrt[1 - c^2*x^2]/(c*x*S
qrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])), Int[SimplifyIntegrand[v/(x*Sqrt[1 -
c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} \\
&\quad -\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}\left(-8-4c^4x^4-3c^8x^8\right)}{30c^{12}x\sqrt{1-c^2x^2}}\,dx}{c\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} \\
&\quad -\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}\left(-8-4c^4x^4-3c^8x^8\right)}{x\sqrt{1-c^2x^2}}\,dx}{30c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} \\
&\quad -\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}} \\
&\quad -\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}\left(8+4c^4x^2+3c^8x^4\right)}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{60c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} \\
&= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{3c^{12}} \\
&\quad -\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{sech}^{-1}(cx)\right)}{10c^{12}} \\
&\quad -\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{8\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}+\frac{4c^4x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}+\frac{3c^8x^3\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\right)dx,x,x^2\right)}{60c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}
\end{aligned}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3c^{12}} -\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{10c^{12}}-\frac{\left(2b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} -\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\frac{x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\,dx,x,x^2\right)}{15c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} -\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\frac{x^3\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\,dx,x,x^2\right)}{20c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3c^{12}} -\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{10c^{12}}-\frac{\left(2b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+c^2x}}{x}\,dx,x,x^2\right)}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} -\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int x\sqrt{1+c^2x}\,dx,x,x^2\right)}{15c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} -\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int x^3\sqrt{1+c^2x}\,dx,x,x^2\right)}{20c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^{12}} +\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{3c^{12}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{10c^{12}} -\frac{\left(2b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} -\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\left(-\frac{\sqrt{1+c^2x}}{c^2}+\frac{(1+c^2x)^{3/2}}{c^2}\right)dx,x,x^2\right)}{15c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} -\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\left(-\frac{\sqrt{1+c^2x}}{c^6}+\frac{3(1+c^2x)^{3/2}}{c^6}-\frac{3(1+c^2x)^{5/2}}{c^6}+\frac{(1+c^2x)^{7/2}}{c^6}\right)dx,x,x^2\right)}{20c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$
\begin{aligned}
&= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} \\
&\quad - \frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} \\
&\quad - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\,\text{sech}^{-1}(cx)\right)}{2c^{12}} \\
&\quad + \frac{(1-c^4x^4)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{10c^{12}} \\
&\quad - \frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{15c^{15}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} \\
&\quad - \frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} \\
&\quad - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\,\text{sech}^{-1}(cx)\right)}{2c^{12}} \\
&\quad + \frac{(1-c^4x^4)^{3/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}\left(a+b\,\text{sech}^{-1}(cx)\right)}{10c^{12}} \\
&\quad + \frac{4b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.45

$$
\int \frac{x^{11}\left(a+b\,\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \frac{-105a\sqrt{1-c^4x^4}(8+4c^4x^4+3c^8x^8) + \frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^4x^4}(768+36c^2x^2+78c^4x^4+5c^6x^6+35c^8x^8)}{-1+cx} - 105b\sqrt{1-c^4x^4}(8+}{3150c^{12}}
$$

```
[In] Integrate[(x^11*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*Sqrt[(1 - c*x)/(
1 + c*x)]*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x^6 + 35
```

```
*c^8*x^8))/(-1 + c*x) - 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8)
*ArcSech[c*x] + 840*b*Log[x*(1 - c*x)] - 840*b*Log[1 - c*x - Sqrt[(1 - c*x)
/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/(3150*c^12)
```

Maple [F]

$$\int \frac{x^{11}(a + b\ \mathrm{arcsech}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

```
[In] int(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
[Out] int(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}\left(a + b\mathrm{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{105\,(3\,bc^{10}x^{10} - 3\,bc^8x^8 + 4\,bc^6x^6 - 4\,bc^4x^4 + 8\,bc^2x^2 - 8\,b)\sqrt{-c^4x^4+1}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - (35\,bc^9x^9}{}$$

```
[In] integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/3150*(105*(3*b*c^10*x^10 - 3*b*c^8*x^8 + 4*b*c^6*x^6 - 4*b*c^4*x^4 + 8*b
*c^2*x^2 - 8*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
+ 1)/(c*x)) - (35*b*c^9*x^9 + 5*b*c^7*x^7 + 78*b*c^5*x^5 + 36*b*c^3*x^3 + 7
68*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 420*(b*c^2*x^
2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))
 - 1)/(c^2*x^2 - 1)) - 420*(b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 +
1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + 105*(3*a*c^10*x
^10 - 3*a*c^8*x^8 + 4*a*c^6*x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 - 8*a)*sqrt(-c^
4*x^4 + 1))/(c^14*x^2 - c^12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\text{Timed out}$$

```
[In] integrate(x**11*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{11}\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\int \frac{(b\,\text{arsech}\,(cx)+a)x^{11}}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sq
rt(-c^4*x^4 + 1)/c^12) + 1/30*b*((3*c^12*x^12 + c^8*x^8 + 4*c^4*x^4 - 8)*lo
g(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-
c*x + 1)*c^12) - 30*integrate(1/30*(30*c^10*x^21*log(c) + 60*c^10*x^21*log(
sqrt(x)) + (60*c^10*x^21*log(sqrt(x)) + (3*c^10*x^10*(10*log(c) + 1) + 3*c^
8*x^8 + 4*c^6*x^6 + 4*c^4*x^4 + 8*c^2*x^2 + 8)*x^11)*e^(1/2*log(c*x + 1) +
1/2*log(-c*x + 1)))/((c^10*x^10*e^(log(c*x + 1) + log(-c*x + 1)) + c^10*x^1
0*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^{11}\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^11*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^11*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.187 $\int \frac{x^7\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 316

$$\begin{aligned}\int \frac{x^7\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = &-\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}+\frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{18c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}\\&-\frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/2}}{30c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8}\\&+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8}\\&+\frac{b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}\end{aligned}$$

```
[Out] 1/6*(-c^4*x^4+1)^(3/2)*(a+b*arcsech(c*x))/c^8+1/18*b*(c^2*x^2+1)^(3/2)*(-c^
2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)-1/30*b*(c^2*x^2+1)^(5
/2)*(-c^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/2)+1/3*b*arctanh
((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c/x)^(1/
2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^9/x/(-1+1/c/x)^(1/2)/(1+1/c
/x)^(1/2)-1/2*(a+b*arcsech(c*x))*(-c^4*x^4+1)^(1/2)/c^8
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {272, 45, 6444, 12, 6874, 862, 52, 65, 214, 797}

$$\int \frac{x^7\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^8}+\frac{b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{c^2x^2+1}\right)}{3c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right)^{5/2}}{30c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}+\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right)^{3/2}}{18c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}-\frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{3c^9x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

```
[In] Int[(x^7*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^9*Sqrt[-1 + 1/(c*x)]*Sqrt[1
 + 1/(c*x)]*x) + (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(18*c^9*Sqrt[-1
+ 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))
/(30*c^9*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (Sqrt[1 - c^4*x^4]*(a +
b*ArcSech[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2)*(a + b*ArcSech[c*x]))/(6*c^
8) + (b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(3*c^9*Sqrt[-1 + 1/(c
*x)]*Sqrt[1 + 1/(c*x)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6444

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcSech[c*x], v, x] + Dist[b*(Sqrt[1 - c^2*x^2]/(c*x*S
qrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])), Int[SimplifyIntegrand[v/(x*Sqrt[1 -
c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{6c^8x\sqrt{1-c^2x^2}}\,dx}{c\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}\,dx}{6c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}\left(2+c^4x^2\right)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{12c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{2\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}} + \frac{c^4x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\right)dx, x, x^2\right)}{12c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{6c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\,dx, x, x^2\right)}{12c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx, x, x^2\right)}{6c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int x\sqrt{1+c^2x}\,dx, x, x^2\right)}{12c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{6c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int \left(-\frac{\sqrt{1+c^2x}}{c^2}+\frac{(1+c^2x)^{3/2}}{c^2}\right)dx,x,x^2\right)}{12c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{18c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/2}}{30c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{3c^{11}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} + \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{18c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/2}}{30c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{6c^8} + \frac{b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.56

$$\int \frac{x^7\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$$

$$= \frac{-15a\sqrt{1-c^4x^4}\left(2+c^4x^4\right)+\frac{b\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^4x^4}\left(28+c^2x^2+3c^4x^4\right)}{-1+cx}-15b\sqrt{1-c^4x^4}\left(2+c^4x^4\right)\operatorname{sech}^{-1}(cx)+30b\log(}{90c^8}$$

```
[In] Integrate[(x^7*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
[Out] (-15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[
1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c*x) - 15*b*Sqrt[1 - c^4*x^4
]*(2 + c^4*x^4)*ArcSech[c*x] + 30*b*Log[x*(1 - c*x)] - 30*b*Log[1 - c*x - S
qrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/(90*c^8)
```

Maple [F]

$$\int \frac{x^7(a + b\ \mathrm{arcsech}\,(cx))}{\sqrt{-c^4x^4 + 1}}dx$$

[In] int(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)

[Out] int(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.06

$$\int \frac{x^7\left(a + b\mathrm{sech}^{-1}(cx)\right)}{\sqrt{1 - c^4x^4}}\,dx = -\frac{15\,(bc^6x^6 - bc^4x^4 + 2\,bc^2x^2 - 2\,b)\sqrt{-c^4x^4 + 1}\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - (3\,bc^5x^5 + bc^3x^3 + 28\,bcx)\sqrt{-c^4x^4 + \cdots}}{}$$

[In] integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")

[Out] -1/90*(15*(b*c^6*x^6 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*b)*sqrt(-c^4*x^4 + 1)*lo
g((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (3*b*c^5*x^5 + b*c^3*x^
3 + 28*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 15*(b*c^2
*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^
2)) - 1)/(c^2*x^2 - 1)) - 15*(b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(-c^4*x^4
+ 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + 15*(a*c^6*x^6
 - a*c^4*x^4 + 2*a*c^2*x^2 - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c^8)

Sympy [F]

$$\int \frac{x^7\left(a + b\mathrm{sech}^{-1}(cx)\right)}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{x^7(a + b\,\mathrm{asech}\,(cx))}{\sqrt{-\left(cx - 1\right)\left(cx + 1\right)\left(c^2x^2 + 1\right)}}\,dx$$

[In] integrate(x**7*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)

[Out] Integral(x**7*(a + b*asech(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)

Maxima [F]

$$\int \frac{x^7(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arsech}\,(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

[In] integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")

[Out] 1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*b*((c^8*x^8 + c^4*x^4 - 2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8) - 6*integrate(1/6*(6*c^6*x^13*log(c) + 12*c^6*x^13*log(sqrt(x)) + (12*c^6*x^13*log(sqrt(x)) + (c^6*x^6*(6*log(c) + 1) + c^4*x^4 + 2*c^2*x^2 + 2)*x^7)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^6*x^6*e^(log(c*x + 1) + log(-c*x + 1)) + c^6*x^6*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x))

Giac [F(-2)]

Exception generated.

$$\int \frac{x^7(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^7\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

[In] int((x^7*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)

[Out] int((x^7*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)

3.188 $\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 159

$$\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x} - \frac{\sqrt{1-c^4x^4}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{2c^4} + \frac{b\sqrt{1-c^2x^2}\,\mathrm{arctanh}\left(\sqrt{1+c^2x^2}\right)}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

```
[Out] 1/2*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^5/x/(-1+1/c/x)^(1/2)/
(1+1/c/x)^(1/2)-1/2*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^5/x/(-1+1/c/x)
^(1/2)/(1+1/c/x)^(1/2)-1/2*(a+b*arcsech(c*x))*(-c^4*x^4+1)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {267, 6444, 12, 1266, 862, 52, 65, 214}

$$\int \frac{x^3\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\,\mathrm{sech}^{-1}(cx)\right)}{2c^4} + \frac{b\sqrt{1-c^2x^2}\,\mathrm{arctanh}\left(\sqrt{c^2x^2+1}\right)}{2c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/2*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^5*Sqrt[-1 + 1/(c*x)]*Sqrt[1
 + 1/(c*x)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcSech[c*x]))/(2*c^4) + (b*Sqrt
```

```
[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(2*c^5*Sqrt[-1 + 1/(c*x)]*Sqrt[1
+ 1/(c*x)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 6444

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcSech[c*x], v, x] + Dist[b*(Sqrt[1 - c^2*x^2]/(c*x*S
qrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])), Int[SimplifyIntegrand[v/(x*Sqrt[1 -
c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}+\frac{\left(b\sqrt{1-c^2x^2}\right)\int -\frac{\sqrt{1-c^4x^4}}{2c^4x\sqrt{1-c^2x^2}}\,dx}{c\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}\,dx}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{4c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx,x,x^2\right)}{4c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{4c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{2c^7\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{sech}^{-1}(cx)\right)}{2c^4}+\frac{b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{2c^5\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.88

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{a\sqrt{1-c^4x^4}+\frac{b\sqrt{1-c^4x^4}}{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}+b\sqrt{1-c^4x^4}\text{sech}^{-1}(cx)-b\log(x(1-cx))+b\log\left(1-cx-\sqrt{\frac{1-cx}{1+cx}}\sqrt{1-c^4x^4}\right)}{2c^4}$$

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/2*(a*Sqrt[1 - c^4*x^4] + (b*Sqrt[1 - c^4*x^4])/(Sqrt[(1 - c*x)/(1 + c*x)
]*(1 + c*x)) + b*Sqrt[1 - c^4*x^4]*ArcSech[c*x] - b*Log[x*(1 - c*x)] + b*Lo
g[1 - c*x - Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/c^4
```

Maple [F]

$$\int \frac{x^3(a+b\,\text{arcsech}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

```
[In] int(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(135) = 270.

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.75

$$\int \frac{x^3\left(a+b\text{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \frac{2\sqrt{-c^4x^4+1}bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{-c^4x^4+1}(bc^2x^2-b)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)-(bc^2x^2-b)\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{c^2x^2-1}\right)+\cdots}{4\left(c^6x^2-c^4\right)}$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(-c^4*x^4 + 1)*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*sqrt(-c^
4*x^4 + 1)*(b*c^2*x^2 - b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*
x)) - (b*c^2*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2
- 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + (b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(
-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) - 2*sq
rt(-c^4*x^4 + 1)*(a*c^2*x^2 - a))/(c^6*x^2 - c^4)
```

Sympy [F]

$$\int \frac{x^3 (a + b\mathrm{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}}\,dx = \int \frac{x^3 (a + b\,\mathrm{asech}\,(cx))}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

```
[In] integrate(x**3*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)
[Out] Integral(x**3*(a + b*asech(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

Maxima [F]

$$\int \frac{x^3 (a + b\mathrm{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}}\,dx = \int \frac{(b\,\mathrm{arsech}\,(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*b*((c^4*x^4 - 1)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 +
1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^4) - 2*integrate(1/2*(2*c^2*x^5*log(c) +
4*c^2*x^5*log(sqrt(x)) + (4*c^2*x^5*log(sqrt(x)) + (c^2*x^2*(2*log(c) + 1)
+ 1)*x^3)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^2*x^2*e^(log(c*x +
1) + log(-c*x + 1)) + c^2*x^2*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqr
t(c^2*x^2 + 1)), x)) - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

Giac [F]

$$\int \frac{x^3 (a + b\mathrm{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}}\,dx = \int \frac{(b\,\mathrm{arsech}\,(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsech(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{sech}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^3*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^3*(a + b*acosh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.189 $\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\mathrm{Int}\left(\frac{a+b\mathrm{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\mathrm{sech}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a + b\,\text{arcsech}\,(cx)}{x\sqrt{-c^4x^4 + 1}}dx$$

```
[In] int((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x)
[Out] int((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arcsech(c*x) + a)/(c^4*x^5 - x), x)
```

Sympy [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\,\text{asech}\,(cx)}{x\sqrt{-\left(cx - 1\right)\left(cx + 1\right)\left(c^2x^2 + 1\right)}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x/(-c**4*x**4+1)**(1/2),x)
[Out] Integral((a + b*asech(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.85

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*inte
grate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(sqrt(-(c^2*x^2 +
1)*(c*x + 1)*(c*x - 1))*x), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

Mupad [N/A]

Not integrable

Time = 5.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\,\mathrm{acosh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1 - c^4\,x^4}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

3.190 $\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\text{Int}\left(\frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

```
[In] Int[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

```
[In] Integrate[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\ \text{arcsech}\left(cx\right)}{x^5\sqrt{-c^4x^4+1}}dx$$

```
[In] int((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
[Out] int((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\text{arsech}\left(cx\right)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arcsech(c*x) + a)/(c^4*x^9 - x^5), x)
```

Sympy [N/A]

Not integrable

Time = 24.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{sech}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\text{asech}\left(cx\right)}{x^5\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

```
[In] integrate((a+b*asech(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
[Out] Integral((a + b*asech(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
)), x)
```

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.77

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x^5}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2
*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x)
 - 1) + 1/(c*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(c*x - 1))*x^5), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arsech}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x^5}\,dx$$

```
[In] integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a + b\text{sech}^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\,\text{acosh}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1 - c^4\,x^4}}\,dx$$

```
[In] int((a + b*acosh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*acosh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-ty
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```